PROCEEDINGS OF THE 47th CONFERENCE OF THE INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION

Auckland
Aotearoa New Zealand
July 17-21
2024

EDITORS
Tanya Evans
Ofer Marmur
Jodie Hunter
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VOLUME 1

Plenary Lectures
National Presentation
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PME-47

RETHINKING MATHEMATICS EDUCATION TOGETHER

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PREFACE

Professor Jodie Hunter

Kia ora and welcome to PME-47, the 47th Annual Conference of the International Group for the Psychology of Mathematics Education. We are excited to be hosting the PME conference in Aotearoa New Zealand for the first time. We recognise the PME conference as one of the most significant annual international conferences in the field of mathematics education and are privileged to be hosting PME participants. The PME community includes researchers across all levels of mathematics education with diverse fields of interest and expertise, including cognitive, social, cultural, and emotional components of mathematics education. This year's conference has approximately 380 participants from 41 countries from all over the world. We are looking forward to the 318 presentations that will be made during the conference.

We hope that participants of PME appreciate and enjoy experiencing the bicultural heritage and nature of our country. Aotearoa is the traditional Māori name for New Zealand meaning the land of the long white cloud. This refers to the cloud formations that were used by early Pacific navigators to find land. Aotearoa New Zealand is a welcoming, diverse country with a history of Māori, European, Pacific Island, and Asian immigration. This rich blend of cultures, combined with geologically fascinating landscapes and unique flora and fauna, makes New Zealand an exciting country to explore. Aligned with the location of the conference, we selected the theme of the conference to be described as "Rethinking mathematics education together". This theme has been chosen to emphasize mathematics education research as a shifting field of knowledge which is developed as a collective body of research. The theme has relevance both to the local Pacific/Oceania context with collectivism being one of the key cultural values of both Māori and Pacific peoples and given the ongoing focus in Aotearoa New Zealand on the development of an equitable educational system building on our bicultural heritage. The theme also connects to the broader international context with ongoing debates related to mathematics education and research and shifts in both curriculum and pedagogical practices.

We are excited to welcome distinguished international plenary speakers and panellists to deliver plenary addresses at the conference. All of the speakers represent different research backgrounds, experiences, and contexts and we are delighted to engage with varying theoretical perspectives. The program of PME 47 will include a variety of types of sessions including Research Reports, Oral Communications, and Poster Presentations, along with Working Groups, and a Seminar. A National Presentation will provide insights into both curriculum development and research studies that have been undertaken in Aotearoa New Zealand. The four volumes of the proceedings are organized according to types of presentations. Volume 1 contains the Plenary Lectures, Plenary Panel, Working Groups, Seminar, National Presentation, Oral communications and Poster Presentations abstracts. Volumes 2, 3 and 4 contain Research Reports.

PME 47 is a joint conference being hosted from colleagues at Massey University, University of Auckland, and Auckland University of Technology. Three committees have been involved in organising the conference: The International Program Committee for PME 47, the International Committee of PME together with the PME Administrative Manager, and the Local Organizing Committee. We wish to thank all of those involved in the organisation of the conference for both their time and support. We are also appreciative of all of the PME 47 participants for travelling to our country to attend the conference and for the contributions from the wider PME community in the reviewing process.

We hope that PME 47 offers new learning opportunities for all participants and an opportunity for us to rethink mathematics education together. We also look forward to both old colleagues and new colleagues connecting during the conference through existing friendships and the development of new friendships and possibilities for future work together. As expressed in a Māori proverb: "*Nau te rourou, naku te rourou, ka ora te manuhiri" translated this means "with your food basket and with my food basket, the people will thrive". Together we can create something greater than ourselves.*

Jodie Hunter

PME 47 Conference Chair

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BUILDING AND ENGAGING COMMUNITIES: THE CASE OF THE MENTAL STARTERS ASSESSMENT PROJECT

Mellony Graven

Rhodes University

Speaking to the theme of 'Rethinking Mathematics Education together' in this plenary talk I share how a recent national intervention in South African Grade 3 classrooms (the Mental Starters Assessment Project – MSAP) emerged from the ongoing collaboration of the two South African Numeracy Chairs (myself and Prof Venkat) with multiple education communities (researchers, teachers, teacher educators, departmental personnel, and learners). In telling the story of this emergent and gradually upscaled intervention I focus on the critical role of multiple communities that participated in: conceptualising MSAP; iterative MSAP design cycles; the implementation and trialling of MSAP (local, then 3-provinces, then all nine provinces); and continuous dialogue about MSAP across multiple conference and education platforms. The Chairs were established in 2011 partly because South Africa's Early Grade Mathematics (EGM) research and development was thin on the ground. Feasibility at a national scale necessitated building and engaging multiple 'communities of practice' where participating members developed mutual understanding around key aspects of EGM enabling leading edge learning and innovation (after Wenger, 1998). Capacity for national implementation of MSAP was built over time across multiple levels (from the classroom level to the Department of Education's district, provincial, and national teams) and with research and intervention design teams and teacher educators. Developing and expanding MSAP thus created multiple opportunities for ongoing capacity-building of members in multiple overlapping communities.

INTRODUCTION AND CONTEXT

It is an honour to be speaking to the theme of 'Mathematics Education Together' especially with PME 47 being hosted in New Zealand. The work of New Zealand mathematics educators and researchers, particularly in terms of understanding issues of equity in mathematics education and advocating for greater access for marginalised learners, has inspired my work in the South African context for many years. I have chosen to share the story of the emergence and development of the Mental Starters Assessment Project (MSAP) in South Africa because it powerfully illuminates how orchestrating multi-stakeholder engagement and collaboration over time enables innovative high-leverage interventions, feasible for national implementation. The creation of these multi-stakeholder intervention-focused communities supports the maintenance of intervention fidelity and positive impact as the innovations expand and become increasingly independent of the initial design team.

The term 'mental starters' refers to the use of mental mathematics (practising number facts and mentally solving calculations) as a 'warm-up' at the start of primary

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mathematics lessons. The South African curriculum recommends the use of mental mathematics for the first ten minutes of lessons and suggests teaching several mental strategies (e.g., bridging through ten, re-ordering, etc.). The inclusion of 'assessment' in the project's name is because mental mathematics pre- and post-tests form part of the project design. The MSAP is described in more detail in the following section.

In 2011 two National Research Foundation (NRF) 'South African Numeracy Chairs' were created (myself at Rhodes University and Prof Hamsa Venkat at Wits University). This followed the establishment of four Secondary Mathematics Education Chairs in 2010. These six Chairs differed from most NRF Chairs insofar as their mandate was to intervene for positive change in mathematics teaching and learning (particularly in euphemistically called '*previously* disadvantaged' schools) alongside the mandate to conduct research and design sustainable scalable interventions. Another mandate for our Numeracy Chairs was to establish and grow the field of South African Early Grade Mathematics (EGM) research and development as, compared to secondary mathematics education, this was under-developed.

Merging research with development (R&D) enables a powerful dialectical relationship between the two. This has been elaborated elsewhere (Graven & Venkat, 2015; Graven, 2023) and is illuminated here in the specific story of the MSAP. Blending research with development through partnerships with school communities is ethically important, given that much mathematics education research reports 'the problems' without considering researchers' ethical obligation to contribute possible solutions. Negative research findings are often reported in the South African press in ways that blame teachers and their lack of mathematical competence for the 'mathematics education crisis'. (As an example, Govender's 2023 article in the Sunday Times newspaper explicitly headlined the maths crisis as due to poor teaching.) This sort of negative storying of maths teachers tends further to marginalise them from opportunities to engage with research and development that could address challenges. Participating in research could mean colluding in the production of negative narratives about themselves, so exacerbating low teacher morale. If we were to shift focus from understanding and reporting problems to feasibly intervening for change our Chairs needed to forge meaningful long-term partnerships with teachers, schools, learners, and district advisors. The long-term funding model of up to three consecutive five-year terms (following regular and rigorous evaluation of whether Chairs met key R&D outcomes) supported the development of ongoing, and continually strengthened partnerships with teachers, schools, districts, and provincial and national Department of Basic Education (DBE) personnel.

In our first five-year term our Chair teams had established various early grade mathematics professional development (PD) communities in our respective provinces with linked classroom- and learner-based interventions (e.g. lesson mental starters and number talks, mental math games in after-school maths clubs). At the start of our second term Prof Venkat and I met to consider scalable interventions to the number sense challenges we had experienced and responded to in our first term. Thus in 2016 we formed a working group comprising our Chair teams (researchers and project members) who brought experiences from highly diverse contexts in terms of wealth, rurality and educational outcomes and EGM international research experts we had collaborated with (Professors Mike Askew and Bob Wright). We further included representatives from: our professional teacher Association for Mathematics Education of South Africa (AMESA); our Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE); Non-Government Organisations and district, provincial and national DBE teams.

During a five-day think-tank, the key question we aimed to answer was: What possible high-leverage, feasible, and scalable intervention should we design to improve number sense teaching and learning in South African schools? Sub-questions guiding our design decisions were: What problem most urgently needs addressing? What perpetuates this problem? What is the curriculum space for addressing this problem? What classroom space is optimal for the intervention? What policy space might support the intervention going to scale?

THE MENTAL STARTERS ASSESSMENT PROJECT

Engaging with these questions and coordinating the broad range of perspectives of the diverse collaborators/stakeholders, led to the Mental Starters Assessment Project (MSAP). While it is beyond the scope of this paper to fully describe the MSAP and results of various iterations of piloting (see Graven & Venkat, 2021; Askew et al., 2022), here I briefly engage with how answering these questions led to the specific design of the model as follows:

For each of six mental strategies described in the curriculum, (e.g., jump strategy, rounding and adjusting etc.) Grade 3 teachers:

* Administer a 5-minute pre-test;

* Teach eight 10-minute lesson starters focussing on developing the strategy (and linked fluencies) using key representations (e.g., empty number line) to aid understanding and serve as mental image tools to assist learners;

* Administer a 5-minute post-test;

* Focus learners on their pre- to post-test learning gains (vs raw scores).

In answering the questions, we considered the key challenge that most urgently needed addressing was persistent unit counting for calculating across primary grades (well beyond appropriate number ranges) — even for calculations such as 2 + 98 or 10 + 10 + 10 (see Graven et al. (2013) and Weitz & Venkat (2013) respectively). This problem has been widely reported since Schollar’s (2008) large-scale research indicating that almost 80% of South African Grade 5 learners relied on simple unit counting to solve calculations. He further argued that learners’ weak place value understanding and inability to manipulate numbers was the biggest cause of poor math performance in schools. Our Chair research indicated that despite increased reporting on this problem, unit counting for calculation persisted in classrooms often followed by erroneous implementation of algorithms (e.g., 92-87=15) that seemingly circumvented the need for number sense (Graven et al., 2013). Like Schollar (2008), we considered that the ability to manipulate numbers through a good understanding of place value and a structural understanding of number could be the key to leveraging improved mathematical trajectories for early grade learners.

Mental mathematics strategies were considered opportune curriculum content for developing the ability to manipulate numbers. Several strategies are listed in the curriculum, though seldom taught. The recommended 10-minute mental maths warm-up to lessons (DBE, 2011) was identified as an ideal space for establishing fluencies and developing strategic manipulation of numbers for efficient calculation. Venkat and Naidoo (2012) had powerfully illustrated how this warm-up practice was randomly and incoherently used rather than being focused on establishing particular facts, patterns, or noting relationships. Grade 3, the final year of the Foundation Phase, was considered a good place to begin. Moving forward, we are working to expand to other primary grades.

Our working group member from the National Department of Education Assessment Unit, whom we had engaged with at various national platforms in our first term, indicated that there was a call in the policy space for the development of diagnostic assessments to support teaching. This followed the scrapping of Annual National Assessments written in every primary grade each year-end. Teachers refused to participate in these, objecting that they simply highlighted poor performance in most schools without giving any support for improving teaching and learning. The policy appetite for diagnostic assessment led to the inclusion of the 'diagnostic' pre-test into our model as a means of assisting teachers in establishing what learners knew. This, combined with the post-test, nudged teachers to shift their focus from curriculum coverage or raw scores to learning gains.

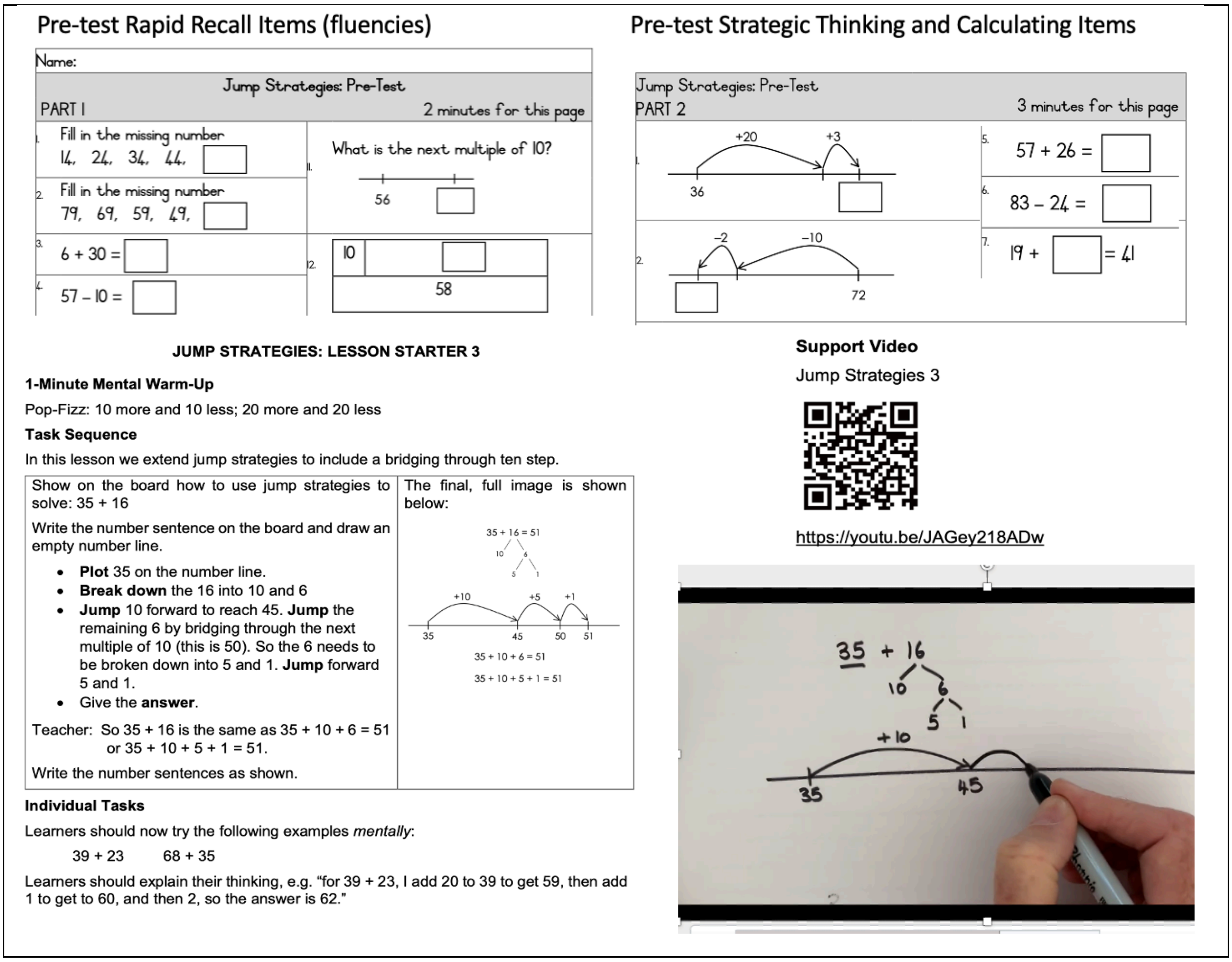
MSAP was thus designed as both a learner and a teacher intervention. The learner and teacher goals are summarised in Table 1.

Image 1 shows screenshots from the Jump Strategy materials in the Grade 3 MSAP Teacher Guide in English (Graven et al., 2020, pp. 27-28; 33-34). Screenshots include examples of: pre-test items; the scripted beginning of a mental starter (with 1-minute basic number facts warm up game); the QR code for a 70 second video illustrating the teaching script; and an image from the video at the 50th second. Post-test items closely mirrored the pre-test items. See https://www.education.gov.za/MSAP2022.aspx

Learner Goals	Teacher Goals
• Support learner progression from unit counting towards strategic thinking, number fluency and number sense. • Develop key number facts and fluencies. • Develop structural understanding of number. • Use representations as tools for thinking: empty number line, bar model, and arrays. • Focus learners on personal learning gains – Did I improve? Am I calculating more efficiently?	• Effective use of the 10-minute warm up time for teaching efficient methods of calculating • Understand the relationship between specific number facts and strategies (e.g., bonds to 10 for bridging through 10) • Use key representations as tools for effective teaching and explanation (empty number line, bar model, and arrays). • Pay attention to learner calculation methods (encourage efficient strategies) versus accuracy of answers. • Focus on learning – from Did I teach X? to Did they learn X?

Table 1: MSAP Learner and Teacher Goals

Image 1: A collation of screenshots from the materials on Jump Strategy



Moving forward, I draw on Lave and Wenger (1991) and Wenger's (1998) Communities of Practice (CoPs) perspective to frame the story of this evolving project.

FRAMING THE STORY: A COP PERSPECTIVE ON 'RETHINKING MATHEMATICS EDUCATION TOGETHER'

A CoP perspective informed the design of multiple projects and interventions in the Chair as well as the framing of this story. For Lave and Wenger (1991), learning resides in the process of co-participation and with increasing access to participation. Participating in the practices of communities is critical because learning involves becoming a fully participating member. My Chair work is guided by the notion that strengthening learning in EGM requires supporting researchers, teachers, teacher educators, departmental personnel, communities, and learners to have access to engaging with quality resources in CoP spaces constituted by members from multiple overlapping communities (Graven, 2019). These CoPs provide access to "a wide range of ongoing activity, old-timers, and other members of the community; and to information, resources, and opportunities for participation" (Lave & Wenger, 1991, p. 101). The power of the various project CoPs formed (MSAP being the one in focus here) is that they are constituted by members who belong simultaneously to "other tangential and overlapping communities of practice" (Lave & Wenger, 1991, p. 98). This creates the need for synthesising multiple perspectives and different member experiences in the development of shared understanding and repertoire necessary for productive mutual engagement around the joint project enterprise of the CoPs. Wenger (1998) notes:

> A history of mutual engagement around a joint enterprise is an ideal context for this kind of leading-edge learning, which requires a strong bond of communal competence along with a deep respect for the particularity of experience. When these conditions are in place, communities of practice are a privileged locus for the creation of knowledge. (p. 214)

In the Chair PD CoPs all members are learners bringing different kinds of experiences and expertise to engagement, with teachers' 'particular' experiences of diverse learner needs and classrooms being especially respected. Furthermore, the Chair PD and MSAP CoPs were professional communities in that they were organised professionally, meeting regularly, with a shared sense of purpose, a co-ordinated effort to improve EGM learning, and engaged in collaborative professional learning with collective control over key decisions (Secada & Adajian, 1997). The co-ordination of multiple experiences and perspectives and collective control over key decisions supported buy-in and commitment to MSAP goals from members of different professional communities. This facilitated broader buy-in to MSAP goals from the communities of stakeholders that the members represented.

BUILDING AND ENGAGING COMMUNITIES THROUGH MSAP

In telling the story of this emergent and then gradually upscaled MSAP intervention, I focus on the critical role of multiple communities that participated in: conceptualising

MSAP (discussed above); iterative MSAP design cycles; implementation and trialling of MSAP (local, then 3-provinces, then all provinces); and continuous dialogue about MSAP across multiple conference and national education platforms.

Following the 2016 working group meeting, a smaller MSAP design team was formed to prepare the assessments and teaching materials for the established model. This team was led by the two Chairs with volunteer members from the working group. The team met regularly, sharing insights from their trialling of ideas with learners in clubs and classrooms. Various modifications to original teaching materials and pre-and post-tests were implemented in this process. For example, initially we had assumed that some basic calculations (e.g., adding a single digit to a multiple of 10 (e.g., 50 + 8 = 58) would be rapidly answered by learners. We therefore had not built games and activities that practiced what we saw as relatively trivial number facts into teaching the strategies that required them. However, when trialling the mental starters with learners we found most learners used their fingers to add on a single digit to a multiple of 10. For example, in the case of 50 + 8, holding out 8 fingers and counting-up in ones from 50 while tapping or closing the 8 fingers. We thus revised the materials to ensure that we included the practice of *every* type of basic fact that learners would need to know for use in the strategy. Indeed, a strategy like jump strategy with bridging through ten only becomes efficient if learners can fluently add a single digit to a multiple of ten and rapidly recall other basic facts (Askew, 2009).

Once materials were considered sufficiently developed for use by teachers in our partner schools, an initial pilot was run (in 2017) involving learners and teachers in some of these schools. Positive improvements of 15-17 percentage points across schools (see Graven and Venkat, 2021) led to an expansion of trialling beyond our partner schools across three provinces in 2019. Here, we supported district subject and phase advisors appointed by the DBE (some of whom we had already worked with) to train teachers to teach one of the mental starter strategies and to implement the pre-and post-tests. An average improvement of 17% points in learners' post-test scores indicated feasibility for MSAP to be scaled-up and rolled out by district personnel with teachers. This led to an all-province familiarisation trial that included four of the six strategies in which we supported provincial coordinators in online training. They then worked with a sample of district personnel in their provinces who, in turn, worked with a sample of Grade 3 teachers and their classes. Analysis of these results indicated statistically significant improvement across provinces and strategies (Askew et al., 2022), indicating feasibility for the national roll-out across schools (currently underway). This all-province trial provided opportunity for familiarisation of department personnel across provinces working at various levels to gain experience in implementing MSAP. It also provided opportunities for feedback on the rollout. Generally positive feedback indicated appetite among teachers, district advisors, and provincial personnel for using the materials.

It is important to note here that at each iteration of scaling up, the core MSAP design team became a step further removed from teachers. In the first pilot the Chairs directly supported teachers in the implementation; in the second the team supported district advisors who supported teachers; in the third the team supported provincial coordinators who supported district advisors who supported teachers. While I tend to be sceptical of cascade models for teacher professional development, in the case of the MSAP the slow and steady building up of buy-in across stakeholders (Venkat & Graven, 2022) with the quality and accessibility of the materials (supported by QR-coded videos) enabled maintenance of teaching fidelity, so enabling the positive results across the third trial.

Across each trial the MSAP team was able to meet regularly (either face-to-face or online) with those involved to gather feedback on their experiences (teachers, district advisors, provincial coordinators). Our teams collaborated with DBE personnel in presenting a plenary panel on our work at our SAARMSTE conference in 2017 and again in a SAARMSTE symposium in 2022. We also jointly presented this work at our annual 'Mathematics Education Chair Community of Practice' forums hosted by the National Research Foundation (NRF) and the DBE. Through collaboration in presentations, we further strengthened our own learning and joint commitment to our goals, invited input from a wide range of stakeholders attending these forums, and held ourselves accountable to these communities in terms of reporting whether we were achieving our mandates. A wide range of other MSAP-linked presentations across local and international conference platforms (e.g., PME, ICME, MERGA, CERME) have been shared by various team members.

Ongoing dialogue and reporting about MSAP across national platforms led to interest from pre-service teacher educators as well as the Department of Higher Education and Training (DHET) in using and adapting the MSAP materials for EGM pre-service teacher education. Research shows that primary teachers' mathematics content knowledge is generally well below what is needed (Venkat & Spaull, 2015), with more recent research turning the spotlight on pre-service teacher education (PTE). Bowie et al.'s (2019) research across three higher education institutions showed that there was little difference between the mathematical content knowledge of the first- and fourth-year primary pre-service teachers (PSTs). The study highlighted that the challenges of poor mathematical knowledge of PSTs were not being effectively addressed in PTE. In 2021 we were approached by the DHET to consider a parallel project for teacher educators. While several teacher educator colleagues in our institutions were using aspects of the MSAP in their lectures, the formation of a regularly-meeting CoP of lecturers across multiple universities and private institutions, enabled the joint and national exploration of how MSAP may support PTE.

The Mental Mathematics-Work Integrated Learning Programme (MM-WIL) was thus formed in 2022, with participants from nine institutions meeting up with the Numeracy Chair MSAP teams in annual 3-day working sessions. A range of innovative approaches to using the materials have been shared and researched leading to the team presenting a SAARMSTE symposium in 2023 on: "Incorporating attention to Mental Mathematics in pre-service teacher education". From this symposium a Special Issue

(SI) proposal for the South African Journal of Childhood Education was developed. Contributions from the MM-WIL team and other MSAP researchers are currently under review. Snapshots of these papers were also presented in a 2024 SAARMSTE symposium on: "Mental Mathematics for Number Sense in the Early Grades".

Figure 1 captures how MSAP emerges from the collaboration of members of multi-stakeholder communities in Chair-led professional development CoPs and linked classroom/club focused interventions — extending in 2022 to a network of EGM pre-service teacher educators across higher education institutions, while constantly engaging in dialogue with national and international professional, research and education communities.

Figure 1: MSAP emerges from CoPs, builds new CoPs and engages CoPs

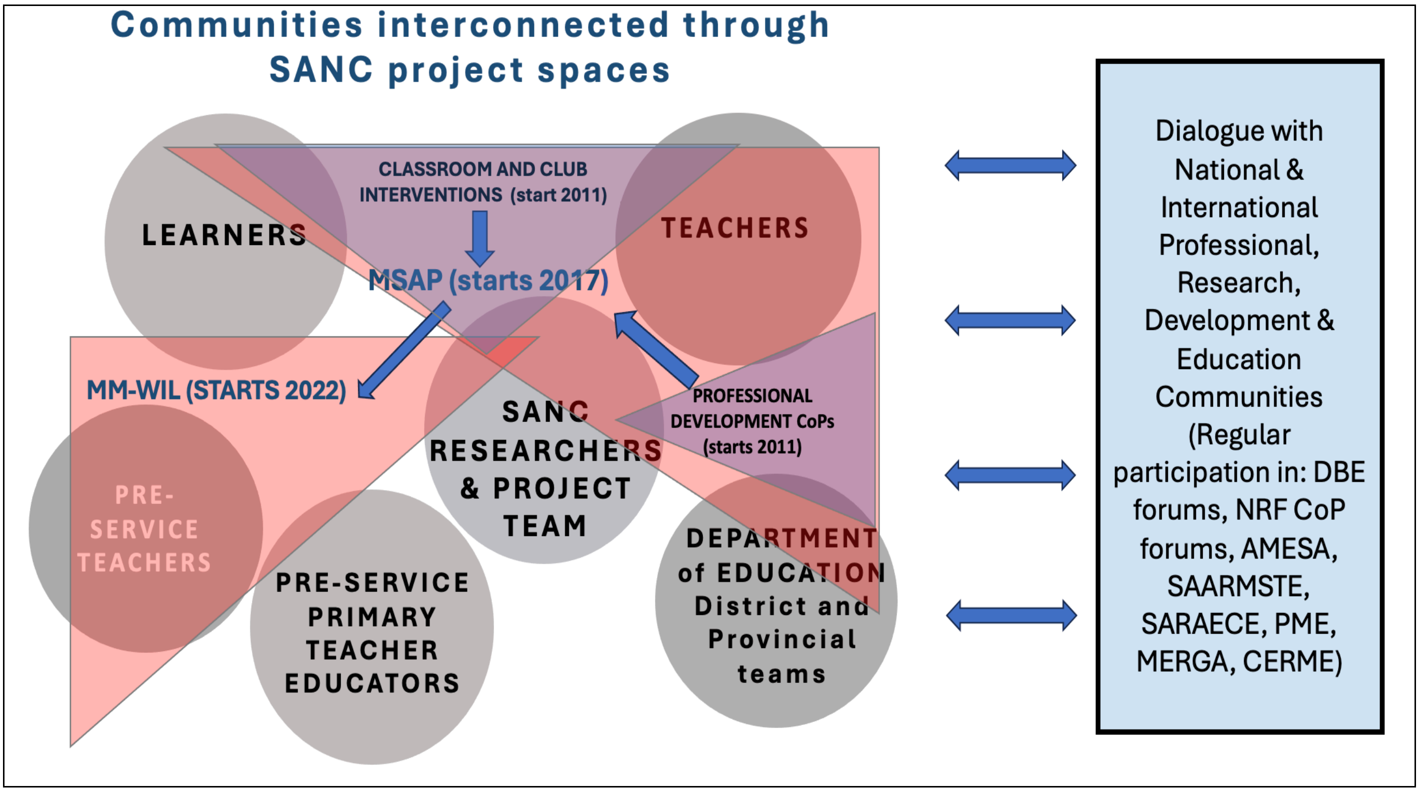
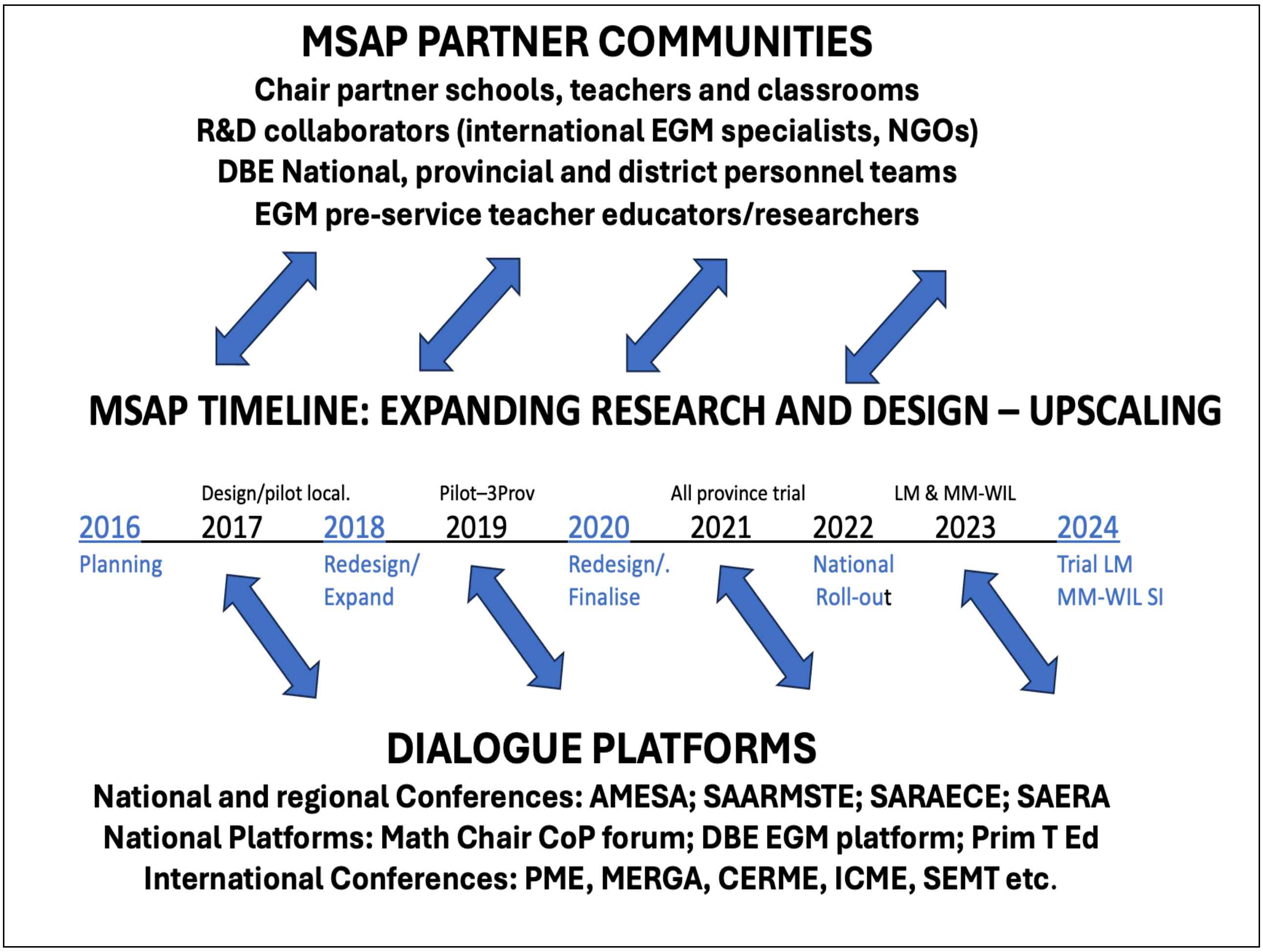


Figure 2 captures the way in which MSAP's ongoing dialogue with partner communities and members of multiple communities coming together in various national platforms and national and international conferences informed and were informed by the evolution of MSAP. Ongoing reflection on materials and their translations (supported by post-graduate students' research) continues to inform possible future revisions.

In 2023 in consultation with the DBE it was decided that our MSAP team would prepare Learner Materials (LMs) for supporting in-class and take-home work. The latter will provide learners access to practising the strategies and fluencies, and watching the QR-coded materials, in their homes. Research proposals are being developed to explore teacher and learner experiences of these 'take-home' materials.

This research will likely then inform their re-design. Thus, another MSAP learning cycle unfolds. Similarly, the planned expansion into other primary grades will result in further research and development learning cycles.

Figure 2: MSAP timeline - ongoing dialogue with partners and platforms



CONCLUDING REMARKS

Speaking to the theme of 'Mathematics Education Together' I have shared how long-term collaboration between two Research and Development (R&D) Chair teams and multi-stakeholder communities enabled the establishment of various professional CoPs (with multi-stakeholder membership). These professional CoPs enabled the emergence and gradual upscaling of MSAP that aimed at strengthening the teaching and learning of number sense through engaging with mental mathematics strategies at the start of lessons. I highlighted how ongoing collaboration and engagement with, and accountability to, members of multiple stakeholder communities support both the quality of the intervention and the feasibility for up-scaling.

I thank Professor Venkat, our MSAP team, DBE partners and teachers, and members of the many communities we have engaged with for the powerful learning journey. Through this journey we have experienced the pragmatic and motivational power of researching and developing ‘mathematics education together’. It has been an enormous privilege to have the resources to be able to navigate this journey with so many diverse collaborators. In this respect it is important to acknowledge the innovative funding model of the NRF Mathematics Education Chairs that enabled this flexible, long-term, research with development journey. If we are to ‘rethink mathematics education together’ we need to increasingly lobby for funding models that enable this theme to shape how we work as mathematics education researchers and educators.

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THE SECRET LIFE OF MATHEMATICAL PROBLEMS THROUGH THE LENS OF RESEARCH-PRACTICE PARTNERSHIPS

Boris Koichu

Weizmann Institute of Science

In this plenary talk, I address two longstanding challenges for mathematical problem solving as a teaching and learning practice for all: (1) the many potential (and divergent) roles of a teacher in problem-solving instruction; (2) the vast diversity in intentions, goals, and meanings of tasks intended to be problems in different classrooms. In relation to the first challenge, I develop a metaphor of a problem as a living (discursive) creature whose "live" depends on who and how attends to it. This metaphor posits teachers as "revivers" of problems in their classes. In relation to the second challenge, I show how mathematical tasks are transformed in a chain of intended, planned, enacted, and experienced activity, and argue for research-practice partnerships as a useful perspective for making problem-solving instruction feasible.

INTRODUCTION

Aligned with the PME47 theme "Rethinking mathematics education together", this plenary talk is devoted to rethinking the nature, roles, feasibility, and future of problem solving as a teaching and learning practice for all. This practice is endorsed by mathematics educators, and simultaneously it is known to be particularly difficult to implement in a regular classroom (Lester & Cai, 2016; Liljedahl & Cai, 2021; Schoenfeld, 2013, 2024). The title of the talk alludes to mathematical problems as living creatures having their own "secret lives" (by analogy with lives of magic creatures from fairy tales), and then to research-practice partnerships as a theoretical-organizational lens that, as I will argue, is crucial for keeping these creatures alive.

Less metaphorically, I argue for a way of re-examining paradigmatic challenges inherent in a longstanding idea to teach and learn mathematics as a nexus of (broadly understood) problem-solving experiences (Cuoco et al., 1996; Liljedahl et al., 2016). I focus on two interrelated challenges that Lester (2013) posits among the main ones: (1) the diversity of roles of a teacher in problem-solving instruction; (2) the systemic lack of knowledge about what is going on in real classrooms with mathematical tasks intended by their creators or proponents to be experienced as "problems".

Having written the above two paragraphs, I vividly imagined a sympathetic but critical reader who might exclaim: "Wait! Hasn't problem solving been overtly, and even overwhelmingly, represented in mathematics education research in general and at PMEs in particular? There are literally hundreds of studies devoted to cognitive, affective, and socio-cultural mechanisms of problems solving, as well as to learning environments, classroom interventions and teacher professional learning programs designed to support problem-solving instruction."

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“Indeed, – I would agree, – there are hundreds of studies, though problem solving as a research theme has had its ups and downs (Koichu, 2014). In addition, problem solving was a topic of 10 PME plenary talks since 1983. Still, problem solving as an activity based on the experience of *productive struggle* is on the fringe of mathematics education in many countries. To recall, a *struggle* in the context of problem solving means that students must expend effort to figure out something that is not immediately apparent (Hiebert & Grouws, 2007), and *productiveness* is associated with a positive attitude towards challenges and setbacks (Warshauer, 2015).”

“And – the critical reader continues, – haven’t reasons for why problem solving is both an endorsed and elusive activity been analysed from the variety of perspectives?”

“Indeed, – I would agree again, – as early as in 1985, Jeremy Kilpatrick distinguished between four perspectives on problems: psychological (problem is “…a situation in which a goal is to be attained and a direct route to the goal is blocked” (Kilpatrick, 1985, p. 2), mathematical (problem as construction), pedagogical (problem as a vehicle) and socio-anthropological (problems are given and received in transaction). He then aptly noted that ‘[r]esearchers in mathematics education have long accepted the truth that a problem for you today may not be one for me today or for you tomorrow’ (Kilpatrick, 1985, p. 3). Eighteen years later, Frank Lester argued that despite the diversity of the existing perspectives, they are generally "...unhelpful for thinking about how to teach students to solve problems or to identify the proficiencies needed to teach for or via problem solving" (Lester, 2013, p. 248). More recently, a discursively oriented perspective was added to the spectrum of perspectives on problems: a problem is a discursive object that triggers a not-straightforward socio-culturally shaped process of producing a public narrative that can be endorsed as one that achieves predefined goals (Koichu, 2019). The development of this perspective was motivated by an idea that teaching problem solving can become more feasible when considered as a discursive activity (cf. Sfard, 2008) within the reach of a prepared teacher. Namely, a teacher can bring to her classroom mathematical tasks that might evoke exploratory discourse in students while being ultimately feasible for them, and then discursively navigate the struggle while keeping her interventions minimalistic to preserve students’ autonomy (Brousseau & Gibel, 2005)”.

“So – the acute reader gets to the point, – what is it good for to offer a new metaphor on mathematical problems after all these developments? Also, how can a lens of research-practice partnerships add to what we already know about, or have tried to do with, problem-based mathematics instruction?”

To this, my brief answer would be, “as a grateful invitee to give one of the plenary talks at PME47, I feel privileged to rethink things while standing on the shoulders of giants.” A longer, and, hopefully, more convincing answer is unfolded henceforth.

LIVES OF MAGICAL CREATURES AND MATHEMATICAL TASKS

From a discursively oriented perspective, a mathematical task is, simply put, a discursive construct having special characteristics. To this end, a task published in a mathematics book is not paradigmatically different from a magical creature that appears on the pages of a fable. They both are born as texts, and they both can penetrate the experiential reality only via the mediation of a real person, for example, a reader, a teacher, a student. As a proverb says, “words have winds, stories have power.” In addition, we know that some stories are more powerful than others. But what about the opposite direction of influence?

The following idea is developed in many fables: the life of a magical creature introduced in a book depends on how readers perceive the creature. In particular, the more people read the book, the more “alive” the creature becomes, and if the book is neglected or forgotten, or if belief in the creature declines, its life is jeopardized or comes to its end. A story of this type appears in the play *Peter Pan* by J.M. Barrie, where survival of the fairy Tinker Bell depends on the belief of real-world children-spectators in her existence. Another example comes from the novel *Inkheart* by Cornelia Funke where a protagonist Mortimer has the special gift to bring things and creatures out of books – actually, to revive them! – by reading aloud.

I will now play with the idea of reviving mathematical tasks as discursive constructs. Let us consider the following proposition: the more people attend to a task, the more “alive” the task is. For example, a Google search (conducted on April 2, 2024) showed that the equation $x^2 - 5x + 6 = 0$ appears in Internet documents more than 80,000 times, and $x^2 - 29x + 210 = 0$ only once. Thus, the first equation is very much “alive”, and the second one – is very much “dead” or at least “sleeping”. Indeed, there are many reasons for a textbook writer or a teacher to use the former rather than the latter equation. For instance, the roots of $x^2 - 5x + 6 = 0$ can be found either by calculation or by guessing, it has a relatively transparent factorization procedure, and more. (My experience with $x^2 - 29x + 210 = 0$ is outlined in the next section).

Likewise, the cubic equation $x^3 = 15x + 4$ (appears on Google for about 16000 times) is more “alive” than the similar-looking equation $x^3 = 17x + 4$ (appears 8 times). Note that the “technical niceness” of both equations is comparable: the first one has a root 4, and the second one has a root -4; arguably, both roots can be found by guessing. An explanation of the difference comes from the history of mathematics. To recall, the equation $x^3 = 15x + 4$ appeared in Raphael Bombelli’s book *L’Algebra* published in 1572. By comparing different solution methods for this equation, Bombelli discovered that roots of negative numbers that emerge from the use of Cardano’s formula were not just “casus irreducibilis” (“the irreducible case”) but meaningful mathematical entities. This discovery was an important step in a long process of making sense of complex and negative numbers (Merino, 2006).

Let us now turn to questions of *how*, *when*, *by whom* and *why* mathematical tasks are attended to, or, in the other words, to the question “what can the life of a mathematical task look like?” In the next two sections, I discuss these questions by considering a historic example, and then a school-level task, which is particularly close to my heart.

IS FERMAT’S LAST THEOREM ALIVE?

The equation $x^n + y^n = z^n$ is found in a Google search more than 300,000 times, because of its association with Fermat’s Last Theorem. The equation is so famous that it appears on postal stamps (Figure 1). To recall, the formulation of the Theorem in modern terms is as follows: “No three positive integers x, y, and z satisfy the equation $x^n + y^n = z^n$ for any integer value of n greater than 2.”

Figure 1: An image of a postal stamp devoted to Fermat’s Last Theorem

Historians tell us that Pier Fermat, the father of the Theorem, formulated it as a proposition on the margins of his copy of *Arithmetic* by Diophantus. He accompanied the formulation by the comment: “I have a truly marvellous demonstration for this proposition which this margin is too narrow to contain” (quoted from Edwards, 1996, p. 2, in translation from Latin). Accordingly, one can guess that for Fermat it was a not-too-difficult-to-solve *problem*, in Kilpatrick’s (1985) meaning. Given the complexity of the verified proof known nowadays, many believe that Fermat’s mysterious demonstration contained an unnoticed mistake. During the next 250 years, the proposition lived on as a problem to struggle with for thousands of amateur and professional mathematicians, until Andrew Wiles completed the proof in 1993.

I am not going to delve here in the story of Wiles’ struggle (for this, see Edwards, 1996, among other sources) but would like to share an anecdote. When I was a student at the Faculty of Mathematics of the Ivan Franko Lviv National University in the late 1980s, one of my professors, Igor Guran, had a special assignment from the dean. Every year tens of amateur mathematicians sent to the university their “proofs” of Fermat’s Last Theorem, and Guran was in charge of reading the “proofs”, finding mistakes, and tactfully replying to the authors.

When Wiles’ proof had been validated by the mathematics community, the nature of the interest in the Theorem changed. Until today, mathematicians are looking for simpler proofs, as well as for proofs of generalized propositions (e.g., what about $x^n + y^k = z^m$?). Back to the proposed metaphor, all of the above are signals that Fermat’s Last Theorem is “alive.” To stretch the metaphor, one can argue that, as a living creature, the Theorem has parents, birth, periods of development and stagnation, and also offspring. Furthermore, during its long life, the Theorem was attended to by different people in different ways: as a problem to struggle with, as a vehicle motivating the study of advanced branches of mathematics, as a source of inspiration for opening new directions of research, as a cultural artifact for enriching general knowledge, and more. The point is that Fermat's Last Theorem has many more "modes of living" than those offered by the broadly accepted distinction "exercise or problem" (Pólya, 1945/1973; Schoenfeld, 1985), or even by a more recent distinction "exercise, problem or mystery" (Schoenfeld, 2024).

THE SECRET LIFE OF THE MISCOPYING TASK

About 10 years ago, I was routinely searching mathematical books in my office for a nice task to use in a course on teaching and learning algebra for secondary school mathematics teachers. The following task – I will refer to it as the Miscopying Task – caught my attention.

> Two mathematics students attempted to solve a quadratic equation of the form, $x^2 + bx + c = 0$. Although both students did the work correctly, the first miscopied the middle term and obtained the solution $\{-6, 1\}$. The second miscopied the constant term and obtained the solution $\{2, 3\}$. What is the correct solution?

Figure 2: Miscopying Task (Lehoczky & Rusczyk, 2004, p. 87)

The task was presented by the authors of the book as a preparatory task for the US mathematics competitions. (Later I found that a slight variation of this task appears on several websites devoted to school mathematics and pops up in Google search 10 times.) The task was somewhat refreshing for me even though I hardly struggled when solving it. Indeed, the givens readily reveal which equation each student-character solved: the first student solved the equation $(x + 6)(x - 1) = x^2 + 5x - 6 = 0$, and the second one solved the equation $(x - 2)(x - 3) = x^2 - 5x + 6 = 0$. Then re-reading the information about which coefficients were miscopied quickly leads the solver to the "correct equation": $x^2 - 5x - 6 = 0$ having the solutions $\{-1, 6\}$, and we're done. Or are we?

Personal struggle around the Miscopying Task

The result of the Miscopying Task deeply surprised me. I remembered that when in school I noticed that changing the sign of the middle term of a quadratic equation having integer roots is never a "troublemaker" (i.e., the change preserves the property of the equation to have integer roots) but changing the sign of the constant term usually is. This observation shows up, for example, in the equations $x^2 - 3x + 2 = 0$, $x^2 + 3x + 2 = 0$ and $x^2 - 3x - 2 = 0$ that have solution sets $\{1, 2\}$, $\{-1, -2\}$, and $\left\{\frac{3 \pm \sqrt{17}}{2}\right\}$, respectively.

The Miscopying Task highlighted for me a pair of equations $x^2 - 5x + 6 = 0$ and $x^2 - 5x - 6 = 0$ both having integer roots despite the opposite signs of the constant terms. I wondered, are there additional pairs of equations $x^2 + bx + c = 0$ and $x^2 + bx - c = 0$ that both have integer roots? Attacking this question by trial-and-error was unsuccessful for me. I realized that the key part of a systematic search should be tied to the discriminant $b^2 - 4c$ as the "troublemaking" part of the formula $x_{1,2} = \frac{-b \pm \sqrt{b^2-4c}}{2}$ for solving monic quadratics. Accordingly, I formalized the above question as the Double Perfect Square Task (Figure 3).

Find all (or at least some) of the pairs of non-zero integers $\{b,\ c\}$ so that $b^2 - 4c$ and $b^2 + 4c$ are both perfect squares. In other words, find all (or at least some) of integer quaternary $\{b,\ c, p, s\}$ so that $\begin{cases} b^2 - 4 \cdot c = p^2 \\ b^2 + 4 \cdot c = s^2 \end{cases}$.

Figure 3: Double Perfect Squares Task (DPST)

For a while, I attempted to think of this task by recalling facts on properties of roots of polynomial functions that I had studied many years ago (e.g., the Rational Zero Theorem; the Vieta Theorem). This direction appeared to be unproductive. Then I asked myself: what would Pólya advise me to do? With this question in mind, I came back to explore the only case known to me and noticed the following.

Equation $x^2 - 5x + 6 = 0$ has the discriminant $25 - 24 = 1$, and equation $x^2 - 5x - 6 = 0$ – the discriminant $25 + 24 = 49 = 7^2$. Looking for some time at the numbers that showed up, $7, 24, 25,$ I noticed that they form a Pythagorean triple. Indeed, $7^2 + 24^2 = 49 + 576 = 625 = 25^2$. It was a surprise, and it looked promising because Pythagorean triples have many beautiful properties that might support further thinking. After many attempts, I decided to explore more deeply a particular case $b^2 - 4 \cdot c = 1$. The system of equations from the DPST became $\begin{cases} b^2 - 4 \cdot c = 1 \\ b^2 + 4 \cdot c = s^2 \end{cases}$. I transformed it into the system $\begin{cases} b^2 = \frac{1+s^2}{2} \\ c = \frac{s^2-1}{8} \end{cases}$, in which the second equation looked promising: it has an integer on its left side and division by 8 on its right side. This structure led me to a conclusion that s should be an odd number. So, let $s = 2k + 1$. With this, I came back to the first equation of the system.

$$b^2 = \frac{1 + s^2}{2} = \frac{1 + (2k+1)^2}{2} = 2k^2 + 2k + 1 = k^2 + (k+1)^2$$

At this point I experienced an *aha-moment*: I recognized that the structure $b^2 = k^2 + (k+1)^2$ implies that b is a member of special Pythagorean triples in which two smaller members (legs) are consecutive numbers. Such triples are known as *twin leg-leg triples* (e.g., https://mathworld.wolfram.com/TwinPythagoreanTriple.html). The most known triple of this type is 3, 4, 5. From here I could work backwards:

If $k = 3$ then $b = \pm 5$, $s = 2 \cdot 3 + 1 = 7$ and $c = \frac{7^2-1}{8} = 6$. That is, $\{b, c\} = \{\pm 5, 6\}$, which gives us equations $x^2 \pm 5x + 6 = 0$ and $x^2 \pm 5x - 6 = 0$. Apparently, I had merely rediscovered a pair of equations that I had already known, but I felt that I'd made progress, because I now had a method to look for additional pairs of equations based on additional twin leg-leg triples. The next one is 20, 21, 29. The reader is invited to check that this triple leads to the following pair of quadratics:

$$x^2 - 29x + 210 = 0\ (\{14, 15\}),\text{ and } x^2 - 29x - 210 = 0\ (\{-6, 35\}).$$

And the next twin leg-leg triple is 119, 120, 169, and the next one is 696, 697, 985, and so on. These numbers explain why I could not find such equations by trial-and-error!

Intermediate conclusions

What am I trying to say? To me, the above story substantiates two observations. ***First***, the Miscopying Task became a trigger for posing a question that matured into a genuine problem for me. In terms of Kilpatrick (1985), the follow-up question became a hurdle I chose to struggle with. On the way, I contributed to "reviving" the Miscopying Task, first for myself. ***Second***, I deem the above reconstruction of the solution process representative for many additional cases of productive struggle including aha-moments. In Koichu (2018), I attempted to make sense of such struggles by means of *Shift & Choices Model* (SCM) (Figure 4).

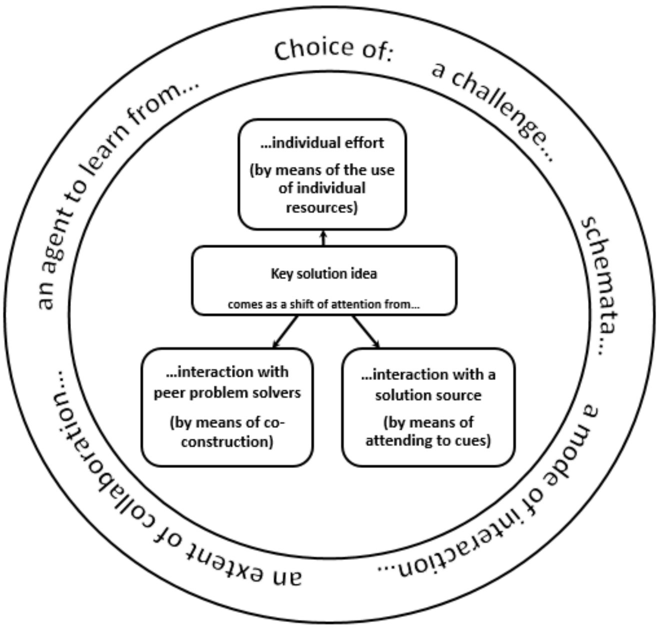

Figure 4: Shifts & Choices Model (from Koichu, 2018)

The model, inspired by John Mason' theory of shifts of attention (Mason, 2008) relies of the premise that a key solution idea to a problem (in my case, an observation about twin leg-leg Pythagorean triples) is constructed in a pathway of shifts of attention stipulated by the solver's individual resources and/or interaction with peers and/or with sources of knowledge about the solution. In principle, the solvers' shifts of attention are unpredictable but patterned, depending on personal, environmental, and socio-cultural factors. This stance has a practical corollary: the successful solution of a problem can never be guaranteed, but it can be facilitated by empowering the solver to navigate among various informed choices, including the choice of a challenge to cope with, of problem-solving schemata, and more (see Figure 4). I will connect solvers' choices with choices of additional actors in the Reflections sections.

Striving to share and its mathematical consequences

If I were a mathematician, I would probably keep thinking about the general solution to the Double Perfect Square Task (DPST). But I am not. Being a mathematics teacher in nature, I desperately wanted to share my aha-experience with somebody. During 2020-2023, I used the above story on several occasions. Two of them resulted in discovering a full solution of the DPST, and others – in further "reviving" the task.

Mathematics first. In one of my meetings with mathematics teachers in 2021, I finished the above story by saying: "No wonder I could not find additional pairs of equations by trial-and-error. Would YOU like to try?" To my delight, Anatoly Polonsky, one of the teachers in the audience, accepted the challenge and shortly presented two new pairs:

$$x^2 + 10x + 24 = 0\ (\{-6; -4\}) \quad \text{and} \quad x^2 + 10x - 24 = 0\ (\{-12; 2\});$$

$$x^2 + 13x + 30 = 0\ (\{-10; -3\}) \quad \text{and} \quad x^2 + 13x - 30 = 0\ (\{-15; 2\})$$

"How?" - I asked. "By trial-and-error," – answered Anatoly with a smile. Examples found by Anatoly served for me as a timely reminder that the solution, which I was so proud of, in fact resolved only a particular case (i.e., $b^2 - 4 \cdot c = 1$.)

In February 2023, I used the story, with an addition from Anatoly, at the conference devoted to the 80th birthday of Avi Berman, my academic father and lifelong mentor. Uzi Vishne, a mathematician from Bar Ilan University, was in the audience. Shortly after finishing my talk, I found an email from Uzi. It began as follows: "I am sitting now at Avi Berman's 80th birthday conference, where I enjoyed your talk earlier. I don't know how much thought you actually spent on the riddle concerning the mistaken coefficients. Here is something you might find useful." This polite introduction was followed by a brief and elegant general solution that Uzi discovered in just a few minutes! The solution – I would be glad to share it upon request – was based on a parametrization for the Pythagorean triples that I overlooked. To my knowledge, Uzi is the only person (besides me) who engaged with the DPST as a problem. Others have usually perceived it as an artifact from someone else's problem solving.

Striving to share and consequences for task designers

At some point, I shared my encounter with the Miscopying Task with my dear colleague Rina Zazkis. Rina appreciated the story and gave me valuable advice on how to communicate it. However, I felt that the story did not kindle the fire I was hoping it would. The same tended to happen when Rina shared with me her mathematical encounters: I greatly appreciated them but usually remained a supportive observer rather than an engaged co-solver. We studied together the pedagogical difficulty of transferring one's joy of problem solving to another person. Eventually, we wrote a book (Koichu & Zazkis, 2021) based on our dialogues around mathematical tasks we designed and loved, and our research-accompanied attempts to use these tasks with our students; part of the story of the Miscopying Task appears there among the others.

I also shared the story with the R&D team of the project *Raising the Bar in Mathematics Classroom* (RBMC) that I have had the pleasure to lead along with my dear colleague Jason Cooper. The RBMC explores ways to encourage teachers to engage their students in increasingly demanding problem-solving activity at an increasing frequency (hence the "raising the bar" metaphor.) In this project, we – a team consisting of mathematics education researchers, mathematics educators and teachers – create mathematical tasks intended to be problems for middle-school students (see Koichu et al., 2021 and Cooper et al., 2023, for details and examples) and lead professional learning courses of various formats for in-service teachers. This is in the hope that some of the RBMC tasks will reach school students. My colleagues were glad for my aha-moment with the DPST but got excited not exactly about this aspect of the task, but rather over the original Miscopying Task. They offered several modifications, and one of them, by Jason, entered the RBMC collection (see Figure 5).

> After teaching how to solve quadratic equations, Ms. T gave her class a surprise test – to solve an equation of the form $x^2 + bx + c = 0$.
> Yasmin found two solutions: $2, 3$. Reef found two solutions: $1, -6$. Don found two solutions: $3, -6$.
> As it happens, all the answers were wrong! However, two of the students made a small copying error, and received 9 out of 10 points for their correct solution of the miscopied exercise. The third student solved incorrectly and received no points. Which of the students solved incorrectly? Can you say what the given exercise was?

Figure 5: Modified Miscopying Task (MMT)

"Your DPST is not appropriate for a 45-minute lesson, - argued Jason, - and the Miscopying Task is a bit too straightforward. My version requires from the solver to add personal interpretations of the givens – it is not given which coefficients are miscopied, they need to decide what constitutes a *small* copying error, and the problem has more than one correct answer, which conveys an important message."

Needless to say, I liked the Miscopying Task I had played with for so long more than Jason's version, and Jason had equally strong sentiment for the MMT. As a result, each of us was using his own version in the RBMC courses. On one occasion, Jason and I presented both versions as a trigger for teachers' problem posing. The teachers appreciated our versions but became truly enthusiastic about their own questions inspired by the miscopying situation. It was amazing to see how far the initial tasks can be taken! (Koichu & Cooper, 2024).

Actually, the same phenomenon occurred with most of the RBMC tasks. In this project, an idea for a new task was usually offered by a member of the team who became the lead-designer of the task. As a rule, the initial idea comes from one's personal struggle.

The rest of the team provided feedback and offered amendments (Cooper et al., 2023). Surprisingly (or perhaps not), when it came to choosing tasks for the RBMC courses for teachers, members of the team tended to use tasks for which they had assumed the role of a lead-designer, and only rarely chose tasks for which they had taken a peripheral role in their design. For me, this is a nice elaboration of the Kilpatrick's (1985) maxim quoted in the introduction: for an individual in a position to choose, a task can become a problem after establishing a personal relationship with the task.

The Miscopying Task as enacted by teachers

Out of about 500 teachers who became familiar with the RBMC collection of tasks, many chose to enact the MMT in their classes, and 45 of them filled out a reflective report following their enactment. From these reports we learned that only three teachers introduced the task with their own modifications. However, the teachers nevertheless "revived" the task in a variety of ways and encountered various surprises. Table 1 contains examples of teachers' responses to the question "was there something [in the enactment] that surprised you?" The responses are (tentatively) grouped in relation to three elements of the teaching triad (Jaworski, 1994): *management of learning* (ML); *sensitivity to students* (SS) and *mathematical challenge* (MC).

Categorizes of the surprises	Teachers' characteristic assertions
ML	"I was surprised that everybody solved the problem, in this or in that way, during the time I allocated for it."
	"I was surprised to discover that some of the students did not understand solving equations in general and quadratic equations in particular. I had to stop and explain this."
SS	"I was surprised that some of the students who, I thought, possessed instrumental understanding, acted in ways that suggested relational understanding. I think that this is because they worked in small group and did not hesitate to make mistakes."
	"I was glad that eventually they succeeded but was surprised how much they needed to think and how much time the task took."
MC	"The students succeeded to solve the question right away, so I asked them to think about additional solutions."
	"Most of the students found three equations but did not understand who had made a mistake. Then I asked the students to write the equations side by side, and everything became clear to them."

Table 1: Teachers' surprises from the enactment of the Modified Miscopying Task

To me, the diversity of the teacher experiences and surprises when enacting the MMT is evidence for the secret life of the task to partially reveal itself.

The Miscopying Task as experienced by students

Let us now zoom in to students' experiences with the MTT. Orna (a pseudonym), a secondary school mathematics teacher having 35 years of experience, chose to enact the task following a meeting led by Menucha Farber, a member of the RBMC team and a Ph.D. student in my research group. Orna enacted the MMT in her 8th grade class with 29 students. The lesson (observed by Menucha) included a brief introduction of the task, students' independent work (Orna allowed the students to choose between working alone or in pairs) and a classroom discussion, in which the students presented their solutions. Some of the students' solutions were surprising for Orna. Eventually, she reflected on the task as more challenging that she had thought before the lesson, and on the entire lesson – as a positive experience for her and her students.

To reveal students' experiences with the RBMC tasks beyond observations and teacher reflections, we designed a student questionnaire to be filled out immediately after a lesson in which a particular task has been enacted. The questionnaire (Koichu, Badarneh & Farber, 2023) has two parts that correspond to the following two queries. Part 1: what did the students struggle with (if at all), and how did they overcome the struggle (if at all)? Part 2: was the struggle productive, and if yes, in what sense?

Following Warshauer (2015), we considered four kinds of struggle: (1) struggle to get started (e.g., confusion about what the task requires one to find), (2) struggle to carry out a process (e.g., encountering an impasse), (3) struggle related to uncertainty in explaining and sense-making (e.g., difficulty in communicating the solution), (4) expressing misconceptions and errors (e.g., difficulty in finding mistakes in the solution). Speaking operationally, we considered *struggle* as a self-reported difficulty that required from the solver effort or help to overcome.

Part 1 consists of 10 Likert-scale items where 2-3 items correspond to each kind of struggle. We interpret low levels of struggle (i.e., 1-2 out of 5 in the Likert scale) in relation to *all* 10 items as evidence that the task was not sufficiently challenging to qualify as a problem for the solver. The intermediate or high level of struggle (3-5) in relation to *at least one* item is interpreted as evidence for the task qualifying as a problem for the solver.

In these terms, 23 students struggled with the MMT as a problem, and six students did not. Four students had difficulty only with understanding the task formulation. The rest experienced struggle of more than one type. For example, 17 students had difficulty with the task formulation; 10 students encountered an impasse, and 10 students struggled to understand whether their solutions were correct. Interestingly, the students reported that they overcame most of their difficulties by themselves or in interactions with peers; the teacher's help was mentioned occasionally. These findings suggest that the students had highly diverse experiences with the same task in the same lesson.

Now about productiveness of the struggle. According to Warshauer (2015), productiveness of a struggle is associated with a positive attitude towards challenges and setbacks and with students' satisfaction with learning new mathematics.

In Part 2 of the questionnaire, productiveness was explored by means of 17 Likert-scale items and six open items. For designing the Likert items, we collected ideas for what middle-school students can perceive as success, or what they can be pleased with, when coping with a mathematical challenge. Examples include: solving the task independently, engagement in collaboration with peers, contributing to the solution by using problem-solving methods taught in the past, resilience beyond one's own expectations, being in position to demonstrate problem-solving skills to the teacher. Examples of open items are presented in Table 2.

By means of the Likert scale, we have evidence that for 27 out of 29 students their engagement with the MTT was productive in some sense. Aspects of productiveness indicated by more than 12 students each were related to: satisfaction with being able to recall relevant mathematical facts or methods studied in the past, understanding why a particular idea led to an impasse, understanding one's own mistakes in the solution, pride associated with not giving up, and the pleasure of being appreciated and encouraged by the teacher. To this I would like to add examples of students' responses to two open items (see Table 2).

Sentence to complete	Characteristic examples of students' responses
While solving the task, I was pleased that…	I succeeded to solve it!
	my first idea was right, I just hadn't thought so.
	I succeeded to understand how to solve the task by thinking about the teacher's hints.
	I coped with the problem as well as I could.
	I understood the solution found by other students.
During the discussion, I was pleased that…	I understood what the others explained.
	there are many good solutions!
	there were students who explained well what they had done.

Table 2: Examples of students' responses to two open items about productiveness

To me, all the above is evidence for the richness of the targeted task's life. In the concluding section, I reflect on why revealing secrets of lives of this task, as well as many others, is a worthwhile pursuit for mathematics education research and practice, and on how this pursuit can be facilitated.

REFLECTIONS

The currently prevalent distinction "problem vs. exercise" (Pólya, 1945/1973; Schoenfeld, 1985) alludes to individual experiences: simply put, if one's coping with a task entails struggle – the task is a problem, and if one finds the solution straightforwardly – it is an exercise. In these terms, an important objective of mathematics education research is to find ways to choose and make use of tasks so that they are experienced as problems for as many learners as possible. This objective has stimulated many projects, including the RBMC (Koichu et al., 2021). Simultaneously, this objective holds the potential of leading to either encouraging or disappointing results when pursued in practice (Lester, 2013; Liljedahl & Cai, 2021; Schoenfeld, 2013, 2024).

Let me suggest that some of the disappointments are because the above distinction is too narrow. In fact, this distinction is productive mainly in the context of cognitive-representational perspectives (Lester, 2013; Schoenfeld, 1985, 2013) that focus on (tacit) processes and attributes of problem solving. The distinction seems to me less meaningful within socio-cultural perspectives (Sfard, 2008; Tabach & Nachlieli, 2016). These perspectives have become influential in research on the learning of mathematical concepts and development of mathematical identities but to the best of my knowledge, not yet in research on mathematical problem solving (Koichu, 2019).

I wonder: is it meaningful to ask whether and for whom a particular task is a problem or an exercise in instructional settings, given the many different modes of living that the task may have? Indeed, as the example of the Miscopying Task shows, a task can act as a problem by invoking struggle in various meanings of the word, as an exercise, as a source of inspiration for posing and exploring new questions, as an opportunity for social interactions, as a vehicle for learning, as an artifact from somebody else's problem solving, and more. I wonder, might it be more meaningful to adapt the Paracels' maxim "*Sola dosis facit venenum*" ("The dose makes the poison"), and put effort into answering the question "what are the modes of living of mathematical tasks that are conceived as problems by their proponents?" Let me argue that the pursuit of this question, which is rooted in the metaphor of a task as a discursive creature having its own "secret life", may lead us to re-think, in practical ways, some of the challenges inherent in problem-based mathematics instruction.

First, this question puts mathematics at the centre and positions mathematics educators as "revivers" of tasks while acknowledging that the tasks' lives depend on many factors and actors. To students' shifts of attention and choices considered in the S&C model (Koichu, 2018) we now can add choices made by teachers, professional development facilitators and task designers that, taken together, stipulate the life of the task.

For example, teacher-choices that influence the task's life may include: a task space (i.e., which tasks the teacher offers the students to choose from), declarative knowledge (i.e., which knowledge is considered as a basis for approaching the chosen tasks), didactical contract (i.e., a system of norms and responsibilities), an agency space (i.e., who or what can serve as endorsed agents to learn from, e.g., a peer student, the last page of the textbook, or GeoGebra), and more. These and other such teacher-choices partially stipulate students' choices but should not be seen as the end-of-the-line. It is appropriate to further inquire which actors and factors stipulate teachers' choices. Following Chazzan et al. (2016), we can acknowledge that teachers' choices, and in turn, the tasks' lives, are influenced, among others, by factors related to: students' learning needs, the teacher self-efficacy, knowledge and preferences, the demands of the educational system the teacher belongs to, the societal demand, etc.

Second, by now the critical reader might wonder: "Well, I heard a lot about 'secret lives' of mathematical tasks, but what about research-practice partnerships (RPP), which appear in the title of this talk?" In response, I would observe that RPPs, or more specifically, a co-learning mode of RPPs (Wagner, 1997), has been implicitly present in the argument from the beginning. The argument implies that the question of mathematical tasks' various modes of living (and in turn, the second challenge pointed out by Lester, 2013, see the Introduction section) can only be approached in an RPP mode of operation. This is because neither teachers nor researchers are powerful enough on their own to reveal lives of tasks intended to be problems when operating separately, at least not on scale.

Indeed, the story of the Miscopying Task began from my personal reflections, and continued based on accumulated reflections of colleagues, teachers, and students. The main role of task designers and teachers in the story was to revive the task, and the main role of the students was to make its life rich. Further, the main role of researchers was to create conditions for accumulating shared knowledge on the task's life and mediate this knowledge for the other actors.

In a series of studies co-authored with in-service teachers (e.g., Cooper et al., 2023; Cooper & Gruenhut, 2023; Koichu et al., 2023), we have begun to explore what researchers and practitioners can co-learn from teachers' reflections and student-data on task enactments. So far, we collected evidence suggesting how researchers and teachers look at the data and reach conclusions. In brief, the researchers mainly produce conjectures about what is worth trying in problem-solving courses for teachers, and theorize phenomena related to teachers' and students' experiences. The teachers are seen to seek insights with relation to what could be improved in their lessons, and also appreciate opportunities to better understand their students. Both parties are frequently surprised by the shared evidence as well as by observations made by the other party. To this end, the researchers' learning, and the teachers' learning seem to be complementary and mutually enriching, and RPP seems to gradually become a way of professional living for both parties, towards handling the aforementioned challenges among others. What can be said with certainty is that new theoretical and practical endeavours aimed at making RPPs an indispensable part of the mathematics education landscape require further experimentation and reflection.

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FIVE WORDS FOR RETHINKING MATHEMATICS EDUCATION

JeongSuk Pang

Korea National University of Education

Drawing on my research and experience over the past 20 years, this plenary lecture offers five words for rethinking mathematics education together: alignment, resources, culture, understanding, and specificity. Each word depicts an area of research that I engaged in over the decades. By looking back at these I offer considerations of how my thinking continues to move forward as a mathematics educator. As such, this talk provides a platform for reflection on my research journey and a way forward on reshaping my earlier thoughts in how we can be responsive to our change in an ever-changing world.

INTRODUCTION

It is a great honour to give a plenary lecture at PME 47. In order to prepare this talk, I looked back on my studies and experiences as a mathematics educator. I have been teaching both pre-service and in-service teachers at Korea National University of Education since 2002. My research areas include teacher education and professional development, mathematics textbook development and research, and various topics in elementary mathematics, especially early algebraic thinking. Reflecting on my research and teaching experiences over the last 20 years or so, I have come up with five key words–alignment, resources, culture, understanding, and specificity–that provide some thought for rethinking mathematics education. For each word, I start with the research background and then present some selected studies to raise implications for mathematics education and our responsiveness in moving forward.

ALIGNMENT

I have been heavily involved in curriculum and textbook development and research, but the word alignment was not really on my radar until recently. In the following section, I describe how alignment emerged in the design of a particular unit in a new textbook (Pang & Sunwoo, 2022). Also, alignment emerged when I was exploring students' perspectives on what they valued in mathematics learning while, at the same time, investigating how students would perceive what their teachers valued in teaching mathematics (Pang et al., 2024).

Three successive alignments from curriculum to student learning

We continue to change our mathematics curriculum to make mathematics better to teach and learn while meeting the new needs of an ever-changing world. We then develop a new series of textbooks to make the ideal or abstract aspects of the curriculum concrete enough to be implemented in the classroom. Alignment between the curriculum and the textbooks is emphasised. However, we often overlook the critical link in the alignment chain that connects textbooks (or any other instructional

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resources) to actual classroom instruction and student learning outcomes. This is especially true in Korea, where research on curriculum or textbooks has been the most common topic in recent years, but the majority of studies have focused on analysing the curriculum or textbooks themselves rather than on how the intended curriculum is implemented in the classroom (Pang, 2022). With this in mind, the following is an example of how alignment was seriously considered in the design of a pattern and correspondence unit to promote functional thinking in a Korean textbook.

In Korea, the national mathematics curriculum requires students in Grades 5 and 6 to (a) identify and explain the pattern from a table showing the correspondence relationship in which one quantity changes and the other quantity depends on it and (b) represent the relationship by an equation using symbol variables. Four key instructional elements were extracted from a review of the literature on functional thinking: correspondence relationships in real-life contexts, various pattern tasks, exploration for a correspondence relationship, and symbol variables to represent a correspondence relationship. The previous textbook unit on pattern and correspondence was analysed through the four elements. The analyses led to the new textbook, including more geometric patterns using shape blocks and additive relationships, giving students many opportunities to explore the relationship between two covarying quantities and to think about the usefulness of using symbol notations to represent a correspondence relationship along with the meanings of such symbols.

Once textbook activities are aligned with the curriculum expectations, the next step is to align the intent of the textbook activities with an actual lesson. The new pattern and correspondence unit was implemented in an elementary classroom (the intervention group) to explore its appropriateness for students to develop functional thinking. As it is important for a teacher to understand the intentions of new textbook activities, a teacher's guide was also developed to include the background knowledge of the unit, such as three modes of student thinking when exploring the relationship between two quantities (i.e., recursive thinking, covariation thinking, and correspondence thinking) and content-specific pedagogical strategies (e.g., using a non-sequential function table). Five lessons in implementing the new unit in the intervention group were videotaped and qualitatively analysed using the four key instructional elements described above. Students were able to (a) notice and articulate various correspondence relationships between objects in the classroom, (b) find a correspondence relationship in both numerical and geometric pattern tasks, (c) use correspondence thinking beyond their initial recursive thinking, and (d) use symbols for variables when expressing a function rule.

A final alignment needs to verify that the implementation of the new unit has resulted in positive student learning outcomes. For this verification, the intervention group (with the new unit) and the non-intervention group (with the same unit from the previous textbook) were compared using two types of written assessment: Type A assessment was to confirm whether the students understood the main content of the unit, while Type B assessment was to compare and contrast the functional thinking of both groups. There were no statistically significant differences between the two groups in the pre-test for the two types of assessment. There were also no statistically significant differences between the two groups in the post-test for the Type A assessment. However, there were statistically significant differences for the Type B assessment, indicating that the intervention group could develop a better understanding of functional thinking than the non-intervention group.

This example is significant because it goes beyond the development research trend of focusing mainly on the alignment between a curriculum and textbook activities, which is often the case, and looks at whether textbook activities are translated into classroom instruction and lead to student learning outcomes. Whenever a new curriculum and its accompanying textbooks are changed, they should be examined to see whether the intentions of the curriculum are ultimately linked to student learning outcomes. In turn, student learning outcomes should be an important catalyst for curriculum revision, if the three types of alignment are done as intended.

Value alignment for effective mathematics instruction

Mathematics educational values are regarded as any attributes of mathematics teaching and learning that are considered personally important (Seah, 2019). As students' values in learning mathematics are key to understanding and facilitating their learning, the importance of considering what students value in learning mathematics cannot be overemphasised. Furthermore, if what the teacher considers important in teaching mathematics is aligned with what students consider equally important, the foundation for effective teaching can be laid.

Korea's participation in the Values Alignment Study (VAS) identified value alignment between students' personal values of learning mathematics and their perceived teacher values of teaching mathematics. A total of 832 Grade 9 students participated. Two specific items of the VAS questionnaire were the following: (a) Think about your own experience of learning mathematics. What do you think is important when you learn mathematics? (b) Think about your own mathematics teacher this year. What do you think is important to him or her in mathematics teaching? Students were asked to list and explain up to three attributes corresponding to what they considered important. In order to capture the richness of the data in the students' writing, we analysed it based on the connectivity between words and the centrality of specific words.

The frequency analyses between students' personal values of learning mathematics and their perceived teacher values of teaching mathematics show a remarkably similar trend: The top three values (i.e., *problem*, *understanding*, and *review*) were the same. The centrality analysis revealed three groups of students' personal values and four groups of their perceived teacher values. As shown in Table 1, the groups had the following values in common when comparing students' personal values and their perceived teacher values: Group 1 included *problem, concept, formula, solution,* and *basics*; Group 2 included *understanding* and *concentration*; and Group 3 included *review* and *preparation*. The value *problem* was centralised as being the most important for students' own learning of mathematics and for their teachers' teaching of mathematics. These findings show an alignment between what the students thought was important for their mathematics learning and what they thought was important for their mathematics teachers in teaching mathematics. However, there were subtle differences, such as more values adjacent to the value *problem* in describing students' own values of mathematics learning, as well as a new group with *oneself* and *lesson* in describing teacher values.

Group	Students' personal values of learning mathematics	Students' perceived teacher values of teaching mathematics
1	problem, formula, concept, thinking, solution, ability, computation, basics, type	problem, concept, solution, formula, basics, method, variety
2	understanding, concentration, persistence	understanding, concentration, thinking, explanation
3	review, preparation, content	review, preparation
4	·	oneself, lesson

Table 1: Students' values of learning mathematics and their perceived teacher values

The striking similarities between students' values of learning mathematics and their perceived teachers' values of teaching mathematics suggest that teachers need to be aware that students' perceived values are related to what teachers consider important. Therefore, teachers need to better understand their values of mathematics education and be able to balance different values when interacting with students to achieve intended pedagogical goals.

RESOURCES

As I said earlier, because of my heavy involvement in the development of instructional resources, primarily textbooks, the word resources clearly came to mind as I reflected on my research and experience. Here, I use the word resources to include not only textbooks, but also workbooks, teacher guides, digital materials, and so on that are developed to support teaching and learning. Effective mathematics textbooks are key to students' learning and teachers' teaching. This is particularly true in Korea, where 99% of fourth graders (the international average was 75%) used textbooks as the basis for instruction, and 97% of eighth graders (the international average was 77%) did so (Mullis et al., 2012). It is not surprising, therefore, that Korea is putting its best efforts into developing effective textbooks. Similarly, research on curriculum and textbooks has accounted for about 15% of all articles published in mathematics education journals in Korea over the past 50 years (Pang & Kwon, 2023).

Textbook development and related issues

In Korea, many textbook analysis studies have been conducted to develop effective textbooks. In fact, about 76% of textbook studies in Korea are textbook analysis studies, mainly focusing on the specific mathematical concepts or the overall organisation of a mathematical content strand, including how and when to teach it (Pang, 2022). Such studies tend to identify the changes in textbooks in relation to the revisions of the national mathematics curriculum in Korea. Also, some of these studies compare Korean textbooks with foreign counterparts in order to identify alternative methods or activities for addressing and presenting specific mathematical concepts.

However, most comparative studies focus on textbooks rather than teacher guides, which makes it difficult to fully understand the intentions of textbook construction and activities, and they are limited to foreign textbooks written in English or translated into Korean. To overcome these limitations, collaborative studies involving foreign researchers need to be activated. In addition, if international comparative studies are conducted not only in textbook analysis but also in textbook use, it will be easy to analyse socio-cultural factors related to the interaction between textbooks and teachers beyond the characteristics of the textbooks themselves.

Another effort to develop effective textbooks in Korea involves multiple experts (e.g., mathematics educators, mathematicians, teachers, and designers) researching, writing, reviewing, and discussing textbooks, which are then pre-tested in actual classrooms to check their appropriateness. This has been possible because Korea has only one set of elementary mathematics textbooks under the national curriculum. However, this government-issued textbook system has recently been changed to a government-approved textbook system for Grades 3 to 6. On the one hand, this change invites the development of diverse and creative textbooks that address and present the same mathematics topics with different approaches. On the other hand, it raises many concerns, such as the decentralisation of research and writing, the weakening of the field review process, the professionalism and transparency of the approval committee, excessive competition among publishers, and textbook adoption influenced by non-mathematical reasons, such as whether textbook publishers have an online platform.

Another recent issue related to textbook development is the development of digital textbooks using artificial intelligence (AI). To be clear, the educational community is already using various digital resources and platforms, but these tools have only served as a supplement to textbooks. However, government-approved AI digital mathematics textbooks will be used for the first time in Grades 3 and 4 next year, so they are currently under development. Much discussion has revolved around how AI digital textbooks will be built, what technologies will be used to maximise student learning of mathematics, and how best to support teachers in teaching mathematics while respecting their ability to plan and implement lessons.

Once AI digital textbooks are developed and deployed in the field, it will be necessary to study how teachers select and use digital textbooks compared to traditional book-based textbooks. Since AI digital textbooks will diagnose individual students' mathematical skills and dispositions, it is expected that a teacher can effectively use AI digital textbooks to provide differentiated and responsive instruction according to students' needs. Similarly, students' actual use or interaction with AI digital textbooks is a research area to be further investigated.

A teacher's guide and related issues

Compared to the development of textbooks, the development of teacher guides has been relatively neglected, mainly because most of the tasks in the textbook are used by a teacher and students in mathematics classes, whereas teacher guides are intended only for teachers and are used selectively by the teachers. Nevertheless, the development of teacher guides in Korea has recently gained importance in terms of teacher learning and professional development (Pang, 2022). On the one hand, pre-service teachers study teacher guides thoroughly in order to pass the highly competitive national teacher employment test, making them an important resource for novice teachers' learning. On the other hand, in-service teachers can strengthen their content-specific pedagogical knowledge by reviewing the intentions of the tasks, understanding the teaching strategies with their rationales, and looking for alternative activities in the teacher guides if necessary.

The Korean teacher guides have both strengths and weaknesses in supporting teacher learning (Pang et al., 2023c). For example, the teacher guides for Grades 3 and 4 are effective in that the mathematical content knowledge for teaching is well provided, and a variety of manipulative materials are presented along with the rationale or strategies for using them, especially in the areas of operations or geometry. However, the sample teacher-student dialogues for each lesson are only useful for getting a sense of the overall flow of the lesson. They do not include a wide range of student responses due to space limitations. Recent changes in the development of the teacher guides should address these weaknesses.

As instructional resources for elementary school mathematics move towards a government-approved system, teacher guides will also be approved separately from textbooks, resulting in richer materials and more research-based guidance for teaching than before. Another change associated with the government-approved system concerns the main textbook publishers, who will provide teachers with condensed versions of the core content of the teacher guides, called teacher textbooks (i.e., teacher editions of student textbooks), to help them use the textbooks more effectively.

The development and dissemination of digital resources can free authors or teachers from the space constraints of typical teacher guides. Korea has a popular online community of teachers, and they have been sharing their own resources voluntarily and collaboratively for over 20 years. In addition, major textbook publishers have recently set up an online platform to provide teachers with almost all instructional resources, including video clips to explain the main concept in each lesson, which they can easily select according to their daily needs.

The increasing availability and variety of instructional resources require teachers to have expertise in their use. No matter how effective the resources are, their effectiveness depends on how teachers use them. Therefore, teachers' interactions with different instructional resources need to be further explored, especially in relation to how recent new features of such resources affect teachers' lesson planning, implementation, and reflection.

CULTURE

It seems obvious that mathematics education should be understood within the socio-cultural context in which it takes place. Nevertheless, I was surprised to find the power or influence of culture in unexpected research contexts, particularly in studies of effective mathematics teaching (Pang, 2012; Pang & Kwon, 2015; Pang et al., 2023b).

Teachers' overall perspectives on effective mathematics pedagogy

One of my main roles as a teacher educator is to help teachers implement effective mathematics lessons. The question of what constitutes effective mathematics pedagogy can be answered from a variety of perspectives, but essentially, it is important to know what the teachers who deliver the lessons think. The initial target participants for my related study were elementary school teachers, but this population was extended to secondary school mathematics teachers.

A questionnaire was developed to explore teachers' perspectives on effective mathematics pedagogy. In the first part of the questionnaire, teachers were asked to describe any aspects that they considered important for effective mathematics pedagogy. In the second part, they were asked to indicate the extent to which they agreed with 48 items related to effective mathematics pedagogy using 5-point Likert scales: A score of 5 meant strongly agree, and 1 meant strongly disagree. The participants were 135 elementary school teachers, 132 middle school mathematics teachers, and 124 high school mathematics teachers. Figure 1 shows the mean scores of the 48 items according to the three groups of teachers (Pang & Kwon, 2015, p. 149).

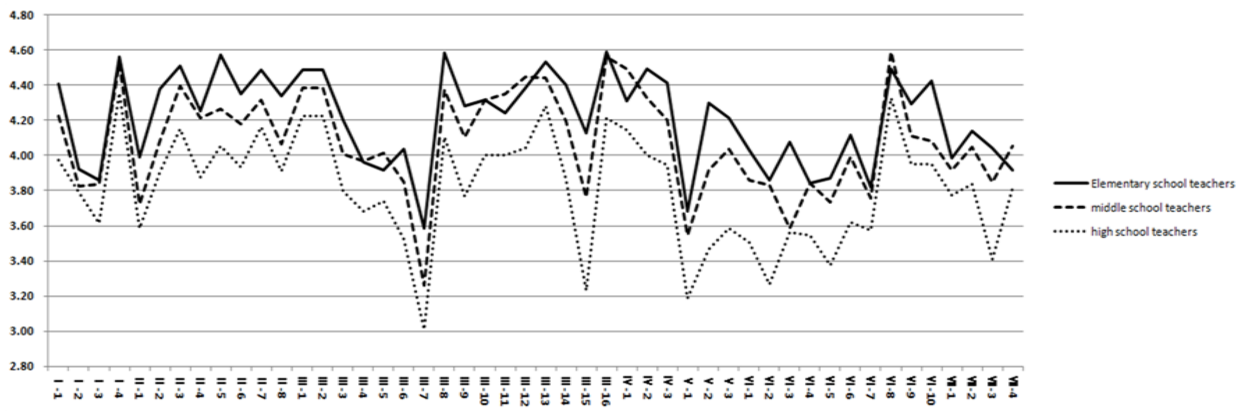

Figure 1: Mean scores of the 48 items by the groups of teachers

The most striking aspect of Figure 1 is the similar patterns across the groups of teachers. To be sure, the extent to which each group of teachers agreed with each item varied. Elementary school teachers tended to agree more than their middle or high school counterparts. In fact, there were statistically significant differences between elementary and middle school teachers on 11 items and between middle and high school teachers on 29 items. However, the overall agreement about what constituted effective mathematics pedagogy was quite similar.

On the one hand, the items the three groups of teachers agreed upon the most included (a) teaching by constructing the curriculum according to students' different levels, (b) teaching based on mathematical communication between the teacher and students, (c) teaching to improve students' self-directed learning ability, (d) providing students with appropriate feedback, and (e) teaching the essential concepts in mathematics. These perspectives are influenced by the recent revisions of the mathematics curriculum in Korea. On the other hand, the least agreed-upon items included (a) teaching students to calculate proficiently, (b) teaching by using technology, (c) having a good physical environment, (d) teaching while managing problematic students, and (e) emphasising human relationships. The similar patterns in Figure 1 suggest that teachers' perspectives on effective mathematics pedagogy were deeply rooted in their socio-cultural contexts. Therefore, it is necessary to explore what effective mathematics pedagogy means when we discuss what we are aspiring to in mathematics education, and we must explore how it is the same as or different from teachers' perceptions of effective mathematics pedagogy, especially among those who actually teach it. This exploration needs to be interpreted within the socio-cultural context of each country.

Changing teaching practices towards effective mathematics teaching

For me, the research context in which the word culture clearly emerged was when I was studying the process by which teachers changed their teaching practices to implement better mathematics teaching. For instance, I analysed how an elementary school teacher (Ms Y) changed her teacher-centred teaching to a student-centred approach by participating in a year-long research project (Pang, 2012). An analytical framework of five dimensions with 24 sub-dimensions was developed, and five lessons of Ms Y's teaching practice were selected to trace her instructional changes. I focused on what had changed and what had not changed in the process of incorporating student-centred instructional approaches (e.g., using instructional strategies tailored to students' differences) into an ordinary teaching practice.

In terms of what had changed, three types of changes were identified: (a) *dramatic changes* were sudden and noticeable changes that occurred in the early stages of teacher change, (b) *substantial changes* were less dramatic but considerable changes that occurred in the middle stages of teacher change, and (c) *gradual changes* were changes that occurred over a longer period. For instance, dramatic changes were noted in the use of manipulatives, the opportunities for students to present their own ideas to the whole class, and the use of small-group or individual activity formats. Substantial changes were noted in the focus on promoting students' mathematical reasoning skills and on soliciting and using students' ideas. Gradual changes were identified for focusing on fostering students' positive dispositions towards mathematics and reducing teacher question/answer and demonstration.

In terms of what had not changed in Ms Y's teaching practice, two aspects were noticeable. On the one hand, there were some positive changes that were not fully achieved as recommended. These less fully achieved changes occurred in the use of teaching strategies tailored to students' differences and in emphasising the importance of mathematical communication. On the other hand, there were unchanged practices throughout the year. Ms Y used many new recommended approaches, but the overall characteristics of her lessons were still consistent, progressive (i.e., from easy/concrete to difficult/abstract forms), and systematic (i.e., a lesson flow included learning motivation, learning objectives, main activities, practice, and evaluation/summary). A notable constant was Ms Y's emphasis on important mathematical content. For instance, she encouraged students to use manipulative materials, but she made sure that such activities were connected to the conceptual structure behind them. She also solicited students' multiple ideas but did not forget to emphasise a mathematically significant idea by orchestrating the path of classroom discourse to explore it or otherwise directly introducing it when students did not come up with such an idea. In other words, despite the change in the form of instructional approaches, the pedagogical priority of Ms Y's teaching was her students' conceptual understanding of mathematical content. The strong emphasis on mathematical content reflects one of the most prominent cultural activities of Korean teaching.

Another similar research context in which the word culture emerged was when I explored the challenges Korean teachers faced in implementing Smith and Stein's (2018) five practices for orchestrating productive mathematics discussions in their classrooms: *anticipating, monitoring, selecting, sequencing*, and *connecting.* The researchers associated these five practices with *setting goals* and *selecting tasks* as a foundation. The research participants were 15 elementary school teachers who were keen to implement the five practices through an iterative lesson study cycle of lesson planning, implementation, and debriefing. They had overall success in implementing the five practices, particularly those related to lesson design (i.e., from setting goals to anticipating student responses), but they also experienced various challenges. Some challenges were similar to those reported in the U.S. context, such as identifying the core mathematical ideas of the learning objectives and planning specific questions and feedback tailored to student responses (Smith et al., 2020).

Other challenges not explicit in the literature were also identified in the study I worked on (Pang et al., 2023b). For instance, one challenge identified was selecting and presenting tasks appropriate to the student levels and the classroom environment, probably due to the specific characteristics of the lesson study. The challenge of taking on multiple teacher roles at the same time may be a problem inherent in monitoring student work. Still other challenges, such as clearly stating the learning goals in sentences, writing a lesson plan for effective use, or visually sharing student presentations, seem to be related to the Korean classroom culture. It was interesting that these challenges manifested themselves when the challenges faced by Korean teachers were compared and contrasted with those of their U.S. counterparts. The process of implementing new approaches, such as the five practices, can reveal subtle but significant differences and variability that exist between cultures. Changes in teaching practices tend to occur within socio-cultural expectations and educational values long maintained in a particular education system. Therefore, further research is needed to identify the different challenges teachers face in implementing the five practices in different education systems and to explore whether these are inherent in the practices themselves or in the socio-cultural context in which they are implemented.

UNDERSTANDING

The importance of understanding in mathematics education cannot be overemphasised. The curriculum that defines what should be taught in school mathematics has been revised periodically in response to societal changes, but what has remained constant is students' understanding of mathematical concepts, principles, or laws. It is important to examine how students understand a mathematical concept. In my recent research on early algebraic thinking, I have been particularly interested in students' understanding of the equal sign and variables (e.g., Lee & Pang, 2021; Pang & Kim, 2018; Pang et al., 2023a).

Students' opposing conceptions of the equal sign

As a basis for developing algebraic ideas, students need to have a relational understanding of the equal sign. In Korea, the equal sign is first introduced in Grade 1 when teaching addition and is then used in many contexts. Using the 27 assessment items from Matthews et al. (2012), I investigated students' understanding of the equal sign, expressions, and equations (Pang & Kim, 2018). Specifically, 695 students from Grade 2 to Grade 6 (ages 7 to 12 years) were included in the study. The results show that students were quite successful in almost all items of three different types (i.e., equation-structure items, equal sign items, and equation-solving items). More importantly, a statistically significant difference was found among the grades except between Grades 5 and 6, indicating that students' understanding improves as their grade levels increase up to Grade 5. Nevertheless, there were some items that needed further consideration.

The most difficult of the equation-structure items was deciding whether the number that goes into the box is the same number in two given number sentences: $2 \times \square = 58$ and $8 \times 2 \times \square = 8 \times 58$. Even among Grade 6 students, about 57% of them got the correct answer, and only about 15% of them could use relational thinking for the answer. Another item that needs attention concerns the meaning of the equal sign. When asked to determine whether the given definition of the equal sign is true or false, more than 80% of the students from Grade 3 onwards chose "true" for the following sentence: The equal sign means "the same as". In contrast, about 22% of the students chose "false" for the following sentence: The equal sign means "the answer to the problem". Students considered the equal sign to mean both "the same as" and "the answer to the problem".

In a separate study I conducted, the coexistence of two conceptions of the equal sign in a single student's mind was found in a whole-class discussion of equations with two equal signs (Lee & Pang, 2021). Thirty students in Grade 4 were studying multiplication and division based on textbook activities. Specifically, students were first exposed to equations with two equal signs while checking $162 \div 20 = 8R2$ and solving $20 \times \square + \bigcirc = \boxed{} + \bigcirc = \boxed{}$. Students came up with different equations, and the teacher guided students to discuss why they agreed or disagreed with each equation. Even after a lengthy discussion of the meanings of the equal sign, as many as 22 students said that they agreed not only with $20 \times 8 + 2 = 160 + 2 = 162$ but also with $20 \times 8 = 160 + 2 = 162$, insisting that the equal sign could be interpreted as both "the same as" in the former equation and "is" in the latter. Note that it was helpful for these students to parenthesise the expression on each side of the equal sign to treat it as an entity or as a whole. It was also helpful for them to explore the structure of the equation or to use the transitive properties of equivalence.

The two studies above indicate that many students have simultaneously opposing (but apparently compatible) conceptions of the equal sign, which are operational and relational conceptions. In this respect, when investigating elementary school students' understanding of the equal sign, it seems a better option to present them with a specific equation and ask them what the equal sign means in that equation rather than to ask them to write a free-form description of the meaning of the equal sign or to give them the meaning of the equal sign and ask them to judge it as true or false. In addition, for concepts encountered across all grades in elementary school, such as the equal sign, it is worth investigating whether an understanding of the concept grows as students progress through the grades, specifically in terms of what misconceptions older students continue to have and what features of equation-structure and equation-solving items they find particularly challenging.

Similarly, to develop students' understanding of the equal sign, it is important to cover a variety of contexts in which the operational or non-relational interpretations of the equal sign do not apply and to encourage students to think about how best to define the equal sign in different contexts. Even if students see the meaning of the equal sign as sameness, it is important to be clear about which components of the equation are equal. Despite numerous studies of students' understanding of the equal sign, the complexity of this understanding remains elusive.

Students' understanding of variables

Variables are an important concept common across the two content areas of early algebra: the generalised arithmetic perspective and the functional perspective (Kieran et al., 2016). Building on the previous finding that Korean students had difficulty representing unknown quantities with variables (Pang & Kim, 2018), I investigated students' overall understanding of variables across the two content areas. Specifically, from the generalised arithmetic perspective, my study's questionnaire included the property of "1", the community property of addition, the associative property of multiplication, and a problem context with indeterminate quantities. From the functional perspective, the questionnaire covered additive, multiplicative, squaring, and linear relations.

A total of 246 Grade 5 students were included in the study (Pang et al., 2023a). The results show that most students could find specific values for variables and understood that equations involving variables could be rewritten using different symbols. However, students struggled to use variables to represent generalised properties or contexts of arithmetic problems. For example, almost all students in the study found a specific value related to the property of "1" (i.e., 10,293 × 1 = 10,293), but only about 66% of them could represent it with a variable (i.e., $\square \times 1 = \square$). Also, about 30% of the students knew that the variable could be not only natural numbers but also fractions or decimals. This suggests that students need to explore whether the properties of numbers and operations remain the same when the range of numbers covered in elementary school extends from natural numbers to fractions or decimals.

The most difficult item in the study was to represent problem contexts with indeterminate quantities in equations with variables. The problem was as follows: Minsoo's current hair length is 7 cm, and his hair grows at a rate of 2 cm per month. Think about how Minsoo's hair will change in the future: (a) How many months will it take until Minsoo's hair is 15 cm long? (b) Write an expression for the length of Minsoo's hair after some ($\square$) months have passed. The study found that 72% of the students were able to answer the first question correctly, but only about 29% of them were able to answer the second question correctly. Students' incorrect responses included writing the rule using a specific number without using a variable (e.g., $7 + 4 \times 2 = 15$), not expressing it in the form of a completed equation (e.g., $7 + \square \times 2 =$), or writing a specific value to the right side of the equal sign (e.g., $7 + 2 \times \square = 15$). While this tendency (i.e., success in finding specific values of variables but difficulty in writing an expression using variables) was consistent for correspondence relations from the functional perspective, it was found to be more challenging for the items associated with the generalised arithmetic perspective. These findings highlight the need to investigate students' understanding of a particular mathematical construct comprehensively by presenting different problem contexts from the two perspectives and within the same perspective.

SPECIFICITY

The last word I would like to mention is specificity. This word emerged as important when analysing the level of implementation of each of Smith and Stein's (2018) five practices as Korean teachers applied them to their mathematics teaching (Pang et al., 2022). Another research context in which the word emerged was in developing a teacher's guide that emphasised "process-focused assessment" (or formative assessment) during instruction (MOE, 2021).

Specificity for higher implementation levels of the five practices

There is a growing consensus that mathematical discussions should be a key part of mathematics teaching. Despite many suggestions to support teachers in transforming a discourse pattern in the mathematics classroom, general pedagogical ideas still make it challenging for teachers to implement productive content-specific mathematical discussions. The five practices make mathematics discussions more manageable for teachers by managing the content discussed and by reducing the burden of improvisations while honouring student contributions (Smith & Stein, 2018).

The five practices were expected to be suitable for Korean teachers to apply in their mathematics classes, as the pedagogical priority in Korean mathematics classes is conceptual understanding of important mathematical content, as mentioned earlier. Indeed, I have conducted several classroom studies with teachers who have attempted to apply the five practices in their mathematics classes (e.g., Pang, 2016; Pang et al., 2022). Overall, the teachers were quite successful as they participated in an iterative cycle of a lesson study. However, a closer analysis of the implementation of each practice led to four levels of implementation based on the extent to which the key components of each practice were implemented: Level 1 was assigned to a study participant's performance in the lesson study when only one component of each practice was considered or undesirable aspects were included; Level 2 when two or three components were implemented or undesirable aspects were included; Level 3 when all the components were implemented but insufficient aspects were included; and Level 4 when all components were implemented faithfully.

Because the teachers who participated in the lesson studies were committed to implementing the five practices, they rarely fell into Level 1 but, instead, implemented each practice at Levels 2 to 4. For example, regarding *task selection*, Grade 6 teachers changed a simple percentage problem to one of mixing two chocolate milks of different thicknesses and finding the thickness of the mixture. The task was aligned with the learning objectives and was cognitively challenging. However, in the first lesson, the students struggled with the task because they did not fully understand the meaning of thickness expressed as a percentage (Level 2). In response, the teachers added a sub-question in the second lesson to explore the meaning of thickness, which helped some students approach the task (Level 3). In the subsequent third and fourth lessons, the teachers adjusted the numbers of the task to make it more accessible to the students by drawing or calculating based on their understanding of percentages (Level 4).

Similarly, regarding *anticipating student responses*, Level 4 was given when a teacher (a) anticipated not only correct approaches to the given task but also errors or misconceptions that students might make in different and specific ways, (b) anticipated responses to student approaches in different and specific ways, or (c) identified the responses that might address the mathematical objectives. In summary, the higher implementation of each of the five practices ultimately depended on how *specifically* they prepared and implemented the key elements of each practice. Given this, specificity is key to maximising the potential of the five practices as content-specific pedagogical practices for orchestrating whole-group discussion.

Specificity of process-oriented assessment in a teacher's guide

Recent Korean mathematics curricula recommend process-oriented assessment as well as assessment of learning outcomes. When a curriculum only mentioned the term process-oriented assessment, the question arose as to how it could be implemented in the mathematics classroom. It was not enough to explain the intentions of the curriculum to teachers in a way that emphasised the importance of students' learning processes or to give brief examples of how process-oriented assessment could be used in a particular unit or lesson in the textbook. Therefore, the teacher's guide has been developed to provide teachers with specific ideas on how to use process-oriented assessment in each lesson.

For instance, in the pattern and correspondence unit for Grade 5 described above, the fourth lesson dealt with representing the correspondence relationship in an equation with symbol variables (Pang & Sunwoo, 2022). In the first task, students were asked to (a) fill in a table with the number of drones and the number of blades when making a drone with four blades and (b) represent the correspondence relationship between the number of drones and the number of blades in an equation by selecting the given cards with words and symbols, such as "the number of blades" or "=". For this task, the teacher's guide presents three possible student responses and gives specific teaching tips. Table 2 represents an example of teaching tips for one of these responses (MOE, 2021, p. 190).

Learning information	Example of teaching tips
Students incorrectly represent the correspondence between the number of drones and the number of blades using symbols and words.	Ask students to check the meaning of their equations. For example, the equation (number of drones) = (number of blades) × 4 means that the number of drones is equal to four times the number of blades. Have students check this using the table. Alternatively, students can plug in the numbers from the table to see if the equation is correct.
e.g., (number of drones) = (number of blades) × 4	This is a typical error response for many students, so it is suggested that you teach this explicitly to the whole class.

Table 2: Part of the process-oriented assessment described in the teacher's guide

Note that for the above task, the teacher's guide includes teaching tips for correct answers as well as incorrect ones. For example, even if students have correctly represented the relationship as (number of drones) × 4 = (number of blades), the guide suggests that a teacher might encourage students to represent it using a different equation and, if so, to compare the apparently different equations. The guide also suggests that students have the opportunity to explore whether the number of drones and the number of blades could be any number.

Process-oriented assessment was abstract in the curriculum, but it manifested in the teacher's guide by presenting teachers with a range of possible student responses to key tasks, along with teaching tips tailored to each response. One of the characteristics of an effective teacher's guide is that it is specific enough to allow teachers to anticipate multiple student responses to each task in the textbook and to provide appropriate feedback accordingly. It was this specificity that enabled teachers to implement the intentions of the curriculum in their classrooms.

CLOSING REMARKS

This talk has addressed five words based on my own research and experience. As such, these words are limited by my research and experience. In other words, the words that mathematics educators consider important may vary depending on their research and experience. What would your five words be based on your own research and experience? How can we rethink mathematics education from the words? According to socio-cultural changes, new content, processes, or ideas should be emphasised in mathematics education to prepare for the future society. So, our mathematics education tries to include more and more. However, we could move in another direction when rethinking mathematics education. Instead of the five words I have chosen, I think it would be possible to condense it down to fewer words or even just one word. Instead of adding new ideas, like Picasso's series of paintings, we could find the most important thing about mathematics education and subtract and subtract until we are left with the essence. I hope that this talk will stimulate your thinking about your own mathematics research journey, the research contributions you have made, and the directions you will take in your continued contributions to the mathematics education journey.

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HOW SHOULD MATHEMATICS EDUCATION RESEARCHERS THINK ABOUT PROOF?

Keith Weber

Rutgers University

In mathematics education research, a proof is often defined as a justification that convinces a student or a mathematician that a claim is true. In this plenary presentation, I present two studies that demonstrate that this perspective can lead researchers to take a deficit-oriented perspective when evaluating students' epistemology. Further, researchers can sometimes critique students for taking the same actions on a proof-oriented task that a professional mathematician would. Finally, I argue that defining a proof as a justification that fulfills a psychological function can lead mathematics education researchers to overstate the importance of proof in mathematics classrooms.

INTRODUCTION

There is a general consensus amongst mathematics educators on three topics:

- Proof is a central part of professional mathematical practice and should play an important role in students' mathematics classrooms as well.
- Students' proof-related competencies and understandings are limited. They have difficulty with writing and evaluating proofs. They also do not understand the nature of proof or appreciate its importance in doing mathematics.
- For these reasons, we need both basic and applied research on proof. As a field, we need to better understand how students perceive proof, and we need to design instruction to help them understand proof better.

While mathematics educators agree that we should investigate and teach proof, there is substantial disagreement on what it means to do so. Indeed, mathematics educators famously disagree on what *proof* even means (e.g., Balacheff, 2008; Czocher & Weber, 2020). For instance, some mathematics educators define proof as a deductive argument that does not admit exceptions, others characterize proof as persuasive arguments that provide mathematicians with certainty, and still others say proof is an argument that is sanctioned by a classroom or mathematical community (Czocher & Weber, 2020).

In this paper, I critically examine one common approach to defining proof in mathematics education. Namely, I consider the consequences of defining a proof as an argument that provides an individual with certainty that a mathematical claim is correct. While this perspective has generated a great deal of research revealing how students think about proof, it has two drawbacks.

First, there is an assumption in this type of research that deductive justifications ought to provide students with mathematical certainty and empirical justifications ought not to provide students with certainty. Research is often conducted with the aim of showing

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that students deviate from these normative expectations. This leads to a deficit-oriented perspective on students' thinking about proof, and negative evaluations of students' epistemologies that are often unjustified. Second, equating proof as a justification that fulfills a psychological function leads mathematics educators to overstate the importance of proof.

The remainder of this paper is devoted to defending these theses. In the next section, I describe how proof gained prominence as an area of research in mathematics education and I discuss why it was attractive to mathematics educators to define proof using a psychological approach. In the two sections that follow, I summarize results from empirical studies that I conducted. In the first study, I defend students who provide empirical justifications when they are given justification tasks. In the second study, I defend students who retain doubts in a theorem after reading a correct proof of the theorem. Finally, I offer some tentative comments about how we should think of proof using the perspective of norms and values.

HOW MATHEMATICS EDUCATORS CAME TO VALUE PROOF

Do we need proof in school mathematics?
Absolutely. Need I say more? Absolutely. (Schoenfeld, 1994, p. 74).

I do not think we would see a quote like this in a mathematics education journal today. It is already widely accepted that we should teach proof in school mathematics, so no one would feel the need to endorse this position. However, in 1994, it was not obvious that proof should be taught in school mathematics. Indeed, some influential policy documents on what should be in the school mathematics curricula scarcely mentioned proof at all (NCTM, 1989; see Knuth, 2000, for further discussion of this point).

The reluctance to teach proof in school mathematics was due, at least in part, to how proof had been treated in the school curriculum (Hanna, 1989; Schoenfeld, 1994). Proofs in classrooms were highly formalized, where often the format of the proof was given more attention than the ideas in the proof. For instance, proofs in United States geometry classrooms were usually written in a highly regimented two-column format. These types of proofs went against the constructivist ethos of the 1990s, which placed a high degree of value on discovery and meaning-making and which was skeptical of compelling students to use institutional representation systems to which students could not attach meaning. Further, a lesson learned from the implementation of the 'new math' curriculum in the 1960s is that a premature focus on formalization and rigor could be harmful to students' mathematical development. These observations could explain why some mathematics educators did not value proof in the classroom.

There was pushback to the de-emphasis on proof, not only from professional mathematicians (e.g., Wu, 1996), but also from mathematics educators such as Tommy Dreyfus, Gila Hanna, and Alan Schoenfeld who had substantial training in graduate-level mathematics. These authors argued that proof was misunderstood by students and mathematics educators, and the "proofs" that appeared in mathematics classrooms were bastardizations of the proofs that appeared in mathematical practice. As Schoenfeld (1994) put matters, proof should not be seen "as an arcane formal ritual, but as a codification of clear thinking" (p. 74). Viewed in this light, proof was seen as an important means to come to understand mathematics. In mathematics education research, student proofs no longer had to be formalized or formatted. Indeed, in one of the most famous student proofs in mathematics education (and the most frequently cited paper that demonstrates that young children can prove), Maher and Martino (1996) present Stefanie's proof by cases as a letter that Stefanie wrote to her friend.

The point that I am making is that mathematics educators' answer to the question "should we teach proof" shifted because the meaning of the question shifted. At first, the referent to proof was highly formal arguments. The referent to proof then came to signify good arguments that students raised to defend mathematics claims. We can all agree that we should be designing classrooms which encourage students to make good arguments. This creates a theoretical quandary. How do we specify what constitutes the type of good student argument that we call a proof?

Mathematics educators have long argued that proof fulfills important purposes in mathematicians' practice and that proof can conceivably fulfil these same purposes in the mathematics classroom as well (e.g., deVilliers, 1990). What some mathematics educators have done is shift from having these purposes be a potential desirable by-product of proof to the constitutive characteristic of proof. For instance, Hersh (1993) titled his paper "Proving is convincing and explaining". This goes beyond the widely accepted belief that a good proof has the power to convince and explain. Rather, a proof *is* what convinces and explains. To emphasize his point, Hersh (1993) argued that a proof should be defined as "an argument that convinces qualified judges" (p. 391). Hersh's account here is consistent with his previous assertion that a proof is what convinces a mathematician who knows the subject. It also aligns with other mathematics educators who have said a proof is what would convince a reasonable skeptic (Volminik, 1990), an explanation that convinces a particular community at a particular time (Balacheff, 1987), or as a convincing argument that answers the question 'why?' (Nickerson & Rasmussen, 2009). In Harel and Sowder's (1998) seminal proof schemes paper, they defined a justification as being a proof to an individual is that proof provided absolute certainty that a mathematical claim was true. With this perspective, proof becomes (by definition) how one believes mathematical claims and is inseparable from learning and doing mathematics.

There are good reasons for defining a proof as a justification that fulfils psychological functions, such as providing certainty. A common appeal to incorporating proof into the classroom is that proof can have psychological benefits for students and can help them come to know mathematical claims (e.g., Schoenfeld, 1994). To an elementary or secondary mathematics teacher, these ends are often more important than the means. The teacher would prefer to wrestle with the best way to persuade and enlighten students rather than address the thorny question of whether a messy argument is technically a proof. This framework also has been tremendously generative for the constructivist research program. Pat Thompson (1982) says that when a student provides an incorrect answer to a problem, “the constructivist asks: what is the problem that the student was trying to solve?” (p. 154), with the point being that the student may have succeeded by their own lights. When students are asked to justify a statement and offer a non-normative response, such as an empirical justification or an invalid deductive justification, these justifications may have succeeded in convincing the student that the statement was true. This leads to the fruitful line of research as to what types of justifications do students find convincing, a topic that has made up much of the proof-related literature in mathematics education (Stylianides et al., 2017). Despite its benefits, I will argue that this perspective can have negative consequences in underestimating students’ epistemologies and overstating the importance of proof.

ON STUDENTS’ PRODUCTION OF EMPIRICAL JUSTIFICATIONS

Imagine the following situation. A high school student was asked to justify that the difference between the square of two consecutive natural numbers was the sum of those numbers. The student submits a justification: “36 – 25 = 11. 9 – 4 = 5. So the difference between the squares of two consecutive numbers is the sum of those numbers”. This task, and the student response, are adapted from Recio and Godino’s (2001) paper, in which 429 secondary students completed a version of this task. About 40% gave an empirical justification like the example above. That is, they verified a general claim about consecutive natural numbers by checking it held for a finite collection of pairs of natural numbers. Several large-scale studies (e.g., Healy & Hoyles, 2000) and numerous smaller studies (see Stylianides et al., 2017, for a review) have shown the same result. Elementary and secondary high school students justify general claims with examples. This result is so robust that the Educational Committee of the European Mathematical Society listed this as one of their “solid findings” in mathematics education research (Nebout et al., 2011).

I am convinced that this result is reliable. What I will challenge is the common interpretation of the result. Students who produce empirical justifications are frequently cited as having empirical proof schemes (e.g., Healy & Hoyles, 2000; Recio & Godino, 2001), or at least believing that empirical justifications are more persuasive than proofs. In Weber, Lew, and Mejía-Ramos (2020), my colleagues and I argue that Expectancy Value Theory (Wigfield & Eccles, 2000) offers alternative explanations. Bandura (1997) distinguished between *outcome expectations* (i.e., the belief that a certain behavior will lead to a certain outcome) and *efficacy expectations* (i.e., the belief that one can successfully perform the behavior to achieve the outcome). When students produce empirical justifications, the claim that they have empirical proof schemes is a critique of their outcome expectations. The students think justifying empirically can bestow certainty. However, their empirical justifications may be due to an efficacy explanation. Maybe students doubt they have the capacity to produce a proof and so they settle for an empirical justification. Expectancy Value Theory posits several reasons why students might settle for empirical justifications—they might not care enough about the claim they are justifying to put in the hard work of obtaining certainty, they may think that searching for a proof is too unpleasant, or they might not think they could produce a proof. We elaborate on these possibilities in Weber et al. (2020).

To see if these ideas for expectancy value theory might offer a useful lens for interpreting students' behavior, my colleagues and I conducted a study in a problem-solving course for mathematics teachers that I was instructing. Early in the semester, in four sessions, we had the teachers work collaboratively to solve problems and justify solutions, where the solutions to the problems admitted empirical justifications. At the end of the problem-solving session, we asked each group of teachers to state the answer to the problem and to justify their answer. We then asked each member of the group how confident they were, on a scale of 0 through 100, that their answer was correct. If they gave a score of less than 100, we asked them why they lacked certainty in the claim. All these discussions were audio-recorded. Additional details about this study can be found in Weber et al. (2020).

	Deductive justification	Empirical justification
# of justifications	13	18
Teachers with 100 confidence	27	7
Teachers with <100 confidence	6	32
Avg. confidence	99.4	68.9

Table 1: Types of justifications and teachers' levels of confidence in them

What we found is summarized in Table 1. In terms of the types of justifications that the teachers produced, our results mirrored those in the literature. The groups of teachers collectively produced 31 justifications across the four tasks, over half of which were empirical. However, Table 1 illustrates that the teachers generally did not gain certainty from the empirical justifications that they produced. In most cases, they doubted that their answer was correct. They did, however, usually gain certainty in their answers if they could support them with a deductive justification.

In the third class meeting, the teachers were asked for what odd numbers *n* could they draw a simple, planar, 3-regular graph with diameter 3. One group claimed that this was impossible if *n* was odd and offered an empirical justification: In all the graphs that they tried to draw with the aforementioned properties, they always had one edge emerging from one vertex that could not be linked to another vertex. After providing their justification, the group was asked about their confidence in their claim:

> Dan: I'm 90 percent positive.
>
> Darlene: I feel like 85.

Interviewer: So then why are you not fully certain in your answer?

Dan: In my experience in math, I feel like there's always one strange exception.

Darlene: I would feel 100 percent confident only if I had a proof telling me as to why it doesn't work, so if I worked out all of the steps of a proof to say, okay, this is why, then I'd be more confident. I think we have the basis for why it doesn't work.

Interviewer: [To Dan] Is there something that would convince you that you were 100 percent on your answer?

Dan: I mean, I guess a proof, yeah. A proof, yeah, a proof.

Interviewer: Okay, so let's see, so you guys said that a proof would give you further confidence. Can I ask you why you aren't seeking that evidence?

Darlene: We were still working on diagrams at that point. So I think that would be our next step. Once we finally exhausted all the diagrams that we wanted.

Interviewer: Okay, so you are saying that if you had more time, you would continue to work on this until you had more confidence in this?

Darlene: Absolutely not!

Dan: Yeah. No, no, no.

Darlene: My motivation has gone down. (Weber et al., 2020, p. 16-17. For the sake of brevity, the conversation was lightly abridged).

This excerpt illustrates a common theme from those presented in Weber et al. (2020). Dan and Darlene produced an empirical justification, but they had the right epistemology about proof. Empirical justifications cannot provide certainty (math has strange exceptions) and proofs can. However, after an hour spent on the problem, they lacked the motivation to search for a proof. The *value* they had for seeking certainty in their solution was not sufficiently high for them to continue expending effort.

Overall, there were 18 instances where a group gave an empirical justification for their solution. In two cases, the group members expressed certainty (confidence of 100) in their answer. In two others, group members were not certain in their solution, but could not imagine a way to increase their confidence in the solution. In the other 14 instance, group members said a proof would be needed to increase their confidence. In two of those cases, the group simply ran out of time to continue working on the problem, but said they would seek a proof if more time was available. In six cases, the students responded as Dan and Darlene did; they lacked the motivation to continue searching for a proof. The value of obtaining certainty was not high enough. In the other six cases, the participants did not think they could produce a proof, or they were not even sure what a proof of an impossibility claim might look like. They settled for an empirical justification because they thought the likelihood of obtaining a proof was too low.

In this study, empirical justifications were common, but empirical proof schemes (i.e., the belief that one can obtain certainty from empirical justifications) was rare. What can we conclude from this? We cannot conclude that teachers, or any other population, do not hold empirical proof schemes. The sample in the study was too small and the context was too specific to make any broad generalizations. Rather, the study is a contribution to theory, showing that if an individual produces an empirical justification, this does not imply the individual has a deficient epistemology or holds an empirical proof scheme. The study also illustrates that considerations of Expectancy Value Theory—specifically efficacy expectations, cost, value, and likelihood of success—are useful theoretical tools for making sense of students' proof-oriented behavior that have not been used by mathematics educators to date.

As a broad point, recall that Thompson (1982) said that when students offer non-normative answers, the constructivist asks what problem the student was trying to solve. Those who would attribute empirical proof schemes to students who generate empirical justifications are saying that students are trying (and succeeding) in obtaining certainty in their mathematical claims. I suggest instead that students probably do not have certainty on their mind. Rather, at least some students are balancing obtaining a high degree of confidence in a claim against the cost of seeking stronger evidence to support the claim, which is moderated by their perceived capacity to produce a proof.

ON STUDENTS' UNCERTAINTY AFTER READING A PROOF

Imagine the following situation. A student is shown a proof of the claim that for every natural number n, $n^3 - n$ is divisible by 6. The student reads the proof and confirms that it is correct. The student is asked if it is possible that this claim might be false for a very large n. The student concedes that it is possible. Finally, the student indicates their confidence in the claim would be increased if they checked it for several examples. I adapted this situation from Fischbein (1982), who found that many high school students acted as the hypothetical student described above.

Fischbein's (1982) study is one of many in which students have access to a proof of a claim, yet still exhibit doubt that the claim is true (e.g., Harel & Sowder, 1998). A common interpretation of these students' behavior is that they do not understand the nature of proof. They do not understand the generality of proof or they do not understand that proofs bestow certainty (see Weber, Mejía-Ramos, & Volpe, 2022, for a review of this literature). I think this result is reliable and the interpretation is essentially correct. However, I argue that students' epistemologies are not deficient. Students are behaving rationally and, in fact, like professional mathematicians.

It may seem intuitively obvious that to mathematicians, proofs guarantee that theorems are true and that students *should* gain certainty from the proofs that they read. As Fischbein (1982) put matters, when mathematicians are marshalling support for a claim, "at a certain point, the search stops and a new situation appears: the mathematician has found a complete proof of his solution or theorem. Such a proof is an absolute guarantee of the universal validity of that theorem. *He believes in that validity*" (p. 17). Many contemporary mathematics educators concur. For instance, Harel (2013) wrote that "clearly a mathematician is certain of a result when he or she proved it or read its proof" (p. 124). Once again, Bandura's (1997) distinction between outcome expectations and efficacy expectations is relevant. For the sake of argument, let us suppose that mathematicians have the outcome expectation that if P is a proof of a theorem T and they can verify that P contains no logical flaws, then they can be certain that T is true. This is an outcome expectation about what epistemic outcomes a good proof and a good proof validation process can yield. However, we still must consider the efficacy expectation. Mathematicians (and students) may still doubt that the proven theorem T is true because they doubt their capacity to check the correctness of P. In Weber et al. (2022), my colleagues and I tested whether this might occur.

We asked 16 mathematicians to participate in a task-based interview. The interviewer first showed them the claim:

$$\sum_{k=0}^{\infty}\left(\frac{-1}{4}\right)^k\left(\frac{2}{4k+1}+\frac{2}{4k+2}+\frac{1}{4k+3}\right)=\pi$$

Participants were asked to rate how confident they were that the claim was true with a score between 0 and 100, where 0 meant they were “absolutely certain” that the claim was false, a 100 meant they were “absolutely certain” that the claim was true, and a 50 meant the claim was as likely as not to be true. Next, participants were shown a correct proof that this claim was true. The proof was computationally complex, but relatively short (one page long) and only used techniques from undergraduate calculus. Participants were given up to 40 minutes to study this proof, and could have more time if they wanted it. Again, they were asked to give a confidence score between 0 and 100 about the truth of the claim. Next participants were shown empirical support that the claim was true in the form of a Microsoft Excel document which demonstrated that the partial sums of the sequence rapidly converged to π. Again, participants were asked to give a confidence rating of the claim. Finally, participants were shown a paper of Adamchik and Wagon (1996) in the *American Mathematical Monthly* which proved that the claim was true, and they were again asked to rate their confidence in the claim. After all the judgments were made, participants were asked to explain their doubt or certainty at each phase of the study. This study was designed to be analogous to Fischbein’s (1982) study with students, where we saw if participants would be certain that a claim was true after reading a relatively short proof of the claim or if further empirical evidence might increase their confidence in the claim. Further details about the methodology are presented in Weber et al. (2022).

As Table 2 indicates, after receiving the claim, no participant was certain that the claim was true. Indeed, no participant gave a confidence rating higher than 70. Collectively, participants’ confidence in the claim increased substantially after studying its proof, but only two participants said they were certain in the claim. Of the remaining 14 participants, 12 stated they were not certain that the theorem was true because they were not sure the proof was correct. This shows how mathematicians may doubt a theorem, even if they were given ample time to study a short proof of the theorem.

Study Phase	Average confidence	Participants with certainty	Participants with Increased confidence
Claim	44.7	0	N/A
Proof	82.4	2	15
Empirical	95.3	6	12
Publication	98.7	8	9

Table 2: Summary of participants' evaluations at each phase in the study

After inspecting the empirical evidence, 12 participants increased their confidence in the claim. Four participants who doubted the claim after reading the proof now indicated that they were certain in the claim. This illustrates how mathematicians may use empirical evidence to increase their confidence in a claim, even after studying a correct proof of the claim. Interestingly, many participants indicated that the proof and the empirical justification worked powerfully in tandem to increase their confidence in the claim. For instance, one participant said that after reading the proof, he was convinced that the summation converged to something involving π, but he had a slight doubt that he missed an algebraic error and the partial sum converged to something other than π itself: "The answer could have been π over 2. The answer could have been negative π". When he saw the empirical evidence, he said the summation "sure looks like it equals π". The only thing involving π that was that close to π was π itself, so the participant said he was absolutely certain that the summation converged to π. Finally, after seeing that this result was published in a respected journal, 9 of the 10 participants who were not yet certain that the claim was true increased their confidence. Two raised their confidence to the point of certainty.

For the sake of brevity, I will describe one other part of the study. We asked participants whether they would be certain that a conjecture was true if their colleague presented them with a proof of the conjecture, they read the proof, and could find no error in the proof; 15 of the 16 participants said they would not be certain. We then asked if they would check the claim with examples after reading their colleagues proof; 15 participants said they would. Many were quite emphatic, saying things like, "Of course. That's precisely what I will do", "Yes, definitely", and "Sure. That's usually what mathematicians do". This suggests our earlier findings were not a consequence of the artificiality of our study, but indicative of mathematicians' actual practice. Further details and excerpts can be found in Weber et al. (2022).

What do these results tell us? First, I find it remarkable that half of the participants were certain that the claim is true by the end of the study. This claim is not at all obvious, and on the surface, there is no reason to believe it. Mathematics really is a special discipline in which its practitioners can gain certainty in complex claims. However, the data paints a nuanced picture into how certainty was obtained. For most participants, the proof alone was not enough to be certain that a claim was true. Participants were aware that they were fallible and may have overlooked the flaw in the proof. It was only when the proof was accompanied by empirical evidence, and in some cases, knowing that the result appeared in a respectable journal, that participants were certain. I argue that these results say we should not judge students who retain doubt in a theorem after reading its proof or who desire to check a proven theorem with examples as having deficient epistemologies. This is what mathematicians do as well. These results highlight a problem of equating proof with certainty. It is difficult for both mathematicians and students to be certain that a proof that they have read or produced does not contain an error. I concur with Doyle et al.'s description of the relationship between proof and certainty:

> The simplest characterization of the purpose of proof writing is that we write a proof in order to place the truth of a proposition—the theorem—beyond doubt. The sort of doubt in question is not actual psychological doubt. It is easy to see that no proof can defeat all lingering psychological doubts. For example, serious doubt in one's logical ability or memory will not be allayed by rehearsing a gap-free, absolutely correct proof. The kind of doubt ruled out by a proof is something more abstract than actual psychological doubt. While rehearsing and understanding a correct proof may drive away doubts and produce psychological certainty in many cases, when it doesn't it isn't a failure of the proof, but a failure of the mind surveying it (paragraph 4; italics were my emphasis).

It is one thing to say that proof is a tool that mathematicians can use to obtain certainty in claims, and perhaps even that proof is a necessary tool to do so. However, we must emphasize that extracting certainty via a proof is a difficult and imperfect process. When mathematics educators decide to equate proof with certainty, we are ignoring the complexity of obtaining certainty from proof, and problematizing students' behaviors and epistemologies when they may be simply acknowledging their fallibility.

HOW SHOULD WE THINK ABOUT PROOF?

The benefits of proof in mathematical practice

Consider the following puzzle, adapted from Fallis (2002). Suppose that I want to know if a specific large number p is prime. One mathematician uses a Miller-Rabin primality test. If p were prime, it would always pass this test. If p were composite, it would pass the test less than one in a trillion times. The mathematician's justification J is that p passed this test so it overwhelmingly likely to be prime. For proof P, a mathematician checked by hand that every natural number n does not divide p. Here we follow Fallis in asking why mathematicians (or mathematics educators) would prefer P to J. It is true that J cannot provide absolute certainty that p is prime, but neither can P. It seems foolish to overlook the possibility that a mathematician made an error in performing so many long division calculations by hand. Both J and P can provide mathematicians with a high degree of confidence that p is prime. While good proofs can provide insight or explanation or otherwise make us wiser, the routine proof by exhaustion P does none of those things. I think this also shows the challenge of defining a proof by the psychological function that it fulfills. What possible psychological function can P fulfill that J does not?

Fallis (2002) argued that there is no epistemic reason to prefer proofs to probabilistic proofs, such as proof of primality using the Miller-Rabin primality test. In Czocher, Dawkins, and Weber (in press), my colleagues and I argued that we can make a case for mathematicians' norm of only sanctioning theorems by proofs with a better understanding of how norms operate in a practice. In our account, which borrows heavily from Herbst et al. (2011), mathematical practice has values. Mathematicians want their practice to have reliability (i.e., their established results are true), consensus (i.e., members agree on what statements are established and which questions remain open), autonomy (i.e., any member can in principle verify established statements on their own), permanence (i.e., established results "stay true"), and fruitful (i.e., the practice makes study progress in answering their questions and forming new ones). Asking mathematicians to be constantly mindful of these values would burden them and slow down their work; it would also lead to dissension in the practice about when a result is really proven or about the best way to balance these values which can sometimes be competing. To manage this, a practice establishes norms to uphold these values. Norms are guidelines that will increase the extent that values are obtained, but are clear enough that mathematicians can determine if they are followed. For instance, a theorem becomes established when its proof is published in a respectable journal. This upholds the values of reliability (i.e., the paper is peer-reviewed so errors might be noted) and permanence (i.e., the journal is a static public record) while being straightforward so mathematicians need not concern themselves with whether the proof has been checked carefully enough. Of course, these results do not guarantee reliability and permanence, only increase the extent that these values are upheld.

Our argument acknowledges Fallis' (2002) point that probabilistic justifications are occasionally epistemically on par with some proofs, but states this is not why proof is beneficial to mathematical practice. The norm in mathematical practice to insist on proof to establish theorems does not benefit individual mathematicians in specific instances. Rather, norms benefit the practice as a whole on aggregate, and one of their virtues is providing clear guidelines, which is vital for obtaining consensus within a practice. There are a wide range of probabilistic proof methods available. It may well be the case that the Miller-Rabin primality test functions well enough to ensure a high degree of reliability (e.g., computers have random generators that produce random natural numbers less than p in a "random enough" fashion). However, there are other methods, such as using many strands of DNA to perform Monte Carlo tests (e.g., Fallis, 1996), whose reliability is more suspect. How do mathematicians decide which probabilistic methods are permissible?

Further, proofs often generate new concepts and methods, which add to the fruitfulness of mathematics, while probabilistic proofs never do. Indeed, in my prior work, I have found that learning new methods is the primary reason that mathematicians read each others' proofs (Mejía-Ramos & Weber, 2014; Weber & Mejía-Ramos, 2011). Insisting on a proof will not guarantee that new concepts or methods are produced. Proofs are not always generative in this way; the proof by exhaustion P that I described above was not. But many are. For exceptionally large numbers, proofs by exhaustion will take too long to carry out, so mathematicians will search for more efficient ways to establish primality. And sometimes they will succeed. The norm of insisting on proof, and not allowing probabilistic proofs, does not guarantee that mathematicians will find new methods, but it does encourage behaviors by which the emergence of new methods is common.

The benefits of proof in mathematics classrooms

Imagine the following classroom situation. Advanced university mathematics students are shown the diagram in Figure 1, which is defined as a square where there is a segment from each corner of the square to a midpoint of the opposite side. They are asked to find the ratio of the area of the larger square to the smaller middle square. One student models the situation on Geometer's Sketchpad to show the ratio is 5:1. Another student plots a unit square on a Cartesian plane, models the situation algebraically, and proves that the area is 5:1. Assuming that the students were all familiar with proof by "algebratizing" geometric situations, what psychological benefits does the proof offer that the Geometer's Sketchpad situation does not? Personally, I have more faith in the Sketchpad argument. I know from experience how easy it is to make an algebraic error! Neither justification offers much by the way of insight or value.

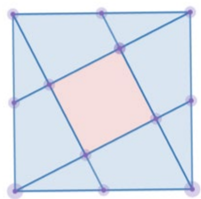

Figure 1: The geometry problem being solved

I think the benefits of insisting on proof simply are not realized for individual students with this particular proof. However, the norm of encouraging proof will benefit a geometry class as a whole. For instance, modelling this particular situation with Sketchpad is straightforward and I would have confidence that my students can do so successfully. However, in general, the faithfulness of a Sketchpad model is not always obvious. There are situations when one might inadvertently 'overconstruct' or 'draw' their geometric models, or rely on Sketchpad's empirical measurements when it is problematic to do so. When one should use Sketchpad is a gray area, and there is no principled way to settle debates on when this is appropriate. In contrast, with proofs, we can ask where assumptions come from or why deductions are valid. If a proof is wrong, we can pinpoint the place where a mistake was made. Proofs therefore are better in aggregate at inducing classroom consensus and they establish a format for debate.

There are benefits that proofs (and the search for proof) can bestow upon a classroom. In mathematics education research, there are well-rehearsed arguments that proofs can provide explanation or introduce new techniques for problem-solving or justifying. Once "proof by algebratization" is well-known to students, the proof described above does not offer these types of insights. It is a "brute force" justification. But a norm insisting upon proof offers students the encouragement to search for, and find, proofs, some of which will offer these benefits. As a final point, in a practice, norms can be broken in a mathematical practice, provided that the transgresser acknowledges the breach and can justify it. I would not want an insistence on proof to be a straightjacket for teachers and students. If there are classroom situations in which a proof would not be convincing (e.g., it was too computationally complex), or it offered no insights (e.g., it was brute force), or a heuristic argument offered more insight, then proof is not necessary. The teacher can simply acknowledge that proof is not sought or presented in this situation and explain how this omission relates to upholding mathematical values.

Classroom proof in terms of norms, values, and obligations

I have sympathy for those who define a proof as that which convinces a student or a mathematician. In Czocher and Weber (2020), my colleague and I contended that there is no single attribute that all proofs share. Those who define a proof as a justification having a specific feature (such as bestowing certainty) are in essence making a value judgment in elevating what they think is most important about proof. It makes sense that many mathematics educators would conceptualize proof as fulfilling psychological ends, as this is to some what teaching is all about. There is a danger in taking this too literally though. We overstate proof's importance. By ignoring fallibility in checking proof, we grant proof the capacity of providing certainty that it cannot possess. By ignoring Bandura's (1997) efficacy expectations, we attribute deficient epistemologies to students when they may simply be acting upon rational beliefs about their capacity to produce or validate proofs. We also ignore the necessity of empirical evidence, as well as social and institutional processes, that mathematicians and students need to obtain certainty (Weber et al., 2022).

One alternative to defining proof as a justification that fulfills a psychological function is to define proofs in terms of values and norms (Dawkins & Weber, 2017). My perspective is to not view a proof as a static object with particular attributes. Rather, a proof is an object that involves obligations on those who advance it. The object itself should rely on true assumptions shared by the classroom community and only employ deductive reasoning (c.f., Stylianides, 2007) and the proof should be written in a way to emphasize clarity. After the proof is advanced, the prover still has the obligation to state why assumptions should be regarded as shared by the classroom community, justify why particular deductions are valid, and explain why any aspects of the proof are unclear to the satisfaction of the classroom community. Finally, the prover has the obligation to concede their proof is incorrect if a counterexample to one of their steps is shown and the proof is incomplete if they cannot justify one of its steps. A proof is a key artifact in the broader activity of proving, which consists of a web of obligations. Here we can see that proof and its surrounding activities have the potential for providing conviction, insight, and consensus for students. Proof is not divorced from its epistemic functions or it psychological benefits. But proof is not equated with them, and students who fail to produce complete proofs or who cannot gain certainty or insight from a proof that they witness are not viewed as epistemically deficient.

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AOTEAROA NEW ZEALAND MATHEMATICS EDUCATION RESEARCH AND CURRICULUM DEVELOPMENT

Jodie Hunter[1], Naomi Ingram[2], Generosa Leach[1], Tony Trinick[3], and Sze Looi Chin[1]

[1]Massey University, [2]University of Otago, [3]University of Auckland

This national presentation will focus on mathematics education research and curriculum development related to Aotearoa New Zealand. We recognise that there are many countries that share similar histories of colonisation. We begin by highlighting that the educational systems in Aotearoa New Zealand have been heavily influenced by colonisation with a resulting negative impact on both Indigenous Māori and Pacific peoples in relation to mathematics teaching and learning (Allen & Trinick, 2021; Hunter & Hunter, 2018; Trinick & Heaton, 2021). With the underpinning of centering indigenous knowledge and developing social justice and equitable mathematics classrooms, the presentation will provide an overview of policy, curriculum changes, initiatives, and research projects that have transformative potential.

During the first part of the presentation, we will provide background information in relation to schooling structures and mathematics education including both historical context, successes, and ongoing challenges. This part of the session will include an overview of the changes in curriculum and policy development over the past 30 years from 1993 until present times. We will then shift focus to examine Māori and Pacific initiatives in mathematics education. This part of the presentation will focus on both policy development and research studies which have centred Māori or Pacific knowledge, language, and ways of being in relation to mathematics education. We will finish with our concluding remarks to summarise the key-points of the session.

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PLENARY PANEL

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CAN WE DRAW ON THE DIVERSE BODY OF KNOWLEDGE OF MATHEMATICS AND, AT THE SAME TIME, DEVELOP THE MATHEMATICS CURRICULUM?

Vilma Mesa[1], Núria Planas[2], Armando Solares-Rojas[3], Tony Trinick[4] and Stefan Ufer[5]

[1]University of Michigan [2]Universitat Autònoma de Barcelona [3]Cinvestav
[4]University of Auckland [5]LMU Munich

INTRODUCTION (ARMANDO SOLARES-ROJAS)

Research on mathematics education has widely documented the importance of recognizing and including diversity of mathematical knowledge in school practices (Asher, 1991; Bishop, 1991; Civil, 2002; D'Ambrosio, 2018; Hoyles, 2010). However, non-canonical mathematical knowledge is rarely used in mathematics classrooms. The discussion offered by this Plenary Panel addresses this issue. Departing from holistic and inclusive approaches that recognize the richness and validity of non-canonical mathematical knowledge to formulate (alternative) school mathematical curricula, we discuss what mathematical knowledge, canonical and non-canonical should students be exposed to, and whether these can be integrated into the school mathematics curriculum

Besides the research that has highlighted the dissimilarities, contradictions, and difficulties in connecting non-canonical mathematical knowledge with school mathematics curriculum (e.g., Lave, 2011; Nunes et al., 1993), many studies have developed and implemented educational proposals to integrate the mathematical knowledge of culturally diverse human groups into the into school educational practices (e.g., Kisker et al., 2012; Knijnik, 2003; Trinick et al., 2017). However, the integration of non-canonical mathematical knowledge is not straightforward. It is often submerged in a complex network of continuities and discontinuities between the activities and knowledge of the cultural groups and those specific to educational institutions, posing challenges such as resistance from traditional curriculum structures and the need for teacher training in these alternative mathematics education proposals.

With these considerations in mind, we have rephrased the original topic question for this plenary panel to: *To what extent should we draw on non-canonical as well as canonical knowledge of mathematics when developing the school mathematics curriculum?* This new formulation allows us to respond to the requested Oxford-style debate for the panel's discussion. We will start the session by framing the topic; then, each panelist will state a brief position regarding the adjusted question. Once the first positions are stated, the chair will mediate a conversation among the panellists.

Some clarifications on the conceptualization of curriculum and (non-) canonical mathematics are needed to understand the panelists' positions better. Firstly, we understand *developing a curriculum* as the process of deciding on the goals of

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mathematics instruction, comprising concepts to be treated, practices to be introduced, and examples and artefacts used to illustrate and engage with these. Secondly, in this paper, we call *canonical mathematics* the kind of mathematics inscribed in journals and book volumes from the international mainstream mathematics research community over a long period (such as Euclid's classical books or the Bourbaki series). Finally, we call *non-canonical mathematics* those that we do not have in textbooks or national curriculum guides; this understanding includes, but is not limited to, ethnomathematics.

In the following sections of this paper, each panelist delves into their respective positions on this topic from different angles and theoretical perspectives.

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WHICH KNOWLEDGE IN SCHOOL MATHEMATICS CURRICULUM? JUST THIS QUESTION MAY NOT BE ENOUGH (NÚRIA PLANAS)

In this section, I will reflect on some of my experiences with the school mathematics curriculum, from a secondary-school mathematics teacher' perspective and as a university researcher in mathematics education. I will aim at coordinating the personal voices of the teacher and the researcher throughout my writing, as well as developing a conversation between these two to support me in making my position on today's debate. My position is that we should draw on non-canonical knowledge of mathematics, when developing the school mathematics curriculum. I have two main arguments for my position: (1) knowledge of mathematics is not just canonical knowledge of mathematics, and (2) mathematics learning is hindered by historical limitations to the knowledge of mathematics that can be experienced in the school curriculum. Equity issues are not less importantly present across these arguments.

Let us begin my positioning by recalling my years as a mathematics teacher in a secondary school of Barcelona. In the teachers' room we used to talk about the "mountains of knowledge" in the mathematics curriculum. The question of whose knowledge was also in focus. This is not surprising because the school was highly multicultural and multilingual, and it had been labelled in the institutional system as a 'centre of preferential educational attention' due to the percentage of learners from families in conditions of poverty. I remember finding it very challenging to co-create and implement, with other teachers, everyday word problems for teaching algebraic equations, spatial isometries, arithmetic proportionality, etc., that were representative of home and out-of-school knowledge (see e.g., Planas & Civil, 2009). Different lesson experiences, however, raised teaching challenges that went beyond everyday word problems, showing other sides of the complexity of the curriculum.

In my context, we have prescriptive curricular guidelines and influential mathematics textbooks, including illustrations of tasks and definitions. For example, proportions are defined as statements of equality of two ratios. This used to be my definition in the introductory lessons on proportionality reasoning. I then used to write the expression $a/b=c/d$ and read the symbols, "*a* to *b* equals *c* to *d*". Still today, the set of guidelines and the majority of textbooks do not emphasize languages of comparison when specific instances of comparison lead to equality. Hence proportions do not tend to be defined as, for example, the comparison of two ratios that are equal. Despite the existing emphases and their prescriptive nature, it is fascinating how some learners make meaning of proportions through languages of comparison (e.g., it-is-as, it-is-as-if, it-is-like), of one ratio to another, rather than through direct examination of the equality relationship. These learners notice that the ratios can be compared, not only the magnitudes from each ratio. They would say, "We can see one to three as two to six," "Two is twice as much as one and six is twice as much as three," "One to three works as two to six," or "If you compare one with two, then you compare three with six."

It was particularly fascinating how a learner in one of my classrooms, Carlos, drew on languages of comparison to make meaning of proportions. His story is about mathematics knowledge that is intrinsically present in the out-of-school practices and cultures, and that is distinct from the "mountains of knowledge" in the school mathematics curriculum. Carlos participated in mathematical cultures of orientation and calculation in connection with his working practices of handing out and charging for butane gas tanks in the neighbourhood (for an interesting discussion of 'street mathematics knowledge', that is, the knowledge of children who work on the street from poor backgrounds, see Vithal, 2003). Carlos' story is also about the politics around the questions of whose knowledge counts, why, and what may it all mean for the diversity of learners and their mathematics learning. Variations of these questions keep puzzling me in my research field work in mathematics classrooms, which started in the form of action research on my own school teaching. By that time, I recorded the lessons using an old video camera with cassettes. Long after that action research, I could recover the content of most cassettes and convert it into digital format. In preparation for this Panel, I spent time watching the videos from Carlos' classroom. In one lesson, Carlos used languages of comparison to talk proportions in a conversation with me, the teacher. On the board, the symbols written were *2/3=4/6*, and I had asked for a volunteer to read aloud the expression. The lesson included the following moment of Catalan-Spanish bilingual talk (my translation in brackets):

Carlos: Dos a tres es como cuatro a seis.

[Two to three is like four to six.]

Núria: Esperava potser dos terços, quatre sisens. Molt bé, Carlos, no són fraccions. Dos a tres i quatre a sis. Només que….

[I was expecting perhaps two thirds, four sixths. Very good, Carlos, they are not fractions. Two to three and four to six. Only that…]

Carlos: ¿Qué?

[What?]

Núria: Dos a tres és igual que quatre a sis. No s'assemblen, són el mateix. Si u és dos, llavors tres és sis.

[Two to three equals four to six. They do not look alike; they are the same. If one is two, then three is six.]

Carlos: Pero un butano no es dos, uno y dos se parecen porque uno se compara con tres y dos... Puedo comparar el dinero que gano si planeo seis butanos.

[But one butane {tank} is not two, one and two are alike because one is compared to three and two... I can compare the money I make if I plan for six butanes{tanks}.]

Carlos was a newly arrival learner in Barcelona from rural Peru, whose home languages were Quechua and Peruvian Spanish, and who was in the process of learning Catalan, the language of instruction. He was meeting the psychologist in the school once per week because he engaged in some repetitive behaviours and insisted on using earplugs when working in small groups. Later in the year, a formal diagnosis of Carlos' autism arrived. The valorisation of Carlos' 'street mathematical knowledge', and the opportunities of engagement with and learning from this knowledge, were therefore complicated in many respects. Either way, this learner's ways of knowing and thinking mathematically clashed with the curricular knowledge and meanings in my teaching.

Some groups and their knowledge have been historically and remain currently marginalised in the school mathematics curriculum, in the sense of being under-represented. The extremely heterogeneous group of autistic multilinguals intersected with conditions of poverty and rurality is one of these. Research, with the help of data and studies across continents (Abtahi & Planas, 2024), indicates that the phenomenon is not local. School institutions do not seem to value in the same manner mathematical knowledge and meanings expressed by people from some groups in society. Issues of ability status, language, social class, race, culture, gender, rurality, and more, are pervasively tied to experiences of marginalisation in more or less camouflaged ways.

Despite the importance of de-camouflaging marginalisation and disadvantage, I have chosen Carlos' story to comment on the optimistic notion of *mathematics learning advantage*, whereby all learners and teachers of mathematics gain opportunities of thinking and learning mathematics by interacting with learners who think and learn mathematics differently. Regardless of which group we feel we belong to, or we are socially placed in, a *mathematics learning advantage* emerges with the opportunities of learning mathematics in educational contexts that are supported but not constrained by the school mathematics curriculum and canonical forms of knowledge in mathematics. In Carlos' classroom, a *mathematics learning advantage* emerged for all for all learners and the teacher with the communication of meanings for proportions in relation to languages of comparison, and prices and quantities of butane tanks.

My overall argument is, however, optimistic, because it assumes that the complexity of the curriculum is an opportunity. Complexity here means that curriculum is complicated to understand without considering its impact on learning. The learning of canonical topics from the school mathematics curriculum can benefit from knowledge and meanings that are not institutionally intended and valued, or at least not valued enough to be named just mathematical. Taking Carlos' story and proportions, for example, the situated mathematical understanding of languages of equality and symbolic expressions can benefit from the interaction and contrast with languages of comparison and meanings developed in a diversity of contexts.

There is still much that remains to be done, but the community of mathematics educators, teachers and researchers is incredibly active in rethinking mathematics education together in directions that consider diversities of cultures and ways of being, learning and knowing mathematics. When I started my teaching career, Bishop (1994) had been published, with the levels of the intended curriculum (what we wish to teach), the implemented curriculum (what we actually teach), and the attained curriculum (what is learned by students). Thirty years later, the level of the *widened curriculum*, for what we can teach and learn more, seems very timely. A *widened curriculum* and the related *mathematics learning advantages* could be enacted for all learners under less restrictive interpretations of the school mathematics curriculum and of the diverse knowledge in mathematics. This leads to one last question: How to prepare mathematics educators, teachers, and researchers who see and value more than a very small part of the mathematics knowledge in the world and society?

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TO WHICH EXTENT SHOULD WE DRAW ON NON-CANONICAL AS WELL AS CANONICAL KNOWLEDGE OF MATHEMATICS, WHEN DEVELOPING THE SCHOOL MATHEMATICS CURRICULUM? (TONY TRINICK)

First, I want to thank my colleagues in this debate who agreed that I could take the position advocating for the integration of non-canonical knowledge of mathematics in school curricula, particularly in the context of marginalized Indigenous groups. Having supported this view for over 30 years, I would look rather silly taking a contrary position in front of the Indigenous community I represent. The perils of misinformation on social media are well-known; I could wake up tomorrow disowned by the Māori-medium community.

That is not to say my position should not be challenged—it should. This sort of engagement with international peers can support continuous improvement and cross-cultural understanding, enhancing the overall effectiveness of mathematics education for communities such as mine. One key issue for groups such as mine however, is who gets to decide the nature of the mathematics curriculum. The Indigenous group or the coloniser? Why does it matter? We are frequently told the canonical knowledge of mathematics will serve all citizens!

In the few minutes that I have I want to unpack why the integration and/or addition of non-canonical knowledge of mathematics in school curricula is important. While indigenous communities worldwide exhibit a great deal of diversity, the groups I am concerned with shared histories of colonisation, dispossession, loss of language, and marginalization by external powers. With the efforts to erase or devalue our languages, cultures, traditions, and practices, this shared history has lasting effects on cultural identity, language revitalisation efforts and overall community well-being. In this country, British colonisers have used canonical mathematics and mathematics education as instruments of assimilation.

Educators believed culture was irrelevant to mathematics content claiming numeracy, was a universal construct that transcended variations in languages and cultural groups (Gerdes (1994). However, several researchers (Owens, et al., 2011) show the way people teach, understand, and use mathematics concepts in daily life is highly dependent on culture. So much so, in fact, that mathematics education has the power to either liberate, colonise, or further marginalise non-dominant culture groups (Owens, et al., 2011; Rodriguez, 2013).

Think about why British colonisers ensured only canonical mathematics was taught to indigenous peoples—it wasn't just about teaching numbers and formulas. For example, when I think about my grandmother and those before her who barely spoke English or did not speak English at all, it becomes clear that the British used mathematics as a way to assimilate us into their culture. It wasn't just about mathematics; it was about control—economic control, administrative control, and assimilating us into their beliefs. By imposing their mathematics, they were reinforcing their power over us, making sure we fit into their system, erasing our traditional ways of knowing. And this isn't just a story for Indigenous peoples. It's the same for many marginalised groups—they use education to control and change us, to fit their ideas of what's right and normal (see Orey & Rosa, 2019).

I've been in curriculum development for over two decades now, and I've seen firsthand how useful Western mathematics can be—we have drawn on it to develop our Indigenous school curricula. But here's the thing—I believe in something more. I'm talking about integrating Indigenous and marginalised mathematical knowledge alongside the canonical. It's not simple; it comes with big challenges. It takes research, resources, and a commitment to change.

But why bother, you might ask?

Think about it this way: by integrating diverse mathematical perspectives into our curriculum, we're not just teaching mainstream mathematics. We're validating cultures that have been marginalized for far too long. We're showing our students that their ways of understanding and solving problems matter just as much as anyone else's.

Imagine a curriculum where an Indigenous student sees their ancestors' ways of mathematics honoured and included. That's empowering. It strengthens their identity and their connection to learning. And it's not just about Indigenous students—this approach benefits all students by broadening their understanding of mathematics and its applications in different contexts.

When we think about mathematics education, especially in places like New Zealand, we have to consider its impact on the social, cultural, health, and overall well-being of students, teachers, and their communities. Despite indigenous groups learning mainstream mathematics, they often still face significant disparities in health, education, and wealth compared to the dominant culture. This highlights a broader issue where mainstream educational systems don't fully address the needs and strengths of marginalized groups.

Many indigenous communities have their own rich mathematical traditions that aren't always recognized in Western educational settings. These traditions hold deep cultural significance, and incorporating them into the curriculum can validate and honour these cultures (Meaney, Trinick & Allen, 2021). When students see their own cultural practices reflected in what they learn, it fosters a stronger connection to education and a sense of pride in their identity (Trinick & Allen, 2024).

For marginalized and indigenous students, learning about their own mathematical heritage can be incredibly empowering. It challenges stereotypes that their cultures are somehow less advanced mathematically and helps build confidence in their academic abilities. Involving the community and elders in the educational process enriches this experience even more. Elders often hold invaluable knowledge that bridges the gap between traditional wisdom and formal education, making learning relevant and meaningful for younger generations.

By recognizing and integrating diverse cultural perspectives into education, we not only make the curriculum more inclusive but also create pathways for better educational outcomes and overall well-being within these communities. It's about acknowledging the strengths that each culture brings to mathematics and education as a whole, ensuring that all students feel valued and respected in their learning journey.

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NI CHICHA NI LIMONÁ (NEITHER CHICHA NOR LEMONADE): THE TROUBLING EFFORTS OF INCLUDING NON-CANONICAL MATHEMATICAL KNOWLEDGE IN THE SCHOOL CURRICULUM (VILMA MESA)

The question guiding our discussion today is "To which extent should we draw on non-canonical as well as canonical knowledge of mathematics, when developing the school mathematics curriculum?" I agreed to take an oppositional stance, to promote debate and explore the values that guide efforts that seek to add non-canonical mathematical knowledge into the school curriculum. My position, as a plea for not engaging in the inclusion of non-canonical mathematical knowledge, is predicated in the need to respect and protect the integrity of ways of knowing of the communities in which the knowledge emerges. By ways of knowing I refer to the epistemology, ontology, methodology, and axiology embedded in what we call "non-canonical" mathematical knowledge. My comments are about the axiology. During the panel discussion, the other three aspects will be addressed.

Mathematics education has had its share of developments that seek add on to the canonical mathematics other types of "mathematics." Under the aegis of Ethnomathematics (D'Ambrosio, 1989), substantial work has been done to identify ways in which the canonical school mathematics curriculum systematically ignores and erases contribution by non-Greek, and non-European mathematicians and thinkers (see Joseph, 1991 for one of the earliest critiques of this process.). Such erasure is said to be detrimental for students' identity formation and motivation to pursue mathematics, which in turn curtails their options for a better life.

The canonical school mathematics embedded in our curriculum is predicated on certain ideas of what constitutes the good life and what we should all aspire to have (or be). We can examine the values (axiology) that support such curriculum:

1. The mathematics that is taught in schools is a powerful tool for modeling real world situations and solving real world problems.
2. Mastering the canonical mathematics curriculum makes it possible for students to access a better life—in the form of being able to choose professions that are better paid paving the way to have more fulfilling lives.
3. All students should experience the canonical mathematical knowledge.
4. All students can master the canonical mathematical knowledge.

But do we know what is non-canonical mathematical knowledge? As we look closely at disenfranchised communities or at communities who may—at some point in time—have figured out a harmonious ways of being in the world, we do so through the lens of our canonical mathematics, via a mathematizing gaze (Dowling, 1998). The core central mathematical practices that Alan Bishop (1988) identified—counting, measuring, locating, designing, playing, and explaining—were noticed because he read their activity with a canonical mathematical lens. This mathematizing gaze pervades every effort that we make to augment the curriculum with the lived experiences students have. Canonical mathematics is used to uplift the practices of these communities, be it in nursing, carpentry, construction, baking, purchasing materials for building bookcases, selling candy, playing basketball or dominoes (Diego-Mantecón et al., 2021; Hoyles et al., 2001; Nasir & Hand, 2008; Saxe, 1988). We 'discover' the mathematical ideas supposedly 'frozen' in the practices (Gerdes, 1997). Once the canonical mathematical processes are identified in the non-canonical spaces, then they become interesting and, moreover, worthy of inclusion into the school mathematics curriculum.

There is symbolic violence, in my view, when we seek to imbue what is done in other communities with our own mathematical perspectives; while we may see ourselves as learners, our ulterior motive is to see how we can identify the underlying canonical mathematical activity; even when we learn the language in which the knowledge is produced, we are still forcing the communities' perspectives, practices, and world views into one perspective governed by our canonical mathematical curriculum. An authentic and respectful use of non-canonical mathematics would require substantial time and effort to learn and uphold the axiology that drives that knowledge, without trying to erase it by subsuming it into our own axiology, and without efforts to translate them into the canonical mathematics vocabulary.

I do not find it justifiable to spend time and public or private funding to wrap these communities' practices and concepts under the canonical mathematical mantle to make them amenable to be incorporated into the school mathematics curriculum! This is a travesty. It is another way for us to support a research agenda that benefits us, the community of researchers. In 1992 George Stanic and Jeremy Kilpatrick, admonished, that mathematics education research as a field evolved thanks to many curriculum development efforts, and that a major failing of those efforts was that researchers never understood that questions driving curriculum development (what should we teach and who should access to that knowledge) are not a technical but "fundamentally moral and ethical questions" (p. 415). Any reformulations of curriculum will fail because researchers ignore the axiology that drives them.

Our axiology—what we value—dictates what is important. An axiology can vary within countries and communities and determines what knowledge is valuable and the nature of the objects that make our world. An axiology will also determine what is considered appropriate for identifying and corroborating what new objects and knowledge are valuable. What we value as a good life (access to technology, financial security through a respectable job) requires that we continue teaching the canonical mathematics; what we value as a good life justifies efforts for writing fundable research; what we value as a good life defines also who should have access to it; today, we say, those who master the canonical mathematics will have such access. Attempting to bring other ways of knowledge will fail as understanding the community's axiology, namely what the community considers is a good life, will eventually be subjugated to the world that canonical mathematics has created. We can't avoid seeing their world with a mathematical gaze.

My position is that the *chicha*, the canonical mathematics should be taught in its own terms. And that the *limoná*, the non-canonical mathematics—whatever that is—should be left alone too, taught maintaining its whole integrity, respecting its origins and purposes rather than trying to disguise with a canonical mathematics costume.

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DES IS KA G'MAHDE WIES'N: WHY INCLUDING NON-CANONICAL KNOWLEDGE OF MATHEMATICS IS NOT NECESSARILY A READILY PAVED ROAD (STEFAN UFER)

Drawing on "non-canonical knowledge" in the mathematics curriculum? How could we, valuing diversity, inclusivity, and equity among the many different groups in our societies and cultures, not agree with this? My main point is not to discourage the idea, which is particularly important. When I agreed to enter the panel, the topic was a quite different one. Now, I am faced with a topic for which I am not confident I am a good person to talk about, for obvious reasons. However, I learned it is appropriate, when being asked, to consider the question and contribute as appropriately as I can.

As said, I consider including non-canonical knowledge in the curriculum a very valuable idea. When I was confronted with the (new) topic, however, I thought, *Des is ka g'mahde Wies'n!* (Bavarian for "This is not a mowed meadow."). For farmers in the European Alps, where mowing a meadow (to have feed for the cattle in winter) on steep mountain slopes in the summer sun, often manually with a scythe, is still very hard work even today. Where I come from, the metaphor *Des is a g'mahde Wies'n* (mostly before approaching a job) means *this is as good as already done*. But beware: The *k* in the *ka* from the title implies a negation. Including "non-canonical knowledge of mathematics" into the curriculum draws up important and sometimes difficult practical questions that need to be answered when realizing the idea. Highlighting some of these questions, I intend to contribute to a productive but still conservative struggle to forward the good intentions behind this idea. Before we start, note that the question "to which extent…" posed to the panels is an *if* question: Should we (more or less intensively) draw on "non-canonical mathematics"? This is different from the question of *how we can do this*? and which effects this might have?

1. The foundation: What is "non-canonical knowledge"?

Following Armando's clarification of the term "canonical mathematics", I asked myself to which extent "the mathematics inscribed in journals and books volumes from the international mainstream mathematics community" or Euclid's and Bourbarki's books are indeed part of the curriculum. There are voices from exactly this "international mainstream mathematics community" that school education poorly prepares students for engaging with university mathematics and that school mathematics has developed into a (for these mathematicians) meaningless collection of superficial practices (London Mathematical Society, 1995). School mathematics is very different from university mathematics, already now, and for good reasons. Defining "non-canonical" mathematics by delineating it from this understanding of "canonical mathematics" does not lead to a meaningful question. I agree that established understandings of "school mathematics" have developed locally, in countries, school systems, school types, etc., comprising a set of mathematical knowledge and practices, ways of doing and knowing mathematics, that are introduced and practiced in school mathematics instruction. I agree that these have some common core internationally. I can work with understanding canonical mathematics as this kind of "school mathematics". What could "including non-canonical mathematics" mean, based on this understanding? In my preparation, I did some search (not re-search) on "indigenous mathematics" and related terms to find answers.

Perspectives: Canonical mathematics is often criticised for its claim to be universal. Mathematics *can* be used to describe a wide range of phenomena from everyday life, nature, economics, science, etc. Canonical mathematics *can* empower learners to engage with many aspects of our modern, globalized societies. Yet, seeing the world solely through this "canonical mathematical lens" often disregards other important perspectives, such as the person's personal, relational connection to the phenomena, such as nature, life, or land. "Drawing on the non-canonical" may mean acknowledging in mathematics instruction the value of both, canonical mathematical perspectives and other ways of relating to such phenomena. Mathematics instruction needs to explicate the specific, different potential and restrictions of both ways of "seeing" the world. Otherwise, it conveys a distorted picture of canonical mathematics.

Contexts: "Non-canonical" mathematics may also refer to the contexts in which previously introduced (canonical) mathematical concepts (such as numbers, the relations between them, and patterns they can describe) are finally applied, for example in exercise phases. These contexts may also be drawn from the student's own cultures, including the local or regional cultures, aboriginal cultures, or home cultures of students with a migration background. This may also highlight the specific mathematical achievements of these cultures, such as Polynesian navigation (Furuto, 2014). Applications connecting to students' own lives have been valued in mathematics education for a long time (Cevikbas et al., 2022). However, this idea runs at the danger of resulting in a superficial connection of mathematics learning to the students' culture and of superimposing a mathematical perspective to phenomena that can rightfully be seen with other perspectives. This double-edged sword needs to be balanced.

Individual, informal knowledge as a starting point: Drawing on the non-canonical may also mean drawing on students' informal knowledge when developing new ("canonical") mathematical concepts or practices with them. Some examples tried to achieve this (Dominguez et al., 2023). Developing mathematical practices from phenomena such as real-world situations has long been seen as an important idea in mathematics education. Plausibly, mathematical ideas might be easier to grasp and to value if these phenomena are taken from the students' cultural background—and not from contexts that might be artificial or meaningless. Connecting mathematical concepts to students' heritage languages may play an important role here. Doing this is necessary but also comes with challenges (see below).

Individual, informal knowledge as learning goal: Non-canonical may also refer to the ways of doing mathematics, that are targeted as goals in mathematics education. I think it is folklore of mathematics education that conveying fixed algorithms (in the worst case without understanding them) should not be the sole goal of mathematics. What constitutes a "good" strategy to solve a problem depends on the learner, the problem to be solved, and the demands of the context (Verschaffel, 2024). I understand that the "if" question posed on the panel has been superseded by research on the question "how and with which effects" this may be done (Heinze et al., 2018).

Non-canonical target concepts and practices: What I rarely found was that uniquely "local" mathematical practices of concepts (such as regionally rooted counting systems or calculation techniques) were proposed for introduction into classrooms (but see, e.g.,, Robinson et al., 2023). While this is powerful to keep inherited cultural practices alive, it results in a dilemma: Aiming to empower students by introducing them to "canonical" mathematics as well as cherishing their cultural roots by introducing them to "non-canonical mathematics" may involve additional challenges for learners, teachers, and the school system.

To summarize, depending on what we understand under "non-canonical" mathematics, we face different challenges and decisions. We need to be clearer on what exactly we want to achieve if we want to advance the curriculum.

2. The practical: How do we weigh and decide on the goals of school instruction?

Raising a call to include "non-canonical" knowledge into the mathematics curriculum only makes sense if it is not already part of the curriculum. Thus, the idea is to augment the curriculum with new goals, that go beyond those that are already set. This has two implications, which are also relevant for innovations beyond the current topic.

The first concerns <u>the role of research</u>: Sending students to school is a decision that is based on a societal consensus. This consensus is closely connected to the goals of school instruction. Making decisions about these goals is not something that can be ultimately made based on scientific methods and theories. Research can argue for potential benefits (or dangers) and aspects to consider when including (or not) certain goals in the curriculum. The decision on the goals itself is a political process that, in the best case, weighs the best available scientific theories and evidence from educational research with other relevant knowledge and wisdom. This does not imply that research (or researchers) needs to be or can be unpolitical. It merely means that the function of research(ers) in our societies is different from that of legislation. As members of our society, we are usually part of this decision process. When we are asked to contribute in with our role as researchers, I feel our society expects us to deliver arguments that have the status of a scientific argument (in some sense)—and hold back our equally important personal opinion. As researchers, we should not attempt to answer the "if" question but provide scientific research and evidence towards the question "how, and with which effects".

The second implication concerns <u>resources</u>: As societies, we devote a dedicated amount of our joint resources (e.g., fund, time, property) to school education. Including additional goals into the curriculum will require additional short-term resources (teacher professional development, material development,…) to implement the change and long-term resources (teacher and student time, regular contact with the bearers of original non-canonical knowledge) to pursue the goals. When we honestly call for additional content in the curriculum, we need to explicate either where additional resources should come from, or argue which other curriculum contents may be erased to make place for the new ones. This argument does not intend to play out one goal against another one. It directs our attention to the restricted resources of ministries of education, school headmasters, individual teachers, learners, and their families.

When calling to include "non-canonical mathematics" into the curriculum (additionally), we need to consider the mentioned questions more intensively. With this in mind: *Auf geht's!* Let's start mowing the meadow.

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WORKING GROUPS

2024. In T. Evans, O. Marmur, J. Hunter, G. Leach, & J. Jhagroo (Eds.). *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). PME.

HUMAN DIGNITY AND MATHEMATICS EDUCATION

Yasmine Abtahi and Richard Barwell

University of South-Eastern Norway; University of Ottawa, Canada

In aiming to understand the challenges associated with issues of equity, inclusion and discrimination in mathematics classrooms, in this working session we focus on the human dignity as one of the fundamental rights for the learners of mathematics.

Issues of inclusion and discrimination have been extensively explored in mathematics education research. We can think, for example, of research focused on discrimination on the basis of race, abilities, geographical borders, language, or genders. We believe that challenging, disrupting and overcoming discrimination and division requires attention to, among other things, human dignity. In a general sense, we understand human dignity as taking care of people across borders and generations, aiming for fairness and equity. We conceptualise human dignity as defined by Nussbaum (2013): unconditional respect for people's powers of self-definition. What might this look like in mathematics education research? The central purpose of this working session is to invite participants to explore and discuss how learners' right to human dignity can be investigated in mathematics education research.

The range of perspectives on the importance of human dignity in mathematics education can be seen from prior research. For example, D'Ambrosio and Rosa (2017) proposed that (ethno)mathematics practices are fundamental to respect for solidarity, human dignity and cooperation with others. Explaining that mathematics education research and practices are neither the causes of nor the solution to intolerance and inequity, Valero et al. (2012) argue that decentralisation of the school mathematics curriculum could be an alternative for a democracy that reclaims human dignity. Abtahi (2022), in considering the place of empathy in mathematics teaching, argues that "unless we can imagine the world from the point of view of others—people and things whose experiences, contexts and ways of living are very different from our own" (p. 157)—we are likely to act (albeit unintentionally) in ways that are detrimental to others' well-being and dignity. In this working session, we will explore methodological strategies to collaborate with youth to strengthen the understanding of and the incorporation of human dignity in mathematics teaching and learning. In particular, we propose the following objectives: a) To explore diverse perspectives about the concepts encompassed by the term human dignity; and b) To consider experiences of teaching mathematics by using the concept of human dignity.

To develop ideas for new and different ways of researching the incorporation of human dignity in mathematics classrooms activities and to explore the possibilities and challenges.

The following question will guide our exploration through various activities and discussions: what methodological challenges and affordance exists to explore the ways

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in which the right to human dignity is (or could be) acknowledged, interpreted and acted upon in mathematics education research? In the two sessions, we will offer a variety of activities: Activity A – Using newspaper articles and social media posts, we provide data or stories in which issues of human dignity are negatively or positivity magnified. We then invite participants to reflect on the stories and on what implications they derive for the learning and teaching of mathematics; Activity B – we select a small number of key readings. We invite participants to analyse them in terms of how they explicitly or implicitly conceptualise the term human dignity; Activity C – we explore methodological strategies for incorporating the right to human dignity into the teaching and learning of mathematics. How could these strategies be conceptualised in various kinds of classrooms? What are the challenges and possibilities. We propose to organise the sessions as follows:

Day 1 90 mins	• Introduction to the guiding question and exploration of the key terms. Q & A (15 min.) • Exploration of current literature, on methodological issues in researching human dignity (15 min.) • Small group working on Activity A (30 min.) • Whole-group reflection on the implications of the activity for mathematics education (20 min). • Gathering themes and questions for session 2 and recapitulation. Q & A (10 min.)
Day 2 90 mins	• A short summary of themes collected in the first session (10 min.) • Small group working on Activity B. (25 min.). • Whole group discussion. Relating the work to our stance towards human dignity (10 min.). • Small group working on Activity C. (25 min.). • Whole group discussion, Q & A, closing remarks (20 min.)

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CRITICAL MATHEMATICAL THINKING FOR SUSTAINABLE FUTURES

Julia Aguirre[1], Chiara Andrà[2], Kim Beswick[3], Alf Coles[4], Sarah Digan[5], Vince Geiger[5], Jodie Hunter[6], Stefan Siller[7], Armando Solares[8], Jennifer Suh[9], Eva Thanheiser[10], Nina Unshelm[7] and David Wagner[11]

University of Washington[1], Università del Piemonte Orientale[2], University of New South Wales[3], University of Bristol[4], Australian Catholic University[5], Massey University[6], University of Wuerzburg[7], CINVESTAV[8], George Mason University[9], Portland State University[10], University of New Brunswick[11]

We will present data from some on-going studies and engage in discussion around what is critical mathematical thinking and how is it relevant to a mathematics education for sustainable futures. Questions of how the mathematics curriculum relates to socio-ecological challenges and precarity are increasingly present in our field (as seen, for example, in "global sustainability" theme of PME46). The Working Group aims to keep conversations going in our community about the impact of issues such as climate change on teaching and learning mathematics.

BACKGROUND

This Working Group will report on ongoing research and provide opportunity for scholars working in the field to make connections between their research that others are conducting across the globe. The Group also welcomes, in particular, scholars who are new to research related to critical mathematical thinking and sustainable futures. Previous Working Groups (2020, 2021, 2022) have conducted discussions concerned with curriculum innovation in light of global changes/uncertainties and associated methodological approaches. In 2023, a Research Forum reported on a Special Issue, "Mathematics curriculum innovation in precarious times" (le Roux et al., 2022). In this Working Group we look to extend previous discussion and draw on new initiatives, through a focus on Critical Mathematical Thinking and how this relates to issues of sustainability, as well as considering alternative frameworks and approaches. It is our hope that these new discussions will build momentum towards further joint writing.

CRITICAL MATHEMATICAL THINKING

The capacity to apply mathematics critically is essential for forming balanced judgements and making prudent decisions about economic, health, environmental and other challenges faced by society. Critical Mathematical Thinking (CMT) involves the application of mathematics to complex real-world problems in a wide range of contexts. Identifying the impact of proposed solutions to real-world problems on individuals and society and how these can be addressed is also a key dimension of CMT. An inability to engage CMT when applying mathematics to real-world problems leads to fewer life opportunities, especially for the marginalised and disadvantaged.

2024. In T. Evans, O. Marmur, J. Hunter, G. Leach, & J. Jhagroo (Eds.). *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 82-83). PME.

Despite its importance, there have been few empirical studies focused on developing CMT in school classrooms, and there is little evidence that current curriculum and pedagogical responses to this challenge have been effective. We know, however, that developing the critical capabilities needed to apply mathematics to real-world problems is a challenging goal (e.g., Geiger et al., 2015). Developing critical mathematical thinking, therefore, is a significant aim in relation to how mathematics education might contribute to sustainable futures for the planet, in addition to other disruptive phenomena (recognising the contested nature of the word “sustainability”).

GOALS OF THE WORKING GROUP

In this Working Group we will be developing responses to the following key questions: (1) What is critical mathematical thinking and how does it develop? (2) How can mathematics education make a positive contribution towards sustainable futures? (3) How is critical mathematical thinking relevant to sustainable futures? (4) What methodologies might support work on the questions above?

ACTIVITIES AND TIMETABLE

Session 1: Introduction to ideas of “critical mathematical thinking” and “sustainable futures” with reference to aims and outcomes of past work at PME (2020-2023). (20 mins); Enabling Critical Mathematical Thinking (Geiger) (15 min). Discussion (with a facilitator): Issues around developing criticality in your context/other related perspectives? (20 min); Strengthening Teachers’ Instructional Capability with Big Data and Sustainability (Siller) (15 min). Discussion (with a facilitator): How might sustainability enter your work? (20 mins); Plenary discussion to collate common themes from small group discussion. “Gap task”: What research questions are provoked? What methodologies/frameworks? (25 mins)

Session 2: Feedback on “gap task” and collation of responses (15 mins); Updates on recent thinking relating to criticality/sustainable futures (Andrà, Coles, Hunter, Thanheiser) (20 min). Discussion (with a facilitator): Common themes and issues (25 min); Small group discussion, around identified issues/common areas of interest. (30 mins); Closing discussion: Feedback and next steps for future collaborations. (30 mins)

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MATHEMATICS IN INTEGRATED STEM: DILEMMAS AND STRATEGIES FOR SUCCESS

AnnaMarie Conner,[1] Kristin Lesseig,[2] Claire Miller[1], and Anna Bloodworth[1]

[1]University of Georgia; [2]Washington State University

This working group will bring together researchers involved in interdisciplinary STEM instruction, focused on dilemmas of mathematics in STEM. We will share challenges and promising strategies for teaching and learning mathematics in integrated STEM settings and work towards productive collaborative efforts among researchers.

It has been several years since a series of discussion and working groups engaged the PME community in examining issues around STEM education. These original working groups resulted in publication of an edited volume addressing international perspectives on STEM education (Anderson & Li, 2020) and spirited discussions about STEM and social justice (Anderson et al., 2019). Since then, researchers have continued to wrestle with the place of mathematics in integrated STEM instruction (Just & Siller, 2022; Makonye & Moodley, 2023). The goal of this working group is to build a network of international STEM educators to begin working toward promising practices that address the challenges of interdisciplinary STEM instruction. To do this, we will surface the dilemmas and challenges of teaching mathematics in interdisciplinary contexts, share promising and successful ideas for teaching mathematics in these contexts, and strategize together about resources or actions that can be taken to build on the potential of integrated or interdisciplinary STEM instruction across grade levels.

There are many real challenges of interdisciplinary STEM instruction, not the least of which is the need for knowledge of content and practices of multiple disciplines (see, e.g., Anderson et al, 2019). Individual STEM disciplines provide different epistemological perspectives, use different vocabularies, and embrace different practices (Reynante et al., 2020); although there is little consensus about what is required for STEM integration, most definitions include the presence of multiple disciplines (Bybee, 2010; Ring et al., 2017). Additionally, despite the longevity of calls for STEM teaching, the place of mathematics within this teaching remains uncertain. School structures that silo individual disciplines contribute to the difficulty of envisioning integrated STEM instruction even while leaders emphasize its importance for future success. Teachers often embrace the ideals of integrated STEM instruction while acknowledging that their knowledge and experiences make the teaching of integrated STEM difficult (Margot & Kettler, 2019).

PLAN FOR WORKING GROUP SESSIONS 1 AND 2

30 mins	**Session 1** – Brief introduction – leaders share overview of STEM perspectives from previous PME conferences and their own experiences

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45 mins	Small group and whole group surfacing of dilemmas and challenges of teaching mathematics in integrated STEM contexts; sharing of challenges emerging from both teaching and research contexts
15 mins	Summary of common dilemmas, challenges, and questions; making connections with scholars with similar interests; begin to brainstorm promising ideas
15 mins	**Session 2** – Summary of Session 1
30 mins	Short informal presentations on research projects and STEM education perspectives, approaches, and research agendas from representatives in the working group
30 mins	Sharing promising and successful ideas for teaching mathematics in integrated and interdisciplinary contexts; strategizing resources or actions that can be taken to build on the potential of integrated or interdisciplinary STEM instruction across grade levels
15 mins	Developing strategies for continued collaboration and communication

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CREATIVE METHODS FOR INQUIRY IN MATHEMATICS EDUCATION RESEARCH

Markku S. Hannula[1], Tracy Helliwell[2], and Andreas Ebbelind[3]

[1]University of Helsinki, Finland; [2]University of Bristol UK; [3]Linnaeus University, Sweden

In this new working group, we will explore the use of creative methods for both analysing qualitative data and reporting research results.

INTRODUCTION

The aim of qualitative research is often to make a hidden phenomenon visible to others. One such phenomenon is other people's experiences. Doing research in this area, we often face the daunting task of first understanding the nature of that experience and then reporting that experience in a way that is relatable to the reader ("Crisis of representation", Richardson, 2000). This is much more difficult when the original experience is of someone very different from the researcher and the reader, for example, because of worldview, social context, or cognitive development. In our working group, we will work with a case of young children (six year-olds) solving a combinatorics problem together. An enumerative combinatorial task where the children are asked to find all permutations (when n = 3).

The difficulty of relating to the experiences of another stems from the embodied nature of experience. Experience happens in and through our sensory body, of which our cognition is only a part. To truly understand the experience, we must get into the skin of the other, to make the observed social elements become personal by taking the role of the other (Prus, 1996). In this workshop, we use creative methods of engaging with data, providing participants with new perspectives to understand the experiences of others.

However, even though we as researchers may have an empathetic connection to the experience of the other, we still face the problem of providing that same connection to those reading our research. Richardson (2000) writes how the power of qualitative research does not primarily depend on its truth, but rather on its *verisimilitude* – that is the phenomenon when the reader feels the story to be lifelike. One example of efforts to provide such verisimilitude in mathematics education research was Hannula's (2003) fictional writing of a first-person view on mathematics anxiety. Another example is Ebbelind and Helliwell's (2023) 'virtual dialogue' between two prospective mathematics teachers. During the working group, we will present some examples of visual, narrative and arts-based approaches used in reporting research.

AIMS AND ACTIVITES

The aim of the working group is for participants to learn and engage in creative approaches to analysing and presenting classroom data. Participants will engage with

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a fixed dataset (presented by the workshop holders), including transcripts, videos, drawings, and solutions from children's work.

Session 1

The focus of the first session is to work creatively with data. We will offer three ways for participants to interact with the particular classroom case: *performing*, *creative writing*, and *illustrating*.

- Introduction by organizers (10 min)
- Watching an extract of video data, discussing and clarifying (15 min)
- Working with the data in groups (performing, creative writing, and illustrating) (40 min)
- Feeding back experiences in re-mixed small groups (15 minutes)
- Discussion (10 mins)

Session 2

The focus of the second session is to experiment with creative ways of presenting research findings in publication reports. We will continue working on the same extract of data as in Session 1. We examine visual and narrative elements to use in reporting, such as characters and relationships, voice, space and time, symbols and metaphors, mood, and plot.

- Review of Session 1 and introduction to session 2 (10 min)
- Organisation of groups around common approaches (10 mins)
- Working in groups (40 min)
- Sharing products and experiences in re-mixed small groups (15 minutes)
- Reflective discussion, future plans, and closing the session (15 mins)

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POETIC METHODS IN MATHEMATICS EDUCATION

Andrew Hare, Rachel Helme, and Susan Staats

Saint Mary's University, Canada; University of Bristol, U.K; University of Minnesota, U.S.A

This continuing POEME working group will consider the layering of poetic forms, embodiment, and emotion in mathematics activity, such as teaching, learning, and researching. We will invite dialogue around the representation of layering that gives attention to poetic form both alongside, and within, embodiment and emotion.

BACKGROUND

This working group is a continuation of the POEME group newly established at PME 46. Previously, the working group considered poetic methods for noticing and representing the layering of mathematical voices in interviews and student-to-student problem-solving conversations. This patterning of mathematical voice called forth the need for poetic methods, but importantly, these are methods for noticing the inherent dialogicality of expression, the generative poeticity lying within all human expression and interaction, helping us to better understand the development of mathematical thinking and understanding over time.

The current working group will examine how poetic methods within mathematics education research can support commitments in the area of socially constructed mathematical identity trajectories and teachers' gestural and ethical practices. We extend previous activities with the question: "Why does mathematical poeticity and its poetic methods require attention to the layers of gesture and to emotion?". We present samples drawn from upper secondary and undergraduate mathematics classrooms, but participants working at any educational level will be able to engage the working group material. The layered and dialogical nature of mathematical poeticity is presented through a specific gestural practice used by an undergraduate abstract algebra instructor, "regarding as," which refers to "expressing with the body and hands the manner in which some writing is to be considered" (Hare, 2022, iv). Just as the dialogical heart of poeticity is invoked through echoing of words and sounds, so it is with repeated "regarding as" gestures, that the instructor used to link mathematical actions with mathematical notations, tables, and diagrams.

Secondly, the group will consider emotional responses, how the language (and hence the educational experience) feels both for speaker and listener (Quinlan, 2016). The complexity of emotion is discussed in mathematics educational research in relation to the teaching and learning experience, but the researcher themselves can be invisible. Op 'T Eynde, De Corte, & Verschaffel (2006) talk about the mathematics education researcher as not an observer but as an actor, acknowledgeing the social nature of emotions. The emotions of the listener, responses to hearing the words of another could itself be considered to be poetic in form. We will consider that mathematical poetic

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methods require developing an attentiveness to one's own emotions while engaging research material, and also, attention to the ways in which emotion is a mobilizing force in educational experiences.

GOALS OF THE WORKING GROUP

Participants in this working group will consider the following questions: 1) What are the implications of giving attention to embodiment and emotion within mathematical poetic methods? 2) How can the poetic layering of words, gestures and/or emotions be represented? 3) What type of collective output would participants value, such as a journal article, special journal issue or edited volume?

ACTIVITIES AND TIMETABLE

Participants are encouraged to bring a 1-page passage from their own collection of transcribed mathematical interactions, or identify a video recording with transcript, for small group practice with poetic methods during session 2. Participants are asked to attend to their ethical agreements as this material may be viewed by others during session 2. The presenters will provide additional transcripts for participants who are not able to bring their own material.

Session 1

- Poeticity, gesture and emotion: outlining goals of the working group. (10 mins).
- Interactive demonstration on poeticity and "regarding as" gestures (30 mins).
- Interactive demonstration on poeticity and emotions (30 mins.)
- Questions, discussion and planning session 2 (20 mins.)

Session 2

- Review and discussion of session 1 (10 mins.)
- Groups construct a poetic artifact from their transcription sample (40 mins.)
- Group presentations (20 mins.)
- Feedback; discussion of collective outputs (20 mins.)

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INTERNATIONAL PERSPECTIVES ON PROOF AND PROVING: RECENT RESULTS AND FUTURE DIRECTIONS

David A Reid[1] and Yusuke Shinno [2]

[1]University of Agder, Norway / [2] Hiroshima University, Japan

This working group is a continuation of the working group on international perspectives on proof and proving at PME-43, PME-45 and PME-46 (Reid et al., 2019a, 2019b, 2022; Sommerhoff & Komatsu, 2023). The aim is to foster research on proof and proving from an international perspective by bringing together research on proof and proving and international comparison. The long-term goal is to present the results of several comparative international studies on proof and proving in a PME Research Forum.

The past two decades have seen a strong increase in research into proof and proving in mathematics education. Much of this has been conducted in single national and cultural contexts, although there have been, and continue to be, comparisons that have compared proof in a few contexts. Several of these have been reported at and supported by this working group. For example, the work of Miyakawa and Shinno (2021) has provided a theoretical frame for comparisons in terms of three aspects: 'structure', 'language' and 'function'. Boero and Turiano (2022) provide an alternative framework consisting of a mathematical entity (for example a definition or a proof) and a rational process aimed at producing an instance of that entity. Empirical studies have been reported by Otani, Reid and Shinno (2022) and Hakamata et al. (2022).

This growing research base on proof and proving from an international perspective is much needed as it remains unclear whether existing research results from single national and cultural contexts are transferable, or, indeed, if the assumptions on which the studies are based are valid elsewhere. Notwithstanding the small amount of existing comparative research on different aspects of proof and proving, comparatively little information exists about the role of proof and proving in educational contexts from an international perspective. Additional international comparisons involving a wider range of countries could shed light on the *teaching and learning of proof and proving* in areas such as curriculum (including textbooks and other teaching and learning resources); student learning and achievements; teaching (including teaching practices, teachers' knowledge, and teacher education or professional development of teachers); and assessment.

As part of the PME 47 Working group sessions there will be brief reports on research carried out in the past year.

STRUCTURE OF THE SESSIONS

Involvement of participants from different countries is essential to the functioning of the group. Over the two sessions the following activities are planned:

Day 1: Introduction to the working group, its aims, goal, and history (20 minutes); Introduction of participants (30 minutes); Reports from PME 46 participants (20 minutes) Reports of other related research: Japan-German comparative research (20 minites)

Day 2: New themes and formation of sub-groups (10 minutes); Sub-group discussions (40 minutes); Reports of sub-groups (20 minutes); Whole group discussion of ways to expand collaborations among researchers (20 minutes).

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SEMINAR

2024. In T. Evans, O. Marmur, J. Hunter, G. Leach, & J. Jhagroo (Eds.). *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 93-94). PME.

WRITING PME RESEARCH REPORTS: A SEMINAR FOR EARLY-CAREER RESEARCHERS

Kotaro Komatsu[1], Peter Liljedahl[2], Sean Chorney[2]

[1]University of Tsukuba, Japan; [2]Simon Fraser University, Canada

INTRODUCTION AND THE GOAL OF THE SEMINAR

PME has provided seminars for the professional development of PME participants on different topics related to scientific activities. The themes of the previous seminars include reviewing PME Research Reports (Gómez & Dreher, 2018) and writing and publishing journal articles (Bakker & Van Dooren, 2019). In this seminar, we extend these PME commitments by organising a program about writing PME Research Reports (RRs).

RRs have two types of papers, namely reports of empirical studies and theoretical and philosophical essays, and this seminar focuses on the first type. The seminar is intended for early-career researchers who are considering writing their first RR proposals for future conferences or have authored one or two RR proposals.

The goal of the seminar is to provide some insights into how to write and publish RRs. This contribution type has played several roles in the development of individual research projects. For example, researchers have shared the preliminary findings of their ongoing studies and used the feedback from the audience in the conference presentations to expand their research and later publish their work as full journal articles. This also applies to PhD students who present their intermediate findings and build on the audience's feedback to complete their theses. Published RRs themselves are regarded as high-quality publications in the mathematics education community. To take full advantage of these opportunities by publishing RRs, contributors need to be familiar with the characteristics of this contribution type. Submitted papers need not be the report of completed research, but papers are reviewed by researchers with multiple RR publications according to several criteria, such as rationale and research question, theoretical framework and related literature, methodology, and results. The participants of this seminar will obtain helpful knowledge for their future RR writing (see also https://www.igpme.org/annual-conference/pre-submission-support/ for pre-submission support for novice or inexperienced researchers).

Participants of the seminar are advised to bring their computers (e.g., laptops) with internet access because we digitally share the seminar's materials.

ACTIVITIES OF THE SEMINAR

The seminar consists of two sessions (90 min each) described below.

First session:

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- Introduction (15 min), including the explanation of general information about RRs.
- Presentation from an experienced researcher (40 min). One of the most prolific and popular parts of the PME Scientific Program is RR presentations. This part of the seminar looks closely at what constitutes a good RR paper and offers guidance for early researchers on how to structure their writing to fit the expectations of what constitutes a PME RR (cf. Liljedahl, 2019).
- Group discussion (35 min). Participants are split into small groups where they share the challenges they experienced, or anticipate experiencing, in writing RRs. If experienced participants join this seminar, they are expected to serve as group mentors, moderating group discussions and sharing their experiences. General comments and questions from the group discussions will follow.

Second session:

- Introduction (10 min).
- Reading and discussing two published RRs with different types (40 min each). Each session proceeds with individual reading, small group discussion, and whole discussion.

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ORAL COMMUNICATIONS

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ENRICHING PRIMARY MATHEMATICS LESSONS THROUGH PICTURE STORY BOOKS: AN OVERVIEW

Ergi Acar Bayraktar

Technical University Chemnitz, Germany

Picture story books have increasingly found their way into primary mathematics classrooms. This article aims to provide an overview of the significant findings of the graduation theses of students in teacher training programs regarding the utilization of picture books in primary mathematics lessons. These theses were supervised as part of my associate professorship studies and focus on integrating picture books into mathematics education. The tasks presented in these theses are related to the use of picture book contexts for fostering social classroom interactions. The data obtained were analysed using qualitative content analysis (Mayring, 2020) and / or interaction analysis (Krummheuer & Brandt, 2001). In line with socio-constructivist theory, I believe that a child's cognitive development is inherently linked to their participation in various social interactions (Brandt & Acar Bayraktar, in press). The initial analyses indicate that integrating picture books into primary mathematics lessons enhances mathematical and conceptual understanding (Köhler, 2023), promotes positive and diverse learning styles (Gießner, 2022), stimulates critical thinking and problem-solving skills (Brandt & Acar Bayraktar, in press). Additionally, it enables educators to better align with curriculum goals and the objectives of mathematics education (Köhler, 2023). Effective lesson planning incorporating picture story books enhances motivation, sustains student engagement, and facilitates a deeper understanding of mathematical concepts. In the presentation, results will be discussed in detail.

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Oral communication

A NEW THINKING OF FRACTIONS ‘FRACTURING’

Aehee Ahn

University of Bristol

This communication proposes a new fractional thinking, termed ‘fracturing’. Fracturing perceptually isolates parts from a whole, and the isolated parts are conceived as entities without the whole. I found this novel approach from an exploration of students’ visual images of fractions. I asked 27 sixth-grade Korean primary students, aged 11-12, to create visual images by drawing any images that came to their minds when thinking about ‘fraction land’. One student named Hannah (pseudonym) depicted incomplete forms of objects in her fraction land. In her drawings, objects labelled as fractions were portrayed proportionally to their fraction names. For instance, a fraction person 2/3 depicted two-thirds of a whole person. Hannah’s action seemed to differ from partitioning in that, in her approach, parts existed independently of a whole. I termed her fraction thinking as ‘fracturing’, signifying the act of breaking down (fracture) a whole and treating the fractured parts as distinct entities independent of the original whole. In essence, fracturing involves isolating parts from a partitioned whole and treating them as a new and separate whole.

I observed fracturing in fraction multiplications in the Korean curriculum, although the curriculum uses fracturing and partitioning without a distinction (see Figure 1).

a. Partitioning	b. Fracturing

Figure 1: Partitioning and fracturing in 1/5 × 1/3

In Figure 1, the key distinction between partitioning and fracturing is on the presence of dotted lines throughout. Rectangles in Figure 1 were partitioned into five equal parts, with one of these parts coloured to represent 1/5. For the multiplication by 1/3, based on the dotted lines, Figure 1.a divides the entire rectangle into three equal segments, while Figure 1.b divides one of the five parts into three segments. Figure 1.b fractures the whole and deals with the part 1/3 of the part 1/5, rather than the part 1/3 of a whole and the part 1/5 of a whole. The significant aspect of fracturing lies in the multiplicity and flexibility it introduces to understanding fraction multiplications. In fraction multiplications, the part, treated as a new whole, is unitised and partitioned again into parts. Throughout this process in fraction multiplications, fracturing generates part-part relationships, offering a way of making sense of multiplicative thinking.

2024. In T. Evans, O. Marmur, J. Hunter, G. Leach, & J. Jhagroo (Eds.). *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). PME.

METACOGNITIVE TOOLS IN MATHEMATICS EDUCATION AS MEANS TO ADDRESS THE IMPOSTER PHENOMENON

Uzuri Albizu-Mallea

University of the Basque Country (UPV/EHU)

The imposter phenomenon, first defined by Clance and Imes, "is used to designate an internal experience of intellectual phoniness (...) particularly prevalent and intense among a select sample of high achieving women" (1978, p. 241). These women tend to attribute their successes to luck or the kindness of those around them, that is, to causes beyond their control. The same phenomena can be observed in mathematics and other STEM areas. This hints at its social and political nature (Breeze, 2018).

For those suffering from the impostor phenomenon, a tiny mistake can represent the exposure of their supposed hidden incompetence. This produces a fear of error and, relatedly, behaviors including paralysis: rather than face the risk of doing something wrong, no action is taken. This paralysis is more common in women than men, and negatively affects women in learning environments (Estes, 2003).

The results presented here are based on one part of a research project carried out with students enrolled in a mathematics subject within the degree of Primary Education. Over the course of the subject, students were taught a methodology based on working through problems in distinct stages. They were encouraged to use it for both in-class and assessment tasks. The methodology facilitated students' metacognitive awareness through self-questioning during mathematical activity.

The extent to which students applied this methodology in the end-of-year exam was observed and quantified, with data disaggregated by gender. The results showed that women made greater use of metacognitive strategies. There is strong evidence to suggest that this had an impact on the rate of paralysis, as all the women sitting the exam attempted problems susceptible to the methodology, while 12.5% of the men made no attempt. It thus appears that metacognition-assisted solving of problems can be a viable tool to address the imposter phenomenon in mathematics.

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THE IMPORTANCE OF THE ELICITATION PROCESS IN FORMATIVE ASSESSMENT: A CASE STUDY

Catarina Andersson, Torulf Palm and Mikael Winberg.

Umeå University, Sweden

Several literature reviews have demonstrated that the use of formative assessment (FA) can yield substantial improvements in student achievement (e.g., Lee et al., 2020), but the effect sizes from individual studies vary substantially. This variation implies that it is important to not only examine whether FA affects achievement, but also how characteristics of FA influence effects. FA is a non-trivial practice, where teacher decisions often occur during a dialogue with the students. The quality of the student–teacher interactions depend on the teacher's understanding of the student's thinking and have consequences for the opportunities to learn (Lee & Cross Francis, 2018).

This paper focuses a mathematics teacher's use of FA when helping students who work individually with mathematics tasks. We focus on the FA processes of teacher elicitation of information about students' learning and thinking (i.e. assessment), the subsequent feedback, and the students' response to the feedback.

The aim of the study is to examine how differences in the teacher's elicitation process influence whether the student-teacher interactions focus on the students' actual learning needs. We worked with an experienced and committed mathematics teacher for six months. Based on audio-recorded student–teacher interactions a researcher provided feedback to the teacher and discussed her beliefs and experiences of implementing FA. The teacher then used the feedback and attempted to improve her FA practice before a new cycle of cooperation started. Characteristics of the elicitation of information and the focus of the student–teacher interactions were identified.

The results exemplify how the characteristics of the teacher's elicitation of evidence of the students' thinking affects how well the feedback accomplishes student thinking that is about their actual learning needs. Thus, the study exemplifies how differences in characteristics of FA components can affect students' opportunities to learn.

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NUMBER LINE ESTIMATION OF FRACTIONS: A COMPARATIVE STUDY

Khemduth Singh Angateeah[1], Anna Neena Georges[2], and Mark Winter[3]

[1]Mauritius Institute of Education, Mauritius, [2]GVM Dr. Dada Vaidya College of Education Jena, India, [3]Unicaf University, South Africa

Understanding fractions is a fundamental aspect of mathematical literacy, yet learners often grapple with the complexities associated with rational numbers. This struggle may even extend into adulthood. Teachers must be adequately equipped to facilitate students' comprehension of rational numbers. This comparative study investigates, using a mixed approach, the number line estimation (NLE) strategies employed by trainee teachers from India and Mauritius, shedding light on the efficiency of these strategies and their implications for the teaching and learning of mathematics. The research, inspired by Van Dooren's (2023) work, engaged 33 trainee teachers enrolled in Postgraduate Certificate in Education programmes, with 18 from India and 15 from Mauritius. The task involved representing 18 fractions on a 200 mm number line with endpoints 0 and 1, and the accuracy of estimates was measured using the Percentage of Absolute Error (PAE) (Schneider et al., 2018). Results revealed a higher average PAE for Mauritian respondents, indicating a less accurate representation of fractions on the number line compared to their Indian counterparts. Notably, both groups struggled more with fractions featuring larger denominators. Mauritian trainees exhibited a more diverse range of strategies, impacting the accuracy of fraction representation. This contrasted with the Indian trainees, whose strategies did not significantly affect accuracy. The implications for teaching and learning are profound. Integrating a diverse set of strategies into teacher training programs can enhance the effectiveness of fraction representation in classrooms. Additionally, targeted interventions should focus on fractions with larger denominators, and customized support for trainee teachers based on their educational backgrounds can address specific challenges. By emphasizing effective strategies identified in the study, teacher training programs can contribute to enhanced accuracy in fraction representation, ultimately fostering a deeper understanding among learners and advancing the discourse on mathematics education globally.

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IMPACT OF AUGMENTED REALITY ON PRIMARY SCHOOL STUDENTS' LEARNING EXPERIENCES OF COORDINATES

Khemduth Singh Angateeah, Divia Renu Ramraka, and Parwez Neeamuth

Mauritius Institute of Education, Mauritius

The integration of technology in mathematics education has opened new avenues for an engaging and effective learning experiences. Traditional methods for teaching coordinates at the primary level have faced challenges in fostering a deep conceptual understanding among students (Radu et al., 2016). This ongoing research delves into the transformative potential of Augmented Reality (AR) in the context of teaching and learning coordinates at the primary level (Duart et al., 2018). Focusing on the often-challenging concept of coordinates, the qualitative study investigates the impact of AR-enhanced instructional methods on students' comprehension and engagement. A convenience sample of five Grade 6 students are considered. AR application from GeoGebra is used to provide an interactive and immersive learning experience for students. The application focuses on visualizing coordinates in a real-world context, offering dynamic and engaging content to facilitate a deeper understanding of abstract mathematical concepts. Qualitative data are collected through, observation, students' reflections, and feedback to gain insights into their learning experiences with the AR-enhanced lessons. The qualitative data will be subjected to thematic analysis to identify patterns and themes. Preliminary findings from piloting with two Grade 6 students showed that students are more interested, motivated, and engaged when using AR in the learning of coordinates as it deals with the dynamic real-world objects rather than static diagrams as in textbooks. Students can experiment, placing objects on a table and read the coordinates from the AR applications. Similar observation for any point or insect on the wall or the roof. The AR application allow the placement of grid and coordinates on real-life objects, thus changing students' perspective of Geometry and thereby enhancing their spatial sense. The findings promise to inform the development of pedagogical strategies that harness the potential of AR to elevate the teaching and learning of coordinates, paving the way for future advancements in primary mathematics education.

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SEEKING EVIDENCE OF GROUNDING IN AN ONLINE MATHEMATICAL DISCOURSE IN COMBINATORICS

Maria Digi Anna Mance Avila and Maria Alva Aberin

Ateneo de Manila University

This pilot study is anchored on the theory of grounding in communication by Clark and Brennan (1991), who argued that discourse requires a conscious and consistent effort from its participants to coordinate the content of what they intend to convey so that they can establish common ground or shared understanding. This coordination process is known as grounding. Due to the fast-expanding use of online environments in facilitating mathematical discourse, a question arises as to whether or not common ground can be reached during online mathematical discourse and what evidence indicates so. As part of an ongoing dissertation project, this pilot study therefore aimed to seek evidence of grounding in an online mathematical discourse to determine whether or not common ground has truly been established by the teacher and students.

Using the aforementioned theory as an interpretive lens and following a qualitative research design, four (4) video-recorded class sessions of an online graduate course in Combinatorics were observed to answer the posed research question. The course was conducted once a week through a video conferencing platform, with each class session lasting for three hours. It was composed of one (1) teacher and three (3) graduate students, who were all pursuing a PhD in mathematics education at a private university in Manila, Philippines. A multimodal conversation analysis (MCA), which involved a transcription of the recordings and an inductive examination of the transcripts, was subsequently carried out to identify the emergence of evidence of grounding.

The analysis showed that positive evidence of grounding was present in the online mathematical discourse in Combinatorics. This evidence came in the form of acknowledgements, adjacency pairs, and continued attention. For instance, the teacher posed questions from time to time to check the students' understanding, to which the students responded either verbally or using embodied and material resources (e.g., head nods, hand gestures, and virtual gestures). Hence, it was concluded that the teacher and students consciously and consistently worked their way towards updating and establishing their common ground during the online mathematical discourse. The findings of this study provided a guide for assessing whether or not shared understanding is achieved between teachers and students as they engage in an online mathematical discourse.

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EXPLORING THE INFLUENCE OF ONLINE HOMEWORK FORMAT ON PROBLEM-SOLVING STRATEGY USE ON RELATED RATES OF CHANGE PROBLEMS IN CALCULUS

Tyson Cassada Bailey and James A. Mendoza Álvarez

The University of Texas at Arlington

An increasing proportion of calculus homework no longer involves traditional paper-and-pencil (TPP) submissions due to the prevalence of online platforms designed for assigning and evaluating homework (Dorko, 2020). For related rates of change (RRC) problems in calculus, the convenience and additional features of online homework may undermine opportunities for students to engage in mathematical sensemaking. However, to successfully solve RRC problems, Engelke (2007) underscores the importance of robust mathematical problem-solving skills. In addition, students' reasoning when solving RRC problems is understudied (Mkhatshwa, 2020). The research questions are: (1) How do students' PSS when working online RRC homework problems compare with their PSS when working TPP homework RRC problems? (2) What influence does the 'view an example' feature in online homework have on a student's PSS when working an online RRC homework problem?

From 318 participants, fourteen first-semester calculus students, selected based upon diverse RRC performance levels, participated in task-based interviews. The interviews included four RRC problems, two sets of paired tasks of a TPP format problem and a similar online problem on the online platform. Using deductive thematic analysis approach, interview transcriptions were coded using a priori codes determined from the problem-solving literature; however, emergent themes also were noted.

Data analysis shows that participants used more PSS when solving TPP RRC problems than when solving online RRC problems. Additionally, participant's use of PSS decreased when using the 'view an example' feature in the online format. Findings suggest several aspects of online homework RRC problems may need to change to ensure that students engage in using PSS in a manner comparable to TPP homework.

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DIMINISHED CONFIDENCE IN THE LIGHT OF MISSED CURRICULUM OPPORTUNITIES

Sarah Bansilal

University of KwaZulu—Natal

In South Africa there have been many changes to the curriculum after the new democratic dispensation. I am interested here in one significant change where the Euclidean geometry strand was initially omitted in an earlier curriculum revision (Curriculum 2005 - C2005) and reinstated in the Curriculum and Assessment Policy Statements (CAPS) for Grades 10-12 mathematics (DoBE, 2011). I examine one of the repercussions of this “off-on” shift (Dakamo, 2023) on the confidence of a cohort of 74 mathematics pre-service teachers (PST’s). Some PST’s (30) experienced C2005 with no Euclidean geometry, while others (44) were part of the CAPs curriculum, which included geometry. The group who did not study the content at school are now expected to teach this content as part of the CAPs curriculum, leaving them apprehensive. The confidence of mathematics teachers forming a crucial aspect of their professional knowledge (Beswick et al., 2012) is explored here with the research question: Is there a relationship between the PST’s confidence and their prior experience in geometry?

The research instrument for this mixed methods study was an open-ended questionnaire which probed participants about their confidence and experience in geometry. Forty-nine of the participants expressed that they were not confident. Furthermore, a Chi-square independence test confirmed that confidence differed significantly between those who had studied geometry at FET level in school or not (χ^2 = 26.8 ; $p<0.001$). The qualitative analysis of the PST’s reflections on their geometry experiences shed further insights into this difference with 55% of the experienced group sharing feelings of anxiety and inadequacy compared to only 14% of the experienced group. This study shows that PSTs who did not study geometry had diminished levels of confidence compared to those who did, and are likely to carry their feelings of apprehension into the classroom.

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PROBLEM SOLVING: THE EFFECTIVE CHOICES OF REPRESENTATIONS

Ana Barbosa[1,2,3] and Isabel Vale[1,2]

[1]Instituto Politécnico de Viana do Castelo – Portugal; [2]CIEC, Research Centre on Child Studies, University of Minho – Portugal; [3]inED, Centro de Investigação e Inovação em Educação, Instituto Politécnico do Porto, Portugal

Representations are an important component of the mathematics teaching and learning process (NCTM, 2014), often characterized according to the nature of the configurations used, which lead to different categorizations (Bruner, 1996; Matteson, 2006; NCTM, 2014). Extending Bruner´s ideas some authors defend a model with five forms of representations associated with the learning of mathematics and problem solving: contextual; concrete/physical; semi-concrete/visual; verbal and symbolic. The terms numerical, graphic, verbal, symbolic and dual representations were introduced by Matteson (2006). Considering these categorizations, we chose to formulate a representational system that articulates five main categories (active, verbal, visual, numerical and symbolic), but also the dual representations resulting from the complementary use of two of the main ones. Not all students have the same thinking preferences, hence we believe that tasks with multiple solutions give students the opportunity to apply and contact with a diversity of strategies and representations that may contribute to the extension of their repertoire.

We conducted a study with 14 elementary pre-service teachers, adopting a qualitative and interpretive methodology. The participants solved some multiple-solution tasks and data was collected through: participant observation, notes of the participants' reactions and interactions, and the written productions regarding the proposed tasks. We identified as categories of analysis: types of strategies; and representations used. Globally, we noticed a predominance of analytical and mixed approaches. Verbal, numeric, symbolic and visual representations were frequently mobilized and in most cases in the sense of the application of dual representations, complementing two of the main categories. There is usually a type of representation that is dominant or essential in attacking the problem. In general, the students resorted to multiple representations, in order to complement their reasoning and reach the solution.

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MATHEMATICAL CONSCIOUSNESS AND THE ECOSYSTEM: AN EXAMPLE OF MATHEMATICAL MODELLING

Richard Barwell

University of Ottawa, Canada

At the heart of most, if not all, of the major challenges facing humans on Earth we find mathematics. We can think of the climate crisis, the biodiversity crisis or widespread pollution, for example, which have their origins in the consumer-capitalist economic system around which societies are organised, broadly speaking, around the world. Mathematics underpins processes of extraction, refinement, design, production and distribution of goods, which have as additional effects various forms of extraction, pollution and exploitation of largely minoritized people, many other species and ecosystems. How does the teaching and learning of mathematics contribute to, challenge, or resist its role in these global, human-created problems?

In this oral communication, I develop the idea that much teaching and learning of mathematics in classroom contexts contributes to forms of consciousness that introduce a separation of actions from their effects and consequently contributes to the ecosystem crisis. Specifically, I argue that common forms of mathematics teaching produce consciousness that alienates learners from other species, other humans and the ecosystem. I think about both consciousness and alienation by drawing on Bakhtin's (e.g., 1993) dialogic theory in which both these notions are about humans in relation with others. I use these ideas to expand aspects of critical mathematics education such as Skovsmose's (2023) examination of mathematical knowing to consider dimensions of consciousness produced through learning and teaching mathematics. In particular, I propose the idea of mathematical consciousness as emerging from experiences of mathematics in relations with others. Through classroom experiences, learners come to see the world in ways that mathematics requires. At the same time, learners are themselves changed by doing mathematics. Thus, in many mathematics classrooms, mathematical consciousness develops with particular qualities or dispositions, such as objectification, categorisation, and a kind of detachment of the mathematising human from the objects of its application.

I illustrate these ideas with an example drawn from research on mathematical modelling, since, as Skovsmose has noted, mathematical modelling is one of the principal ways in which mathematics interacts with social reality. Mathematical modelling is an important tool for understanding climate change, biodiversity, or pollution, for example.

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VALIDATING A MEASURE OF MATHEMATICAL KNOWLEDGE FOR TEACHING COLLEGE ALGEBRA

Mary Beisiegel, Laura Watkins, and Vilma Mesa

Oregon State University, Maricopa Community Colleges, University of Michigan

We report on one aspect of the validation process of our mathematical knowledge for teaching college algebra at community colleges MKT-CCA instrument (Mesa et al., 2024). Using cognitive interviews (Bostic et al., 2021) with a representative sample of 26 community college instructors who recently completed the MKT-CCA instrument, we aimed to confirm whether and how a representative set of the MKT-CCA items captured teacher knowledge as intended. The participants each answered the same five items which varied across content (linear, exponential, rational functions) and tasks of teaching (choosing problems, understanding student work). During Zoom interviews, participants were asked to read the items and express their reasoning out loud and to show their work through annotations on the Zoom screen. We also asked participants to share the mathematical ideas they thought were elicited by each item, misconceptions students might have when working on similar tasks, and whether the scenario in the item was familiar. Our research question is: Do participants use the hypothesized mathematical knowledge for teaching college algebra when working on the items? Our analytic process consisted of: establishing a reliable coding framework that connected participants' responses to the knowledge and mathematical reasoning that we hypothesized they would use, individual researchers coding participants' answers, and confirming reliability of coding both by pairing researchers on multiple responses and having the entire research team code a subset of the responses. For each of the items, we have found participants: (1) answering correctly and incorrectly, (2) using the hypothesized knowledge and also other types of heuristics to answer the items, (3) a preponderance of statements about mistakes students make rather than misconceptions they may have, and (4) familiarity with the content but not with the presentation of the items as participants noted that the items looked novel.

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ETHNOMATHEMATICS: LEARNING FROM REALITY

Karli Bergquist

University of Ottawa, Canada

If ethnomathematics are the strategies of action for dealing with reality (D'Ambrosio, 2018), then paying attention to reality and how people respond to it can reveal the mathematics that matter. Inuit Qaujimajatuqangit, or what Inuit have always known to be true (Karetak, Tester & Tagalik, 2017), is an example of how individuals can deal with reality. It is a perspective based on interconnectedness with others and the non-human world. The research project aimed to document some of the ways pre-service Inuit teachers from Nunatsiavut, Canada, consider mathematics education using a funds of knowledge approach (Moll et al., 1992). Identifying funds of knowledge became problematic when participants said they don't talk about concepts and skills, they talk about common sense. In this oral communication I will present detailed findings of the study and explore the following questions: Does a reference to common sense in what people say about what they do indicate a fund of knowledge? What is the connection between "what you've got to do" and learning mathematics?

My positionality as a non-Inuit researcher required partnership to evolve over time and through community involvement in Nunatsiavut. Using participatory methods, semi-structured interviews were conducted to gather information about mathematics education from an Inuit perspective. Considering activities of daily life described during the interviews, I attempted to identify sources of knowledge, areas of mathematics, and the interactions involved in knowledge generation. Data from nine participants demonstrates two things: how we come to know mathematics matters, and that learning mathematics disconnected from culture, language and the local environment is harmful. The study found that what to learn is as important as how to learn it: family and community interactions, learning naturally and life-experiences were of particular importance. For Inuit in Nunatsiavut, mathematics education is paying attention to common sense strategies of action to deal with reality.

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WHAT IS FUNCTIONAL THINKING? AN ONTOLOGICAL ANALYSIS OF DIFFERENT DEFINITIONS

Anneli Blomqvist and Lovisa Sumpter

Department of Teaching and Learning, Stockholm University, Sweden

Functions and their relationship to algebra have gained increasingly more attention, including functional thinking as a means for learning algebra, as well as studies focusing only on functional thinking (Chimoni et al., 2023). As a mathematical concept, ‘function’ is one of the central ones: “The concept of a function is fundamental to virtually every aspect of mathematics and every branch of quantitative science” (Warren et al., 2013, p.76). Functional thinking can be seen as part of algebraic thinking, and studies have shown that it predicts and explains algebraic thinking to a similar degree to modelling and generalised arithmetic (Chimoni et al., 2023). However, there are many definitions of functional thinking, and it is unclear how and in what ways different theoretical operationalisations of the concept are related. Such understanding lies in the field of ontology, which can be defined as an explicit description of conceptualisation (Obtiko et al., 2004). An analysis of that kind provides information about how a concept is a shared understanding but also helps to see patterns in the theoretical structure of a concept. The present paper aims to present the preliminary results of an analysis of definitions of functional thinking used in papers published between 2002-2022. The papers were identified through a systematic review (n=25). The definitions were analysed using an AI tool. After that, we interpreted the results using a mathematical definition of functions to identify core aspects of the definitions. The analysis generated three main results. The first result is that two specific definitions capture most of the key aspects of functional thinking and that most empirical studies use these key concepts. These two definitions treat functional thinking as products or products and processes. One definition stands out as having a theoretical operationalisation of functional thinking as a process.

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NAVIGATING PROFESSIONAL ROLES: A QUALITATIVE ANALYSIS ON SHIFTS IN TEACHER'S BELIEFS

Jude C. Buot and Lester C. Hao

Ateneo de Manila University

Practices in teaching are influenced by the teacher's beliefs. Moreover, one's beliefs develop and change over time as they gain more professional experience (Eichler et al., 2023). Kasa et al. (2024) summarized a mathematics teacher's belief system to be comprised of four categories: (1) nature of mathematics, (2) mathematics teaching, (3) mathematics learning, and (4) role of teacher. From the perspective of the lead author, we describe how varying professional roles may possibly contribute to shifts in beliefs. In particular, his roles as (1) teacher-in-training, (2) instructor, (3) course coordinator, and (4) textbook co-author are pertinent to the implementation of the course Mathematics in the Modern World (MMW). It is a required general education course for undergraduate students in the Philippines, and its content focuses on the nature of mathematics, mathematics as a language, and mathematics as a tool.

Aside from some documents (e.g., syllabi, student outputs), data for this study come from the lead author's reflection that cover five years of handling the MMW course and informal interviews conducted by the second author. Results of the qualitative analysis, conducted by the second author and subsequently validated by the lead author, reveal that having a reflective disposition towards teaching and learning while stepping into varied roles, takes part in shaping and initiating shifts in beliefs. Through these different involvements, it was found that he has been introspective of the course's identity and its relevance in the undergraduate curriculum. From delivering the course as being content- and performance-focused, he now perceives it to be one that encourages a deeper appreciation of mathematics through understanding and exploration, providing access to students who gravitate away from mathematics. He likewise cited that being a textbook co-author allowed him to rethink the nature of the course. Instead of viewing the topics under "mathematics as a tool" separately, he now espouses a problem-solving mindset throughout, in addition to posing real-world, data-driven problems for students to explore. His current role as course coordinator has therefore accorded him the opportunity to impart his beliefs to new teachers through teacher-training and mentoring.

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TRANSFORMATION OF A MATHEMATICS TEACHER'S KNOWLEDGE FOR TEACHING THE CONCEPT OF LIMIT

Cristián Bustos Tiemann and Elisabeth Ramos Rodríguez

Pontificia Universidad Católica de Valparaíso, Chile

Several studies show that the complexity and difficulties of the concept of limit are not only for students but also for teachers (Irazoqui and Medina, 2013). The aim of this study was to investigate the knowledge transformation (KT) manifested by a teacher in teaching the concept of limit of sequences when participating in a continuous training program. We considered for this work the theory of transformative learning (Espejo and González-Suárez, 2015), which understands transformation as the result of reflective processes. If reflection takes place at the level of content or resolution process we will be working with meaning schemes, but if reflection leads to the questioning of more fundamental premises or personal paradigms we will be at the level of transformation of meaning perspectives.

The paradigm is qualitative and the design is a case study. The case is a high school teacher who participated in a Diploma Course in Didactics of Calculus, conducted virtually, where the participating teacher together with his colleagues faced the problem of how to introduce the concept of limit of sequences in high school students from the design of a class. A pre-test was applied via mail before starting the course, and then a post-test was applied during the course after the teachers discussed with their peers and trainers the lesson plan of the participating teacher. In the pre-test the teacher reported knowledge about definitions from intuitive approximations, as well as considering the passage to the limit as a tendency or approximation, i.e., from potential infinity. In the post-test, the teacher showed knowledge about the importance of knowing the difference between potential and actual infinity when introducing this mathematical object. This conscious change arises from the interaction with peers and trainers and is the product of reflection on perspectives of meaning, since it directly questions fundamental premises such as distinguishing whether the sequence tends to a value or actually reaches it. In this sense we could be facing a KT. It is expected that this study will show how the concept of limit of sequences is introduced and, in addition, it intends to contribute to more specific descriptions of how to approach the KT of the mathematics teacher.

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EMBRACING THE AGE OF ARTIFICIAL INTELLIGENCE: TEACHERS' KNOWLEDGE AND PRACTICE ON THE RESPONSIBLE USE OF AI IN MATHEMATICS EDUCATION

Joseph Ma. Steven S. Cabalo[1], Pablo A. Regalario[2], and Jenny P. Macalalad[3]

[1]Ateneo de Manila University, [2]National University Lipa Campus, [3]Batangas State University Alangilan Campus

As the integration of Generative Artificial Intelligence (GenAI) technologies in education accelerates, understanding student perceptions is crucial; their perspectives can inform policy development around GenAI use. Studies necessitate educator involvement and continuous adaptation to prepare students for the evolving landscape of AI-integrated education (Chan & Lee, 2023; National Council of Teachers of Mathematics, 2024). This study focuses on the responsible and effective use of GenAI tools in mathematics education through the Technological Pedagogical and Content Knowledge (TPACK) framework. The research investigates teachers' mathematical content knowledge (CK), technological knowledge (TK), pedagogical knowledge (PK), Technological Pedagogical Knowledge (TPK), and their Technological Content Knowledge (TCK) on GenAI integration. (Koehler, Mishra, & Cain, 2013).

Utilizing the TPACK framework, this study investigates how senior high school mathematics teachers in secondary schools in the Philippines perceive and integrate GenAI tools into their teaching. While some expressed concerns regarding overreliance and potential misuse, others embraced GenAI for tasks like generating practice problems, visualizing concepts, and personalizing instruction. Furthermore, teachers emphasized the need for professional development opportunities to enhance their TCK. These preliminary findings highlight the need for targeted support to strengthen teachers' TPACK, enabling them to harness the potential of GenAI tools in Philippine mathematics education.

Additional information

This research is currently in progress; to be completed within the third quarter of 2024.

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LEARNING EFFECTS OF MODELS WITH VARYING LEVELS OF ABSTRACTION DURING DIGITAL PRACTICE

Michele Carney, Joe Champion, Angela R. Crawford, and Patrick R. Lowenthal

Boise State University

This study examined if “hints” featuring mathematical models lead to improved learning for middle grades students who are assigned practice items in a digital learning platform (DLP) and if the level of abstraction of the model impacts efficacy (Fyfe et al., 2014). We modified “hints” available to students in the ASSISTments DLP, and students were randomly assigned to a condition with a certain type of model abstraction: informal iconic, formal iconic, and informal symbolic (see Figure 1).

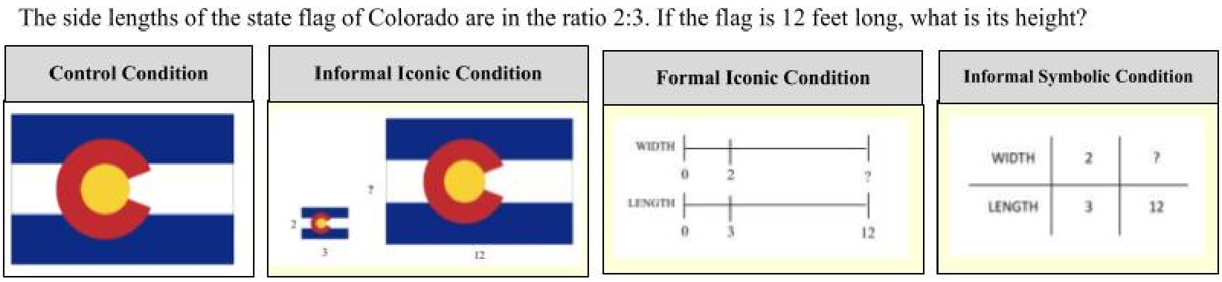

Figure 1: Example item from the study and hints from the experimental conditions.

We found that conditions with mathematical models significantly improved student learning on subsequent similar items when compared to the control condition (see Figure 2). The formal iconic model appeared to be the most effective. However, more work is needed with a larger sample of items and students to further examine and substantiate this finding.

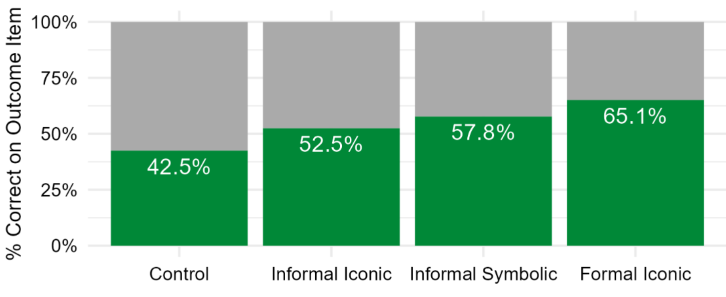

Figure 2. Success rates on subsequent items by the experimental condition.

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AN ALTERNATIVE QUANTITATIVE LENS ON LAVIE-SFARD'S DEVELOPMENTAL MODEL OF NUMERICAL DISCOURSE

Christine Chambris and K. (Ravi) Subramaniam

LDAR, CY Cergy Paris Université, France and HBCSE, TIFR, India

The challenging issue of teaching numbers *quantitatively* has long occupied a prominent place on the mathematics education scene. In much of the research in the field, a Set Theoretic Lens on Numbers (STLN), forms an unquestioned background. Our research hypothesis is as follows: The use of STLN in analyses of learners' reasoning hides specific knowledge that is critical in quantitative reasoning, even when researchers study the role of quantitative reasoning in learning 'numbers'. An alternative Quantity Lens on Numbers (QLN) makes visible pieces of invisible or transparent knowledge. This hypothesis is supported by the anthropological theory of didactics (e.g., Bosch, Gascon & Trigueros, 2017) that postulates that humans act as agents of their communities. Most mathematics education researchers, among them Lavie and Sfard (2019, LS hereafter), do not question STLN as a research principle (community 1), and we act as agents of an alternative QLN community (community 2). Our data is from LS and its online supplementary material. We anchor our analyses in LS's discursive framework, but we redo LS's analyses of children's narratives and deeds based on the introduction of "substances of lengths, and areas" as mathematical objects. These 'new' objects enable a new interpretation of many parts of children's discourse. For e.g., when the child responds "in both" to the question "where is there more?" regarding to two 'long' rows that seem to have a similar length, or "tall" / "not tall", LS consider it as progress in learning but not as the individualized use of a routine, while we argue that the child interprets "more" as a 'tallness' (more = tall), and that for the child "more" or "tall" reflect "substances of length" embedded in the rows. Interpreted thus, the use of these words not only appears as a progress in learning, but as meaningful for the child in terms of 'length', thus as testifying the individualization of a (specific) routine. We suggest this is visible with QLN, and invisible with STLN.

Though our mathematization of 'quantities', here present in "substances of length", and of "area", is a work in progress, our alternative analyses of LS data open avenues for further research on quantitative reasoning, notably by seeking knowledge that is not acknowledged as such in learners' reasoning, by STLN researchers in the field.

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GRAPH LITERACY AND MAGNITUDE PROCESSING ABILITIES: A STUDY OF AGES 3 TO 6

Eugenio Chandíaa, Paloma Nahuehuala, Anahí Huenchob, Gamal Cerdaa, and Farzaneh Saadatic

Universidad de Concepción, Universidad Católica de Temuco, Universidad de Chile

Some studies have proposed that an effective graphic interpretation necessitates a robust grasp of its fundamental construction principles. Conversely, other studies have suggested the existence of innate abilities that enable individuals to decipher even complex graphs from a young age (Ciccione & Dehaene, 2021). In this context, studies in the field of non-symbolic magnitude processing have demonstrated that children as young as three years of age already possess the capacity to engage in tasks of comparison and interpretation of stimuli containing continuous specifiers, such as discretised pie charts or bar graphs. This evidence suggests that these skills are present from an early age. In this context, our study employs a comprehensive cross-sectional investigation to evaluate the graph literacy and magnitude processing skills of 29 children aged 3 to 6. Our focus is on their ability to identify structural components across three types of graphs: discrete (as depicted in pictograms), discretized (as observed in bar graphs with ordinal axis markings), and continuous (as exemplified by pie charts utilizing iconic symbols for variable categorization). Our research is guided by the following question: RQ1: How do children of different ages employ a range of graph structural elements and estimation techniques when making comparative assessments, integrating both symbolic and non-symbolic methods?

After conducting semi-structured interviews, the research revealed significant correlations in their graphical comprehension. A positive correlation was found between the children's ability to recognize variables and their use of discrete elements within graphical representations, denoted by Kendall's tau coefficients of $\tau=.21$ and $\tau=.23$, respectively. Furthermore, a robust correlation emerged between the identification of specifiers and the capability for non-symbolic comparison, with a tau value of $\tau=.46$. Correspondence analysis indicated a pronounced preference for the identification of variables and specifiers within pie charts, whereas bar graphs were primarily associated with an emphasis on axes. These findings suggest the presence of consistent and systematic graphic interpretation skills among young children. Such insights have implications for the structuring of early mathematical curricula, as they highlight the potential for nurturing foundational statistical literacy from a young age.

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CATEGORIZATION OF FIGURE-RELATED MATHEMATICS CONTENT AND NAVIGATION FROM SPATIAL ABILITY PERSPECTIVE

Li-Jung Chang and Chen-Yu Yao

National Tsing Hua University, Taiwan R.O.C.

Researchers have consistently claimed that navigation competence is significantly related to mathematics. For example, Lee et al. (2012) indicated that the understanding of navigation systems can contribute to children's development of geometric knowledge. It is often the case that navigation and mathematics (e.g., geometry, function graph) require reading-comprehension of figures, which fundamental competence is spatial ability. Considering that spatial ability includes different constructs such as perception, mental rotation, perspective taking, and visualization, we aim to examine if different figure-related mathematics content and navigation using map require the same spatial-ability construct or different ones.

To achieve this goal, we included 9 tests for this study; among of which four were geometry tests, four were navigation test, and one was function-graph test. All the tests required to perform figure reading-comprehension. The items included in each individual test ranged from 40 to 80. An exploratory factor analysis (EFA) approach was adopted to categorize those tests into major components. In addition, a simple multiplication test was used to assess and validate the analysis structure. Based on 40 high school students' responses to the nine tests, we performed the EFA using Promax rotation method and the criterion that eigenvalues must be greater than 1. The analysis showed two main components and the two occupied up to 74% variation. One of the components included all the four geometry tests and one function graph test, whereas the other component consisted of the four navigation tests. In particular, the multiplication test was not included in the two components which validated the EFA analysis result. Based on the result, first, we confirmed that the two components derived from EFA are related to figure reading-comprehension. Second, both geometry and function belonged to the same component, showing that categorization criterion is spatial ability but not content knowledge. Third, navigation is another figure-related component, indicating that navigation requires another type of spatial ability. We further hypothesize that visualization is likely the key to formulate the first component that includes geometry and function graph, whereas perspective taking is another core to the 2nd component involving navigation tests. Furthermore, navigation experiences can contribute to the development of geometry. However, the contribution is limited as geometry and navigation likely belong to different spatial-ability constructs.

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ENHANCING STUDENTS' IDEAS OF ALGORITHM WITH COLORING BOOK IN MATHEMATICS CLASSROOM

Narumon Changsri, Maitree Inprasitha, Roberto Araya, and Masami Isoda

CRME, Khon Kaen University, University of Chile & University of Tsukuba

The algorithm is the basis of computational and mathematical thinking, which is necessary for 21st century skills. However, in some parts of the world, students cannot access computers. Therefore, coloring books are attractive and powerful tools for learning and writing logical instructions. These instructions are a pseudo-code designed to prepare for developing algorithmic thinking (Araya & Isoda, 2021). Unplugged activities are also useful for reflection, even when a computer is available. This is because it is necessary to think about what and how to solve a problem before starting to code. For this reason, there is always a two-parallel process with coding and computational thinking.

This study aimed to design lesson plans with coloring books and explore students' ideas of algorithms. The target group was 15 sixth-grade students and 8 lesson study team members. Research design based on one month of weekly cycle of the TLSOA model (Inprasitha, 2022); collaboratively plan; collaboratively do with four steps of Open Approach (OA), and collaboratively see. Three sets of coloring sheets (six tasks) were utilised as tools. Data were collected by recording all processes of TLSOA model and from students' worksheets. The results revealed that students' ideas of algorithm with coloring book could be enhanced when the lesson plan run through four steps of OA as follows: (1) posing coloring task; (2) students self-learning by coloring, involving following instructions and designing instruction by themselves; (3) whole class discussion and comparison, students could share their idea and reflect what they do; and (4) the teacher and students summarize through students' ideas. The time taken for each set of coloring sheets should be between 60-90 minutes, depending on grade level. There were two types of students' ideas related to the algorithms; the first was the idea of order, and the second was the idea of condition in instructions. The ideas of the condition included condition with positions, condition with directions, and condition with word quantity. Students appreciate their friends' ideas during whole class discussions. Some students could use others' ideas to improve their instructions.

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Oral communication

A SELF-BASED COLLABORATIVE CONCEPT EXPLORATION APPROACH TO MATHEMATICS TEACHERS' LEARNING

Olive Chapman

University of Calgary

This paper reports on the initial stage of a project that investigates a *self-based collaborative concept exploration* (SCCE) professional learning approach aimed at enhancing teachers' knowledge of mathematics for teaching. This stage addressed the research question: Do teachers share self-based knowledge and curiosity of a mathematics concept that aligns with and support the SCCE to potentially achieve its aims?

The theoretical framework of SCCE consists of 3 elements: *self-based*, *collaboration*, and *concept exploration*. *Self-based* involves what a teacher knows and wants to know about a mathematics concept based on their practical knowledge. *Collaboration* involves teachers from different grades working together to pool their individual knowledge and curiosities about the concept to arrive at or become aware of something different. *Concept exploration* involves studying the concept in a variety of ways, as Usiskin et al. (2003) suggested, including historical development, different definitions/ meanings, multiple approaches/representations, and applications of concept. The self-based knowledge and curiosity drive the collaboration that drives the concept exploration to deepen understanding of the math concept from a practice-based perspective.

This exploratory case study involved a group of 10 experienced teachers from the same school with grades 2 to 12. Six were elementary teachers (grades 3 to 6) and four were secondary teachers (grades 8, 10, and 11). Data collection focused on what knowledge they could share and what they wanted to know about the concept, which are important for the success of the SCCE. The teachers decided to start with zero (the focus here) as a concept common to all of them and responded to prompts including (1) You are learning about zero, an expert on zero is visiting your class, what would you ask to know more about it? (2) Write zero in different ways using a different mathematics concept or topic for each. Data analysis involved categorizing their written responses for each prompt to form themes and the frequency of responses for each theme was computed.

The findings indicated that the teachers posed questions that would be meaningful to explore to understand zero with more depth (e.g., meaning of zero, characterizing zero, division by zero, purpose of zero, history of zero). They also demonstrated collective knowledge of different ways in which zero is used (e.g., as empty set, in operations, point on number line, coordinate point, slope, 2nd derivative). Findings suggest that teachers could share the type of knowledge and curiosity about a mathematics concept to support a SCCE approach to their learning to enhance their knowledge of it.

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A COIN FLIP SEQUENCE CONTENTION: RELATIVE PROBABILITY COMPARISON RESEARCH IS INCLUSIVE

Egan J Chernoff

University of Saskatchewan

This Oral Communication will establish that a task foundational to relative probability comparison research (e.g., establishing which of two or more coin flip or birth order sequences is least likely to occur) changed, logically, when it moved from the field of psychology to mathematics education. This paper preserves said contention in writing.

In psychology, as I will demonstrate, the task is (implicitly) *exclusive* in nature, that is, participants were presented with two sequences and asked to deem which was least likely to occur. In the field of mathematics education, however, as I will also establish, the task is *inclusive* in nature, that is, participants were presented with two (or more) sequences and asked to determine which sequence was least likely to occur *or* if both or all sequences presented were equally likely to occur.

By way of analogy, consider an individual that lives at the gym or lives at home. "Or", in this example, appears exclusive. When I am at home then I am not at the gym, and when I am at the gym then I am not at home. What if, however, one lives at the gym? If I live at the gym, then "or" can be considered inclusive. What if, for years now, research participants, those who correctly answered that both or all sequences are equally likely to occur, were, instead, indicating that both or all sequences being equally likely to occur was least likely to occur? If this could be the case, then a revisitation of a large swath of relative probability comparison research is in order.

I contend this could be the case. Say, for example, I am tasked with determining (using H for "heads" and T for "tails") whether the coin flip sequence HHHHT or HTHTH is least likely to occur or both sequences are equally likely to occur. From an implicitly inclusive interpretation of the task, one could interpret the choice of both sequences being equally likely to occur as the least likely option. Then, I have picked the correct answer but not for the "right" reasons. Picking the equally likely option, while it is supposed to align with the notion that the two sequences HHHHT and HTHTH are equally likely to occur, could, alternatively, be aligning with the notion that the two sequences presented being equally likely to occur is the least likely of the options presented. The equally likely option requires reconsideration, I further contend.

For the record, I do have data that supports my implicitly inclusive idiosyncrasy. As such, while relative probability comparison research has, for the most part, been focused on normatively incorrect responses, which has proved fruitful in terms of the theories, models and frameworks that stem from accounting for incorrect, inconsistent and sometimes inexplicable responses to probability tasks, perhaps a focus on "correct" answers is also in order for research on relative probability comparisons in the future.

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CHARACTERISING TEACHERS' KNOWLEDGE IN TEACHING DIVISION USING SORT-SEQUENCE-ACT (SSA) METHOD

Su Ngin Chia and Ban Heng Choy

National Institute of Education, Nanyang Technological University, Singapore

What a mathematics teacher knows and how knowledge is transformed during teaching is an indication of one's teaching expertise. Researchers have characterised a teacher's knowledge according to Ball et al.'s (2008) framework of mathematical knowledge for teaching (MKT) using pen-and-paper instruments. However, these instruments do not capture the complexity of a teacher's MKT and the interplay among the different knowledge domains during teaching. In our study, we explore the key question: How can a teacher's MKT and the interplay of the different knowledge domains be characterised in ways that reflect the complexity of the teaching practice?

Moving away from a pen-and-paper instrument, we developed a novel SSA method, a task-based interview methodology (Goldin, 2000) consisting of three different tasks which are the sorting task (Sort), the sequencing task (Sequence) and the demonstration of teaching (Act). For this method, sorting cards instrument is designed to reflect different ideas in a topic. For example, to understand teachers' MKT in teaching division, different representations of division concepts using story problem and concrete representations can be printed on the cards. During sorting task, participants would sort and classify the cards by discerning the properties among the representations on the cards. For the second task, the participants would sequence the cards according to how they would introduce the concepts to the students. During the demonstration of teaching, teachers would demonstrate how they would teach a concept that is reflected on the cards that they find difficult to teach.

In this presentation, we will share how our SSA method can be used to characterise a teacher's MKT in the context of teaching division at the primary school level. We will illustrate how SSA method can capture the participants' different knowledge domains and their interplay more deeply to highlight the complexity of teaching practice. We will discuss existing limitations and discuss possible directions for future research.

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UNPACKING THE COMPLEXITY OF TEACHERS' THINKING THROUGH THEIR DESIGN OF INSTRUCTIONAL MATERIALS

Sze Looi Chin[1], Ban Heng Choy[2], and Yew Hoong Leong[2]

[1]Massey University, [2]National Institute of Education, Nanyang Technological University

Conceiving of teaching as a design activity offers an insightful perspective on the intricate nature of teachers' decision-making processes (Brown, 2009). While studies have documented two-step shifts between teachers' lesson plans and their instruction (Amador, 2016), our study explored the complexities in teachers' design thinking when multiple shifts occur. In Singapore, secondary mathematics teachers commonly design their own instructional materials (IMs) as part of their lesson planning, which comprise items that are deliberately selected, modified, created, and sequenced. Designing these IMs involves making goal-oriented decisions that impact how their lessons will unfold, hence, these IMs offer snapshots of teachers' design thinking, and shifts as they occur.

In this presentation, we present the design processes of two exemplifying cases of Singapore secondary mathematics teachers as they designed and implemented their IMs. A flexible frame-size analysis framework (Chin, 2024) was developed and used to document teachers' design decisions, and to unpack their pedagogical goals and design principles across different frame sizes and design cycles. Findings generated from the analysis of their IMs, semi-structured interviews, and lesson observations, revealed four prominent design principles between the two teachers that were utilised to surface and address students' errors. Shifts in teachers' design decisions documented across the design cycles suggest that teachers' evolving goals may not be restricted to only one frame size, demonstrating the complexities of their design processes. In the presentation, more details and implications will be discussed.

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PARENTS AND TEACHERS TALKING ABOUT MATHEMATICS EDUCATION: CONNECTING HOME AND SCHOOL

Marta Civil and Fany Salazar

University of Arizona

This study responds to the need to address power issues between home mathematics and school mathematics in minoritized communities (Takeuchi, 2018). Our research is grounded on the concepts of parents as intellectual resources and funds of knowledge (González, et al. 2005). We have developed tasks and tools to bring immigrant parents and teachers together to engage in mathematical tasks and conversations. In this presentation we illustrate the use of one tool with 18 mothers of Mexican origin and 16 teachers across two research projects in the Southwest of the United States. We look at what are participants' views on teaching and learning mathematics and on each other's (parent / teacher) role? Through this tool participants read quotes from parents and teachers from prior projects and then choose the quote that most speaks to them. They then share the reasons for their choice in a whole group conversation. An example of a quote is, "Latino children, if their parents come from Mexico, then they probably did it a different way. And so, then you're bringing in another way so that they're seeing maybe even a third or a fourth or a fifth way to attack a problem." All the sessions were videotaped. Clips were selected and analyzed in terms of parents' and teachers' views on school ways and home ways of doing mathematics. Findings included teachers' gained appreciation of students bringing their home mathematics, but also some tensions that these different methods may present in the classroom context. The mothers shared with the teachers how they had learned mathematics in school, which led to an exchange around different approaches, the importance of knowing why they work, and not just getting the answer. This study points to the power of using realistic scenarios to promote a two-way dialogue where teachers can recognize parents as resources in mathematics education and where parents can gain a deeper understanding of teachers' approaches to teaching.

Additional information

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DOES AGENCY IMPACT YOUNG CHILDREN’S OPPORTUNITIES TO ENGAGE WITH MATHEMATICS?

Audrey Cooke, Maria Johansson, and Eva Norén

Curtin University, Luleå Tekniska Universitet, Stockholms universitet

Mathematical opportunities for children in early childhood, even very young and preverbal children, are varied. Delacour (2022) describes how teachers in preschool determine the opportunities children have to develop mathematical understandings. These opportunities may involve choices around whether the mathematical experiences are planned (and controlled) by the teacher or spontaneously emerge from the children (and are child-led). In a planned teacher-led experience, the teacher is in control and children follow instructions and steps that are created by the teacher to maintain the focus on the mathematical understanding the teacher wants the children to develop. In a spontaneous child-led activity, such as free play, the teacher follows the ‘lead’ of the children and the children develop the mathematical understanding through their engagement, having choice and agency in the steps and processes they use; this requires the teacher to identify the mathematical understandings the children are developing. Bishop (1988, p. 182) identifies six activities that he considers as both universal and a requirement that underpins the development of mathematical understandings. These activities are counting, locating, measuring, designing, playing, and explaining. Using Bishop’s mathematical activities as a lens can help teachers identify the mathematics children may be demonstrating and developing *in situ*.

This presentation is part of a larger research project that is analysing video collected via a 360-degree camera that was suspended from the ceiling of a room in a Swedish preschool. The video recorded the entire room and captured all participants and materials within that room. The video of focus for this paper involves one educator and two children; the children were aged 3 years and 2 years and were both described as having sufficient language to communicate verbally. To investigate whether a child’s agency impacts on their opportunities to engage with mathematics, this presentation shares the preliminary findings regarding mathematical understandings that were identified when the 3-year-old child is engaged with the same resources – initially, in a spontaneous play-based episode initiated by the child, and subsequently during a teacher-led episode. The results to be discussed in detail indicate more of Bishop’s six mathematical activities could be identified during the spontaneous episode in comparison to the mathematical activities identified in the teacher-led episode.

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VOLUNTARY MATH CLUBS – A SUSTAINABLE MODEL FOR IMPLEMENTING EDUCATIONAL INNOVATION?

Jason Cooper and Aamer Badarne

Weizmann Institute of Science

Many projects in mathematics education discontinue when funding ends. Raising the Bar in Mathematics Classrooms (RBMC, Koichu et al., 2022) is an implementation project whose goal is to encourage middle school mathematics teachers to incorporate problem solving in their teaching. Teachers solve problems, discuss pedagogies, and implement them in their classrooms for professional accreditation. When its four-year funding ends, professional development will not be viable, and we are seeking models for sustaining impact. Difficult circumstances in Israel in October 2023 created severe educational challenges, including a few weeks of distance learning. To help teachers cope, our research group initiated an online mathematical “club”, where we introduced problems, many from the RBMC project, that teacher could use for "special lessons". The club continues to meet weekly, and over 150 teachers have attended at least once. Our research investigates this club as a model for sustaining the project's impact.

The club has met 21 times. Education has returned to normal in most parts of the country, yet 16 teachers continue to attend regularly, six intermittently, and 55 have attended at least twice. We interviewed one intermittent and five regular participants to investigate their reasons for participation, and how the club influences the field.

<u>Findings</u>: All interviewed teachers expressed personal enjoyment from participating in the club. Many indicated specific problems and discussions that challenged them to think about familiar content in new ways. All felt that students should engage in problem solving, and that the presented problems are suitable for students, yet none of them brought any of the problems to their own classrooms, mainly due to lack of time to cover prescribed content. Some introduced problems to other teachers, many of whom expressed enthusiasm, yet they too did not enact them in class. While these findings are disappointing, we nevertheless believe that teachers' voluntary participation in such a club holds potential for sustained impact, though perhaps not as we have previously conceptualized impact (Cooper & Koichu, 2021). Future research will seek new conceptualizations of impact under which to investigate the club.

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DOES EVERYONE HAVE TO KNOW BY HEART HOW MUCH 8 TIMES 7 IS? A CROSS-SECTIONAL STUDY ON STUDENTS' VIEWS AND PERFORMANCE

Csaba Csíkos[1], Ildikó Bereczki[2], and Fanni Biró[1]

[1]University of Szeged, MTA-SZTE Metacognition Research Group [2]ELTE University

Arithmetic skills are essential components of learning and doing mathematics. The term 'skill' refers to more or less automatized processes that include number facts and quick computational processes. From subsequent research, a 3-second limit appeared for measuring and identifying number facts (Jordan et al., 2003). As for the curricular presence of number facts from the multiplication table, Olfos et al. (2020) compared several different countries' approaches, and rightly asked "Why do some countries divide the multiplication table into several grades and others do not?" (p. 4) In general, by the end of the 4th grade of schooling, children are required to learn the multiplications with one-digit numbers up to 100. In this research, there were two research questions formulated (1) What is the connection between students' performance and their views on the importance of knowing number facts by heart? (2) What is the connection between students' mastery of number facts and their arithmetic skill level? In our cross-sectional developmental study design, 3rd, 5th, and 7th grade students took part, altogether 254 students. Three measures were used. *Number facts*. Ten items were administered in multiple-choice format; three additions, three subtractions, two multiplications, and two divisions. *Students' views on multiplication facts.* The question was: Does everyone have to know by heart how much is…? *Arithmetic skills.* This subtest consisted of 24 items in open-ended formats addressing the four basic arithmetic operations. Our results suggest that the multiplications required by the curriculum are judged to be important to know by heart with average values above 4 on the five-point Likert-scale. There is one anomaly deserving special attention: the judged importance of knowing 7 times 8 by heart. Furthermore, the changing role of number facts in arithmetic skill performance could be revealed in this study.

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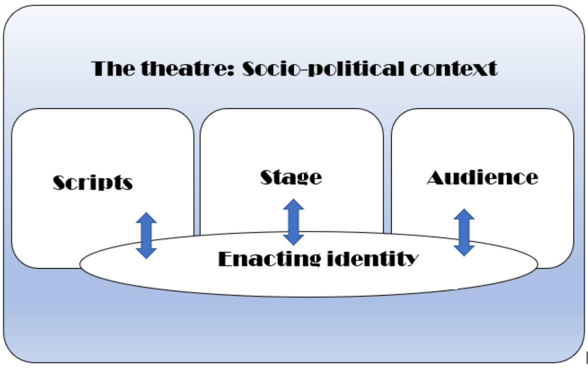
NEW INSIGHTS FROM "IDENTITY SITES": OPERATIONALISING MATHEMATICS LEARNER IDENTITY WITH A THEATRE METAPHOR

Lisa Darragh

The University of Auckland

Mathematics learner identity (MLI) continues to be a topic of interest within mathematics education research and whilst the construct has become more consistently defined in research of late, it remains a challenge for researchers to operationalise MLI in a theoretically coherent manner (Graven & Heyd-Metzunayim, 2019). In this paper I present a theatre metaphor for operationalising MLI that offers multiple perspectives on the construct. I begin by situating identity within the wider socio-political context: the theatre. Then, in order to understand identity performance, we might consider an examination of: the *scripts* i.e. the societal discourses available to the performer; the *stage,* or local context of the performance; and the *audience,* who recognises identity in the performance. I suggest these are three key *sites* for identity; that is, identity plays out in the interactive spaces depicted by each arrow in the figure. Performing the mathematics learner self requires drawing from, ignoring, or improvising with various societal narratives. Performing the mathematics learner self is enabled or constrained by the nature of the local stage, such as the norms for mathematics learning defined in the classroom. Performing the mathematics learner self requires the audience (e.g. the teacher) to recognise the mathematics learner that is performed. Each site highlights a different aspect of the identity performance (and understanding of the individual), and thus a focus on any one of these identity sites misses the opportunities afforded by the others.



To elaborate on this framework, I will present findings from a multiple embedded case study of MLI in online and in-class settings. Analysis of two very different Year 10 mathematics classes (13-14 year-olds) highlight different aspects of the wider socio-political context that interact with identity. I suggest these three different *identity sites* may inform a rich discussion into research methodology on the construct of MLI.

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CHATGPT AS A TUTORING TOOL FOR NONPARAMETRIC STATISTICS: A COMPREHENSIVE ANALYSIS

Joel L. De Castro and Dennis Lee Jarvis B. Ybañez

University of the Philippines Open University

In the past few years, Artificial Intelligence (AI) technology has started to transform education by providing cutting-edge resources for individualized learning. The convergence of AI and data analysis has specifically sparked interests in its potential to offer new approaches in teaching statistics. One tool that is widely used by students is OpenAI's language model, ChatGPT, which provides a significant promise as tutoring tool due to its capacity to produce text responses that resemble human responses. However, despite its abundance in math tutoring resources by virtue of the massive data it contains, there remain concerns about its efficiency and accuracy (Deng & Lin, 2023). As an initial attempt to understand how ChatGPT can be effectively used in learning statistics, this study aims to investigate the use of ChatGPT as a tool for conducting nonparametric statistical analysis, which is valuable in various fields for providing data-driven insights without strict distributional assumptions. Specifically, this study discusses the accuracy of ChatGPT in performing nonparametric statistical analysis, creating null and alternative hypotheses, and providing clear and coherent explanations and interpretations of the analyses. Seven nonparametric tests were conducted using both ChatGPT and different statistical software applications (e.g., Jeffreys's Amazing Statistics Program) to evaluate ChatGPT's accuracy by determining the percent error of the computed values. The differences in the formulated hypotheses statements and statistical interpretations were also considered.

Results show that ChatGPT demonstrates a reasonable level of accuracy in computing test statistics for nonparametric tests such as Analysis of Variance (ANOVA), Wilcoxon signed-rank test, and McNemar. The computed values show only a small percentage of error indicating that the model can perform well in the numerical aspects of these statistical tests. Additionally, it is worth noting that ChatGPT excels in formulating hypotheses and identifying critical values and degrees of freedom, demonstrating a solid grasp of the theoretical and conceptual aspects of nonparametric statistical tests. While ChatGPT may have room for improvement in reducing the small percentage of error in numerical computations, its accuracy in articulating the foundational concepts of nonparametric statistics suggests a promising capability for assisting with hypothesis formulation and interpretation of statistical results. Further refinement of the numerical inaccuracy found in this study could enhance ChatGPT's overall performance as a tutoring tool in performing nonparametric statistical analyses.

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COMPARING CHINESE PRIMARY STUDENTS' CONCEPTIONS OF IMPROPER FRACTIONS AND MIXED NUMBERS

Xixi Deng[1], Rui Ding[1], Xinchun Wang[1], Michael Sun[2], Bingqian Wei[3], and Ron Tzur[3]

[1]Northeast Normal University, China; [2]Englewood High School, USA; [3] University of Colorado Denver, USA

Students often struggle with understanding the conversion of improper fractions (e.g., 10/7) to and from mixed numbers (e.g., 1 3/7). Their reasoning about such units may differ. In this study, we further examine this difference while addressing the research problem: How may Chinese primary students' reasoning about improper fractions and mixed numbers differ in relation to their multiplicative and unit fraction reasoning? Tzur and Hunt (2022) have postulated an 8-scheme progression organized in two clusters, one based on iteration and the other on recursive partitioning. This study focused on the *Equipartitioning Scheme* (EPS) and the *Iterative Fraction Scheme* (IFS) of the first cluster. The EPS involves anticipating multiplicative relations between two units, one considered as the whole and the other iterated to fit *n* times within that whole. The IFS involves iterating a given unit fraction (*1/n*) *m* times to produce a composite fraction ($1/n*m=m/n$) that is larger than the whole ($m/n>n/n$). According to the 8-scheme progression, fractions, as multiplicative relations, are produced by the iteration of units. Thus, we also link the EPS and IFS, two iteration-based schemes, with multiplicative reasoning (MR) about whole numbers. We use quantitative analyses of data collected from 4^{th}-6^{th} graders in China (n=217) who responded to a valid and reliable, 35-item measure. The findings indicate that student responses to improper fractions (33.0%) outdo those to mixed numbers (16.6%). Furthermore, we analyzed the extent to which MR moderates EPS impact on each category – improper fractions and mixed numbers. MR was set to be a moderator with three levels: the mean and one standard deviation below (-1SD) or above (+1SD) the mean. The β-value indicates the slope of each regression line for -1SD, the mean, and +1SD. These are, respectively, 0.15, 0.30, and 0.42 for mixed numbers; and 0.38, 0.53, and 0.65 for improper fractions. This moderation is statistically significant for mixed numbers (F=4.41, p=0.037) but not for improper fractions (F=3.54, p=0.061). Besides, the proportions among slopes at the -1SD and +1SD levels (2.8 for mixed numbers, 1.7 for improper fractions) show the difference in moderation. Based on these findings, MR differs in its moderation of the impact of EPS on improper fractions and mixed numbers. Thus, we claim that this study indicates a conceptual difference, rooted in students' MR, between reasoning about improper fractions and mixed numbers.

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PEDAGOGICAL PRACTICES: PISA 2022 DATA ANALYSIS OF INQUIRY-BASED AND TEACHER-DIRECTED APPROACHES

Marcel Derkum and Tanya Evans

University of Auckland

In recent decades, the implementation of inquiry-based learning in mathematics classrooms has garnered attention for its potential to enhance students' understanding and critical thinking. However, its effectiveness compared to traditional teacher-directed methods remains debated (Evans & Dietrich, 2022; Kirschner et al., 2006). This study examines New Zealand's education data from PISA 2022, focusing on 15-year-old students' mathematical literacy. Through statistical analysis, it investigates the impact of two pedagogical approaches—Inquiry-based and Teacher-directed—on students' mathematics scores while controlling for socio-economic factors.

Based on the student questionnaire, two variables, IBMETHOD and TDMETHOD were created. IBMETHOD represents an Inquiry-based approach, wherein students are encouraged to explore new-to-them mathematics without explanations given by a teacher beforehand. The teacher acts as a facilitator, providing resources and encouragement while refraining from direct explanations. Emphasis is placed on engaging students in everyday life problem-solving scenarios and prompting students to discover new ways to solve problems. TDMETHOD represents the contrasting Teacher-directed approach, characterised by teachers providing explicit explanations of new concepts, demonstrating algorithms and worked problems, and guiding students through activities to replicate and practice demonstrated procedures.

Hierarchical multiple regression analysis revealed that both IBMETHOD and TDMETHOD significantly contribute to predicting mathematical literacy scores, even after controlling for socio-economic status. Notably, the addition of each IBMETHOD and TDMETHOD to the model leads to significant improvements in predictive accuracy. The main finding is that students exposed to Teacher-directed instruction demonstrate significantly higher mathematical literacy scores compared to those in Inquiry-based classrooms, even after accounting for socio-economic differences. However, the methodological limitations of this analytical approach will be discussed.

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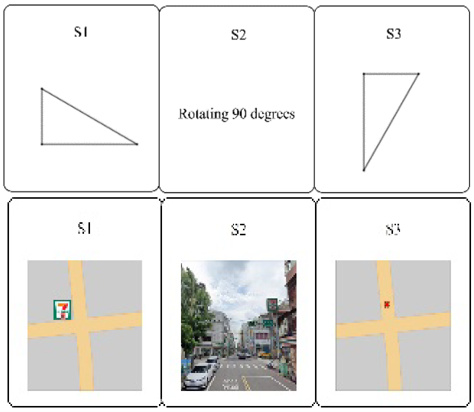
Oral communication

CHARACTERIZATION OF SPATIAL ABILITY: AN ERP COMPARISON BETWEEN GEOMETRY AND CARTOGRAPHY

Tsu-Jen Ding, Hui-Yu Hsu, Yin-Hsuan Yeh, and Chen-Yu Yao

National Tsing Hua University, Taiwan R.O.C



Spatial ability is essential to development of different subject domain areas including geometry and cartography. Dissociating spatial ability into distinct sub-constructs is one of the goals in psychological research. In this study, we hypothesize that geometry and cartography, especially related to self-localization by map, require different types of spatial-ability. That is, geometry problem-solving often has students to perform mental-rotation actions, while self-localization necessitates the use of perspective-taking ability. To examine the hypothesis, the ERP (Event-Related Potential) techniques were adopted. High school students were randomly assigned to geometry group (37 students) and cartography group (39 students). We designed two tests. One is geometry test that asks to perform mental rotation based on mental images of triangles created by students. Self-localization test needs one to perform perspective-taking ability to identify the relationship between map symbols and real-world landmarks. The ERP brainwaves were analysed based on Gunia et al. (2021) as they indicated that spatial ability often occurs in parietal-occipital brain areas. The analysis showed that both geometry group and cartography group all had P300 component in the POz channel located between parietal and occipital brain areas. This is aligned with what Gunia et al. have found in their study. In particular, we found that RRN (rotation-related negativity) component occurred in geometry group but not in cartography group, which confirmed geometry requires the performance of mental rotation actions. We further checked Cz, C3, and C4 channels in central brain area as literature indicated that the three channels are crucial to check if one performs perspective-taking ability. In addition, the negativity peaks in C3 and C4 allow to understand ones' motor intension of body-movements oriented to either right side (C3) or left side (C4). Our analysis demonstrated that cartography group had significant negativity peak in Cz but not geometry group. C3 and C4 also only occurred in cartography group but not geometry group. The ERP analysis results confirmed that geometry and cartography require different types of spatial ability. Moreover, the ERP techniques can be a useful tool in distinguishing the two types of spatial ability.

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IS TRANSPARENT ALSO BEAUTIFUL? FEATURES OF GRAPHS THAT MAKE THEM ACCESSIBLE IN STUDENTS' VIEWS

Andrea Amico and Luca Doria

Università del Piemonte Orientale, Italy

Considering Ainley's (2000) definition of transparency and the importance that reading a graph has in acquiring mathematical skills, we want to investigate which aesthetic features can improve the readability of a graph, since it seems that one of the fundamental characteristics of a graph, able to help students perform a little better, is its aesthetics (Amico & Doria, 2023). This communication is a follow up to a study presented in 2023 at the annual PME 46 International Conference held in Haifa. We highlighted a connection between the aesthetics of a graph and its transparency, but the direction of the connection was not clear (Amico & Doria, 2023).

The study consists of a lecture delivered three times to 88 students from high schools and university, who were then asked to reply to a questionnaire and the responses were analysed. The characteristics, two for *aesthetics* and two for *transparency*, which are appreciated by the students, are:

(I) *order*: the spatial location of the elements that make up the graph, they, in fact, according to their disposition can bring the graph to be harmonious in its forms or can make it very messy and disrupted to the eye;

(II) presence of *colors*: a graph designed to be composed of a greater number of data and with a distinct color for each of them leads the observer to prefer it aesthetically to another with less colors;

(III) presence of the data: consequent easiness of individuation on the diagram, that is how much data is placed in a clear and visible way inside of the diagram;

(IV) comparability with the scale: if in the background of the graph there are lines, which correspond to the scale of the corresponding axis, it amplifies the ability to clearly define the numerical value of the data one is considering.

It was possible to isolate four distinct characteristics (two for aesthetics, two for transparency) that could be crucial in students' accessibility to "hidden" data within a graphic representation. This is, in our view, the main conclusion we can get.

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PROSPECTIVE TEACHERS' MOVES FOR FOSTERING STUDENTS' UNDERSTANDING IN RELATION TO THEIR DIAGNOSTIC JUDGEMENTS

Jennifer Dröse and Lena Wessel

Paderborn University

For student-centred teaching teachers' diagnostic judgements of students' obstacles as well as their moves to foster students' understanding have been shown to be important. To investigate the relationship between diagnostic judgements and teacher moves for fostering students' understanding, we build upon the distinction of three cognitive processes (*perceiving, interpreting and decision making)* from an information-processing perspective (Loibl, Leuders, & Dörfler, 2020). To pursue a content-related approach on these processes they are investigated concerning two dimensions of the mathematical content structure: (1) *conceptual and procedural knowledge elements* and (2) *knowledge elements of the current and prior learning content*. As prospective teachers have been shown to struggle to identify conceptual obstacles and to propose relating concept-based interventions for students (Son, 2013), the following research question is investigated topic-specifically for the topic of multi-digit multiplication of natural and decimal numbers: *Which knowledge elements are addressed in prospective teachers' moves and how are they related to their diagnostic judgements?*

For assessing diagnostic judgements and teacher moves a transcript vignette (Buchbinder & Kuntze, 2018) was used. The data (n = 33) was collected in two university mathematics education courses for German prospective middle school teachers in written format and afterwards coded concerning the addressed knowledge elements.

First results concerning the knowledge elements prospective teachers address in their moves indicate that nearly as many prospective teachers proposed a concept-based move as a procedure-based move. In Addition, the proposed move relates to the main point of the prospective teachers' diagnostic judgement being either conceptual or procedural but not necessarily to their focus on the prior or current learning content.

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INQUIRY- AND MODELLING-BASED LEARNING IN AN ARCHAEOLOGICAL SITUATION

Pere J. Falcó-Solsona, Carlos Ledezma, Gemma Sala-Sebastià, Vicenç Font, and Adriana Breda

University of Barcelona

The investigation presented is focused on the relationships established between the inquiry and modelling processes emerging in a learning situation (LS) based on a realistic and authentic archaeological context: how both processes mutually enrich, promoting the use of mathematical models more accurate and closer to reality than they would have been without the previous development of inquiry tasks.

Inquiry-Based Mathematical Education is defined by Artigue and Blomhøj (2013) as the learning model in which students are encouraged to act as scientists and mathematicians do when they solve real-world problems. Modelling has been defined as a mathematical process that describes the translation of a problem from a real context into mathematics and its return to reality. We take as a reference the modelling cycle proposed by Blum and Leiß (2007), which explains the phases and transitions an individual goes through to solve a modelling problem.

A case study was conducted based on a LS based on an archaeological context (Roman ceramic pieces found) designed to promote the development of inquiry and modelling processes. It was implemented at a secondary education school in Badalona (Spain), with 93 students (divided into three groups of 31 students each) aged 12–13.

The students needed to apply a mathematical model representing the C-14 degradation process to date the pieces together with some data available in the preliminary report provided by the Badalona Museum; they discovered that the pieces were made during the 1st century A.D. They also built a model to reproduce the original shape of the pieces which were solids of revolution, and their sections were circumferences, they obtained the complete circumference figure profile and reproduced their original shapes in 3D representations. When a problem situation is dealt with, neither the inquiry nor the modelling cycle alone is sufficient to find a plausible solution. More specifically, both cycles need to be combined, enriching both processes.

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THREE-PHASE CYCLICAL FLIPPED CLASSROOM MODEL: DESIGN AND EFFECTS ON PRODUCTIVE DISPOSITION

Patrick John M. Fernandez and Angela Fatima H. Guzon

Ateneo de Manila University

Flipped classrooms have been popularized by Bergmann and Sams (2012), but interest in flipped classrooms, both in practice and research, significantly increased since the COVID-19 pandemic. Research on flipped mathematics classrooms predominantly employs quantitative-leaning methods to evaluate the effectiveness of flipped classrooms through the academic performance or perceptions of students; rarely does flipped classroom research focus on students' productive disposition toward mathematics. Moreover, designs of flipped classrooms in the literature are rarely explicitly grounded in theoretical frameworks. Building upon the results of previous studies, this study addresses these gaps by designing the three-phase cyclical flipped classroom (3PCFC) backed by a network of theories such as Transactional Distance Theory (Moore, 2005) and the Theory of Didactic Situations (Brousseau, 1997). To examine the role of activities and interactions in a flipped classroom in shaping students' productive disposition toward mathematics, the 3PCFC model is implemented in two classes of an undergraduate first course on applied calculus in a private tertiary institution. Thematic analysis will be employed on responses to structured journal logs, which are collected twice a week from 60 participants. Expected outcomes of the ongoing study include a positive development of students' productive disposition toward mathematics. Dynamic and collaborative classroom activities are anticipated to increase students' tendency to see themselves as effective learners and doers of mathematics. Additionally, out-of-class activities are expected to develop students' self-regulated learning strategies and foster their belief that steady effort in mathematics pays off. In a volatile world, the effects of a flipped classroom on students' learning strategies, especially self-regulation, are as important as the effects of a flipped classroom on students' academic performance. Flipped classrooms can equip students with the soft skills necessary to adapt to distance learning due to any disruption in education. The 3PCFC model will be revised according to the results of the study to serve as a framework for implementing flipped mathematics classrooms.

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ADAPTING STRATEGIES DUE TO REFUTATIONS IN CALCULUS

Michael Fischer

University of Graz

Since mathematics is a proofing science, argumentation and proof are integral to mathematics curricula worldwide. Therefore, a descriptive framework for proving is essential to better understand students' thought processes and to design and implement appropriate guidance based on their existing knowledge and abilities. Although numerous frameworks exist, only a few studies have either utilized an existing one or developed a new one to explore student activities related to proofs and refutations, examining how students respond to counterexamples and adjust their hypotheses and proofs accordingly. Komatsu (2012) established a framework based on Lakatos's heuristic rules (1976) to address this. This framework outlines desirable behaviors when addressing local and global counterexamples, with the former relating to a sub-lemma of the proof and the latter to the complete theorem.

According to the spiral principle in mathematics education, learners should repeatedly encounter argumentative challenges and mathematical proofs. However, many theorems in secondary-level Calculus can only be proven using traditional methods, often referred to as formal, which are impractical in a school context due to the absence of necessary mathematical tools. Instead, algorithmic strategies are commonly employed in calculus tasks. These strategies are understood as individual steps in a problem-solving process. They are not necessarily applied linearly, but are connected in a specific manner and ideally based on geometric-intuitive reasoning (Swidan, 2022). I extend therefore the framework developed by Komatsu (2012) to strategies and the underlying argumentation of individual steps within the context of problem-solving in calculus. This framework is then applied in an interview study with first-year students concerning the task of graphical differentiation.

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TEACHER ACTIONS TO SUPPORT MATHEMATICAL REASONING THROUGH THE USE OF A CONCEPTUAL STARTER IN UPPER PRIMARY CLASSROOMS

Lauren Frazerhurst

Massey University

For students to effectively reason with mathematical concepts, teachers need to provide rich opportunities for conjecturing, investigating, justifying, and generalising. Carefully planned tasks coupled with discussion can offer students space to share their thinking and compare others' ideas with their own, allowing them to develop an understanding as to why particular concepts may be true or false without relying on simply being told by a teacher (Lannin et al., 2011). However, starter activities used in upper primary often focus on the rehearsal of basic facts, with games or tests used as isolated events and little opportunity for students to engage in mathematical reasoning (Tait-McCutcheon et al., 2011).

The focus of this study was to explore how teacher action can support students to develop reasoning skills during the use of a purposefully planned conceptual starter of choral counting. The study was conducted with 30 students aged nine to 12 years old in two classrooms in Aotearoa New Zealand. Six choral counting lesson starters in each classroom were observed and video recorded. Both the teacher and student discourse were analysed, as well as the specific actions taken by the teacher to support student reasoning and whether this resulted in students engaging in mathematical practices.

The preliminary findings suggest the potential of the use of purposefully planned choral count conceptual starters to facilitate discussion amongst students. Initial analysis of the data showed that the students were able to make connections to patterns within multiplication and division. However, the depth of student reasoning during discussions between teachers and students varied and depended on how thoroughly the teacher pressed for further detail. Emerging findings also highlight that to develop student reasoning using planned conceptual starters effectively, teachers need to have a strong understanding of the concepts that students will be exploring. This is necessary to support student generated explanations and to extend student thinking.

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SENSE OF BELONGING IN GENERAL EDUCATION MATH

Michelle Friend, Andrew Swift, Betty Love, and Nicole Infante

University of Nebraska at Omaha

Sense of belonging – a feeling of acceptance in a setting – is associated with positive academic outcomes (Osterman, 2000). Feeling that one belongs is important for members of underrepresented minority groups (Walton & Cohen, 2007). The sense of belonging to math scale has been used with students highly affiliated with math (Good et al., 2012) but not with non-STEM students. This study examines sense of belonging to math in a general education mathematics course.

Students in a general education math class (n = 536) completed a Qualtrics survey of attitudes at the beginning and end of the semester-long alternative to algebra. The course emphasized creativity, active learning, and in-class collaboration. Data include grades and survey results relating to math interest, math confidence, mindset, and the sense of belonging to math scale.

Confirmatory factor analysis of the sense of belonging to math scale resulted in a similar structure to the original construct, yielding six components:19.9% (Affect), 15.4% (Membership), 13.5% (Acceptance, positive), 11.8% (Acceptance, negative), 9.2%, (Desire to Fade), and 7.1% (Trust) of the total variance, respectively.

As hypothesized based on the class's emphasis on collaboration and a supportive environment, students' sense of belonging increased from the beginning to end of the semester (n=346, p=5x10^{19}). The observed increase for the average of all sense of belonging questions from beginning to the end of the semester was almost half a point on the five-point Likert scale (3.53 to 3.96). In the presentation, further results including the relationship between sense of belonging to math and other variables including grades and confidence will be discussed.

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KNOWLEDGE OF PATTERNING AMONG JAPANESE KINDERGARTEN TEACHERS

Atsuya Fukuzawa

Tokyo Gakugei University

Patterning is a foundational cognitive skill, and fostering patterning promotes the acquisition of various mathematical concepts. These patterns have not been discussed in Japanese early childhood education yet. The Japanese view of kindergarten teachers ("teachers") as "comprehensive guidance through play" is unique. It is believed that Japanese teachers are not familiar with patterning, but How about their knowledge? This study clarifies the status of Japanese teachers' knowledge of patterning. We define early childhood patterning as the activity of capturing patterns and structures as per Mulligan & Mitchelmore’s (2009) "Awareness of Mathematical Pattern and Structure." Kindergarten teachers with 6–31 years’ career and classroom experience participated in the survey, which was conducted in February 2024 using focus group interviews and multi-vocal visual ethnography about 90 min. The data were analyzed qualitatively using the Ueno method of qualitative analysis (Ueno, 2017). We have extracted 94 Ueno formula cards and 22 metadata pieces. Japanese teachers' actual knowledge of patterning: (1) accepting that patterning is a burden, (2) development of a need for patterning with which they were previously unfamiliar, and (3) inadequate understanding of patterning. Teachers were not au fait with patterning, so they experienced a "fear of the unknown" when presented with it. Teachers may be limited to discussing patterns and structures based on their own interpretations.

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HOW DO JAPANESE TEACHERS INTERVENE STUDENTS' MATHEMATICAL META-RULES?

Yuka Funahashi

Nara University of Education

This study aims to clarify how the teacher provokes students' modification of mathematical meta-rules in discourses in Japanese mathematics classrooms. As a case study, I analyzed teachers' narratives, mainly focusing on *subjectifying* (Sfard, 2008), from a second-grade mathematics classroom and post-lesson interviews about simultaneous equations in junior high school.

To identify a mathematical discourse by its external characteristics, Sfard (2008) presents four properties. One is the routine, a set of metadiscursive rules that describe recurrent discursive patterns. She points out that meta-rules are dynamic structures that are constantly created and recreated in the course of interaction and that "modifications in metarules occur as a result of unintended deviations, of other interlocutors, and of intentional redesigning" (Sfard, 2008, p.221).

In the classroom, the teacher, with their expertise and relative authority in the discourse, greatly influences the creation of mathematical discourse rules. However, it is not a unidirectional process but develops as a product of the collaborative work of the teacher and students. In order to capture this collaborative activity, it is important to understand what the teacher said and to whom it was directed. Therefore, in this study, the characteristics of the teachers' narratives were described in the triple structure of $_{B}[\ A\ ]_{C}$, where A is the subject, B is the author, and C is the recipient (Sfard & Prusak, 2005), and their characteristics were examined.

The results revealed that there was a duality of recipients in the teachers' narratives. During the student's explanation of the solution, the teacher formed a response act toward the student. In this case, therefore, the recipient of the teacher's narrative was the student who was presenting the solution. On the other hand, the teacher's narratives in the subsequent utterances in the lesson and the post-lesson interview revealed that, while the teacher was responding to the presenting student, the recipients of his narratives were all the other students in the classroom. These teacher narratives function as interventions into the students' meta-rule of learning mathematics in the classroom. These narratives were considered to constitute *designated identity* (Sfard & Prusak, 2005) and influence the students' identities.

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TEACHING AND LEARNING THE DERIVATIVE TO FUTURE COMMERCIAL ENGINEERS

Maritza Galindo Illanes[1], Adriana Breda[2], Gemma Sala-Sebastià[2], and Vicenç Font[2]

Universidad San Sebastián (Chile); Universitat de Barcelona (Spain)

Several studies concerning the complexity of its meanings, its multiple representations, the teaching and learning processes, the suitability of the meaning of the derivative in the different curricula and the partial meanings in university teaching texts for engineering (Galindo Illanes & Breda, 2023). Working on the different meanings of a mathematical object is an aspect proposed by the Ontosemiotic Approach (Godino et al., 2007) to achieve a better understanding and competence of students in solving a greater variability of problems, which proposes to analyze the complexity of mathematical objects through their many meanings (partial meanings). Based on the above, this paper aims to present the results of a teaching and learning process of the derivative with future commercial engineers, which considers diverse epistemic configurations in the situation-problems on tangents, the calculation of derivative following derivative rules, applications of the derivative for the calculation of maximum and minimum and analysis of function graphs.

The research involved 90 commercial engineering students from the Faculty of Economics and Business of a Chilean university. For this study, three types of problem fields were considered: tangents; the calculation of the derivative from their rules and theorems; and applications of the derivative for the calculation of maximums and minimums, and analysis of function graphs. The results show that the students who participated in this implementation, although they presented some limitations, such as difficulty in sketching the utility function or the application of the derivative for decision-making, learned the relationship between the tangent to a curve and the geometric interpretation of the derivative and the application of the derivative in marginal analysis, showing, in addition, an improvement in the construction of the Cartesian conception of the tangent line.

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EXAMINATION OF GENDER DIFFERENCES IN THE IMPACT OF TEACHING QUALITY ON YEAR 9 STUDENTS' MATHEMATICS INTEREST IN NEW ZEALAND: A MULTI-GROUP COMPARISON OF 2019 TIMSS DATA

Huayu Gao[1], Tanya Evans[1], and Gavin T. L. Brown[1]

[1]University of Auckland, New Zealand

Lent's Social Cognitive Career Theory (SCCT) posits an *Interest Model* that highlights the causal relationship between environmental factors and interest, with confidence and value playing crucial roles (Lent et al., 1994). In this context, confidence emerges as a particularly significant factor. Lent et al. (1994) also suggest that gender differences exist within this model. This study seeks to validate Lent's *Interest Model* using New Zealand data from the 2019 TIMSS dataset, comprising 2,977 boys and 2,862 girls.

Our analysis examines whether maths confidence (MC) serves as the primary mediating factor, apart from maths value (MV), in the relationship between teaching quality (TQ) and maths interest (MI), as well as whether gender differences are present. Employing a structural equation model under SCCT, we investigated gender-specific pathways. Our findings reveal that confidence and values significantly mediate the relationship between teaching quality and maths interest. Notably, no substantial differences were observed between the mediating effects of confidence and values in any pathway: examining MC minus MV, we determined that both MC and MV contribute equally to the mediation effect for boys (β = -0.057, Cohen's d = 0.032) and girls (β = 0.029, Cohen's d = 0.130), nor were there significant gender disparities (TQ-MC-MI: β = 0.040, Cohen's d = 0.076; TQ-MV-MI: β = -0.045, Cohen's d = 0.082; and TQ-MC-MV-MI: β = 0.004, Cohen's d = 0.025). This suggests that individual differences may outweigh gender-group disparities in maths. Therefore, the prevalence of significant gender differences in motivational factors noted in some studies could be influenced by social stereotypes, potentially skewing research conclusions.

While this study partially confirms and expands SCCT, it is crucial to recognise that motivational factors are subject to ongoing fluctuations. Future research should prioritise longitudinal studies to offer more comprehensive and consistent evidence over time.

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IMPACT OF THE PEER INSTRUCTION ON THE ATTITUDE AND ACHIEVEMENT AMONG GRADE 8TH STUDENTS IN CHINA

Huixin Gao

University of Auckland

This study focuses on addressing the challenges in learning Unary quadratic equations in Chinese middle schools by investigating the effectiveness of implementing a Peer Instruction (PI). The research aims to determine whether the PI model can significantly improve students' attitude and achievement by fostering active learning, problem-solving skills, and deeper conceptual understanding. Therefore, the research question is: 1. Does the implementation of PI significantly affect students' attitudes towards mathematics as compared to the conventional instruction?2. How each component of mathematic attitudes affects students' mathematics learning outcomes following the implementation of PI?3. What are the perceived benefits and challenges encountered by students when exposed to the PI learning strategy? This study employs a mix research approach. The study compares two intact high school classrooms, each with 50 eighth grade students. The experimental group receives a three-months intervention using the PI, while the control group undergoes traditional instruction. In our study, we assessed the impact of a new teaching method on students' attitudes towards math and their performance. We used Topia's (1996) Attitudes towards Mathematics Instrument (ATMI) to measure students' security, value, motivation, and enjoyment in math, finding high reliability (Cronbach's alpha = 0.963). The study included a control group and an experimental group, with the latter experiencing the new teaching pedagogy.

Students' performance was measured using their midterm exam results, based on China's national scoring rubric for math exams. This provided objective data on their academic achievement. To gain deeper insights, we interviewed students from the experimental group, asking about their experiences, understanding of math concepts, and challenges faced.

The quantitative ATMI results, alongside midterm scores and qualitative interview data, gave us a broad view of the new pedagogy's effectiveness on both students' attitudes and achievement in mathematics.

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STRUCTURING PLCS FOR COLLEGE-LEVEL INSTRUCTIONAL CHANGE

Jessica Gehrtz and Priya V. Prasad

University of Texas at San Antonio

Instruction that provides opportunities for students to actively engage with content and collaborate with their peers can lead to increased student learning and conceptual understandings. Yet, didactic lecture remains the most common form of instruction in STEM courses in the United States. Instructional change can be difficult to catalyse, but professional development through professional learning communities (PLCs) can be one way to support instructors in using more research-based teaching practices.

The Practicality Ethic framework by Doyle and Ponder (1977) helped us to structure PLCs for our context (NSF #2116187), as this framework identified factors teachers consider when deciding whether innovative curricula is practical in the context of a classroom. We leveraged the Continuous Improvement (CI) cycle (Berk & Hiebert, 2009), the incremental lesson improvement strategy, to guide gradual course changes for more sustained implementation of research-based practices, while also leveraging the knowledge, experience, and priorities of instructors to guide these changes.

This research investigates what factors of the PLC support college algebra instructors in implementing more research-based teaching practices in their teaching, and the impact of these instructional changes on student outcomes. We collected the following video data over the span of three academic years: interviews with instructors (N=8), PLC meetings, and instructors' classes. We also collected data about student outcomes (e.g., final grades, pre- and post-test results).

Thematic analysis of the interviews revealed that instructors found peer observations, available resources (provided by course coordinators and created as part of the CI cycles), and the dedicated time to revise lessons and talk about pedagogical best practices enabled instructors to create more opportunities in class for students to actively engage with the mathematics. Systematic analysis of the class video data documented a shift away from didactic lecture to more student-centred activities. Quantitative analysis of the student data also showed evidence of improved student grades and conceptual understandings. This work has implications for structured, incremental, and sustained improvement of college mathematics instruction.

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Oral communication

21ST CENTURY COMPETENCIES: DIDACTICAL INSIGHTS ON CHANGES IN THE MATHEMATICS CURRICULUM

Evelin Anna Geszler and Janka Szeibert

Eötvös Loránd University

Implementing a comprehensive curriculum revision to the public education system, which considers every aspect, is a highly complex and time-consuming procedure. On the contrary, our rapidly changing world would require just the opposite.

In Hungary, we are facing the same obstacles, where public education is regulated at different levels. One of its most fundamental elements is the National Core Curriculum, the first version of which was introduced in 1995. One of the most criticized aspects of Hungarian education at the time was the excessive focus on lexical knowledge. A major change in 2005 was placing emphasis on application by introducing new topics such as graphs, statistics, and probability theory. Since then, numerous small adjustments have been made, but the core content and basic principles remained essentially the same until 2020 (Gordon Győri et al., 2020).

The current version of the curriculum came into force in September of this year. The undeniable purpose of the latest development was to place even greater emphasis on mathematical modelling and encourage students to apply what they have learned to various, real-world situations. Many countries have seen similar reforms that shifted the focus from procedural knowledge towards the acquisition of competencies such as modelling, reasoning, and problem-solving, which students should be able to master by the end of secondary education (Büchele et al., 2023).

In Hungary, we consider it cardinal that students taking their final exam possess adequate financial knowledge and statistical skills. However, the tasks that can be set within these topics, in the current form of evaluation, are immensely oversimplified models of reality. We have investigated the changes from a didactic perspective and have shaped possible future directions, i.e. the integration of digital tools.

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PRE-SERVICE TEACHERS' CURRICULAR NOTICING: ATTENDING TO FRACTION ACTIVITIES' FEATURES

Juan Manuel González-Forte, Cristina Zorrilla, Pere Ivars, and Ceneida Fernández

University of Alicante (Spain)

Curricular noticing is focused on how teachers make sense of the complexity of content and pedagogical opportunities in written and digital textbooks (Amador et al., 2017). It involves curricular attending (teachers' skills in viewing information contained in curriculum materials to inform teaching), curricular interpreting (a sense-making activity whereby teachers connect the curriculum information to their own mathematical and pedagogical content knowledge for teaching), and curricular responding (teachers' skill of making curricular decisions based on the interpretation of curricular materials). As there are many aspects of curriculum materials to which teachers can attend, we can support pre-service teachers (PTs) in recognising potentially rich mathematical opportunities and in overcoming the limitations of curriculum materials by providing them with theoretical lenses to notice. Students' hypothetical learning trajectories related to a mathematical concept have been shown as potential tools (theoretical lenses) to support PTs' noticing of students' mathematical thinking (Ivars et al., 2020). In the current study, that is part of a larger study with the objective of examining how PTs develop curricular noticing in a teacher training program, we focus on curricular attending. To do so, we examine how 171 PTs attend to noteworthy features (the mathematical elements, the modes of representation, the type of activity, and the type of fraction) of a sequence of five activities about the part-whole meaning of the fraction concept, using a students' hypothetical learning trajectory. Results show that PTs could attend to some of the noteworthy features of the activities about the fraction concept. However, it was not an easy task since few of them could attend to all the features in all activities. These difficulties in attending could affect PTs' recognition of the potential mathematical opportunities and miss-opportunities, and their decisions to overcome the miss-opportunities.

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PEDAGOGICAL FUNCTIONS OF REPRESENTATIONS IN GENERAL AND VOCATIONAL HIGH SCHOOL TEXTBOOKS: THE CASE OF LIMITS AND DERIVATIVES

Pin-Chen Guo and Kai-Lin Yang

National Taiwan Normal University

When students encounter calculus in upper-secondary schools, their prior knowledge and learning objectives may vary significantly due to differences in developmental demands and curriculum design. As multiple representations are necessary means to mathematical objects (Duval, 2000), it is essential to carefully discuss and design the ways of presenting representations tailored to students with different mathematical needs. In response to recent reviews calling for exploring how calculus concepts are represented across diverse disciplines (Biza et al., 2022), this study focused on the intended curriculum, investigating the use of representations of limits and derivatives in general high school (GHS) and vocational high school (VHS) textbooks. Particularly, the *pedagogical functions* (purposes and roles) of representations were highlighted.

In the Taiwanese national curriculum, *Math I* and *Math II* in GHS are designed for students pursuing a future with high (e.g., science) or different mathematical needs (e.g., economics), and *Math C* in VHS caters for students enrolled in majors with advanced mathematical requirement (e.g., engineering). We used an abductive approach to identify the various pedagogical functions of representations in the three programs by analyzing the representations in textbook tasks. After comparing the detailed differences in the ways to present representations, we established categories for pedagogical functions and coded the representations within tasks accordingly. The findings revealed that the graphical representations were used more frequently in VHS textbooks, typically serving as aids to understanding the solving process. In contrast, graphs in GHS textbooks tended to serve as additional confirmations. Many contextual representations catered for students within different programs, where VHS used them as complementary, while GHS used them as additional supplements, to conceptual understanding. We also found that GHS adopted a relatively formal approach, while VHS adopted an informal approach, to the use of verbal and symbolic representations. These findings illustrated various ways to use representations and indicated promising directions for further research for students with different mathematical needs.

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Oral communication

A MULTIPLE-CASE STUDY ON THE PROCESS OF APPRECIATING THE AESTHETIC QUALITIES OF MATHEMATICAL OBJECTS

Hayato Hanazono

Miyagi University of Miyagi

It is well known that mathematicians place great importance on the aesthetic qualities of mathematical objects, but it has been demonstrated that non-mathematicians do not appreciate them on their own (e.g., Dreyfus & Eisenberg, 1986). On the other hand, Hanazono (2021, 2023) have described learners' appreciation process in settings with pedagogical interventions, but these studies have been conducted as single case studies and have not been sufficiently analytically generalized. The purpose of this study is to synthesizes these single-case studies conducted under the same theoretical hypothesis to develop new findings. In accordance with the methodology of Yin (2018), this study conducts a multiple-case study using above two studies to answer the following research questions: how well do the theoretical hypothesis work under different conditions? What empirical supplements to it can be made?

The theoretical hypothesis used in the two single case studies explains the appreciation of the aesthetic qualities of mathematical objects with three axes: the rules of construction that bring "unity in variety" to the mathematical object, the perceiver's cognitive activity with respect to those rules of construction, and the perceiver's sensory activity with respect to the mathematical object. A detailed comparison of the findings of these studies confirms that the theoretical hypothesis is supported in different situations. It was also suggested that the sequence of learning experienced by the learners influences the appreciation process. As an empirical supplement, it was suggested that the appropriateness of the sequence of information presented to learners in pedagogical interventions may vary according to the mathematical object.

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EQUATION-TYPE TASKS IN FINNISH GRADE 3 TEXTBOOKS

Markku S. Hannula

University of Helsinki, Finland

Equations and equality are central in algebra. This topic can further be divided in to three subtopics: 1) Equivalence, Expressions, Equations, and Inequalities; 2) Handling of Expressions, and 3) Modeling and solving word problems (Hemmi et al., 2021).

According to the current Finnish curriculum, students begin to familiarize themselves with algebra in Grades 3-6 and in Grades 7-9 it is a central topic. Before students are formally taught equations and their solution methods, textbooks provide problems that are essentially equations. They may be presented as missing number tasks (e.g., 3 + __ = 8), verbally, or as images (e.g., a scale).

In a joint research project MOPPI, mathematics educators from the universities of Turku, Helsinki, and Jyväskylä are analysing Finnish mathematics textbooks from three different curricula: 1994, 2004, and 2014. One of the aims of this textbook analysis is to identify potential reasons for the decline of Finnish students' mathematics learning outcomes during this period. In this oral communication I will present first results of an analysis of equation type problems in four different grade 3 spring term mathematics textbooks.

In the four examined books, I found 414 missing number tasks, 110 other non-formal equation type tasks, and 51 non-formal system of equation tasks. The total number of tasks in different books was 13, 84, 204, and 274. Most (394) of equation type tasks were solvable with one inverse operation, 90 were of the type $ax + b = c$, and 21 had the unknown on the right side of the equality. In systems of equations, it was typically possible to solve the unknowns sequentially, *i.e.* at least one unknown was solvable easily from one of the equations, and that information made another unknown solvable.

The analysis revealed a big variation in the quantity and quality of equation type tasks across these textbooks. The tasks were almost always solvable using one or two inverse operations and very few tasks would require methods based on balancing. It is also notable that extremely seldom there was an operation with an unknown on the right side of the equation.

The big variation between textbooks encourages to continue textbook analysis to gain a more comprehensive view of how different textbook series might prepare students differently for formal algebra teaching at grade 7.

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DESIGNING OPPORTUNITES FOR COMPUTATIONAL THINKING: LEVERAGING FAMILY STORIES AND COMMUNITY PRACTICES IN TEACHER PROFESSIONAL DEVELOPMENT

Lynn Liao Hodge and Amy Maples

The University of Tennessee

This presentation describes a three-year project that focuses on professional development for elementary teachers in the Appalachian region of the United States. The professional development considers culturally responsive approaches to teaching computational thinking. In the presentation, we draw attention to key aspects of the professional development, their implications for teacher and student learning, and directions for future research. The project, Computer Science for East Tennessee (CSforTN), is a three-year, NSF-funded project (#2219418) that seeks to increase elementary students' access to computational thinking (CT) through the integration of CT and literacy in elementary classrooms. Critical aspects of the project include: 1) professional development for elementary teachers that explores CT and literacy-integrated activities, 2) classroom implementation by the participating teachers to test out and learn from their students' participation in the designed activities, and 3) a computer science teaching network that offers opportunities for learning and networking. The focus of this presentation is on the professional development with attention to how an approach that leverages literacy practices can support or delimit teachers' connections with and learning of computational thinking. The project draws on sociocultural perspectives that view learning as involving shifts in participation reflected in individuals' experiences and changes in trajectory (Nasir, 2012). As individuals learning, they develop a sense of who they are in relation to the practices that come to define a learning context. For this project, identity is defined as how teachers define, the value they place on, and how they view their own competence in relation to CT. The project began in Fall 2022 and has completed one cycle of design-based research on the professional development. We draw on teacher artifacts and interview data to share preliminary insights on teacher learning and identity related to the project.

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EXPLORING STUDENTS' ERRORS ON ESTIMATING LENGTHS

Ricarda Holland and Jessica Hoth

Goethe University Frankfurt, Germany

Analyzing students' errors is an important diagnostic requirement to learn about students' understanding and fostering adaptive learning. In the study, we focus on errors in students' length estimation. Several factors may influence the estimation process: various estimation situations (e.g., size, touchability) or person-specific factors (e.g., prior knowledge) (Hoth et al., 2022). Previous research has shown that students frequently use benchmark comparison and that this leads to higher accuracy (Huang, 2014; Holland et al., 2023). To investigate errors in length estimation, 10 of 1,051 fourth-graders were selected based on their low estimation accuracy in a paper-and-pencil test with 30 tasks. In follow-up interviews about their approaches, seven categories of estimation errors emerged from the inductive coding (Mayring, 2019). 197 explanations were coded as errors and quantified: Incorrect/unsuitable benchmark (52), incorrect standard unit (36), error in counting/multiplying/adding (7), error in indirect/direct comparison (22), visual approach/estimation without a strategy (55), task misunderstood (12), no attention to the unit of the task (13).

The most errors are based on the estimation process, like strategy execution, missing strategy, unsuitable benchmarks, wrong units, and errors in (in)direct comparison. However, errors during estimation impede students' mathematical understanding. Understanding error types is crucial for designing effective instructional strategies, interventions, and assessments. The results imply to talk about and deal with strategies, benchmark knowledge, and foster relevant knowledge and skills involved in the estimation process. Further research could therefore explore the correlation between error categories and estimation accuracy, delving into causative factors.

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Oral communication

A STUDY ON THE DETERMINATION INDICATORS FOR MATHEMATICAL LITERACY QUESTION: FROM THE PERSPECTIVES OF THE SCHOOL TEACHERS

Chia-Jui Hsieh[1], Yu-Zhen Zhang[2], Lin-Chieh Tsai[3], and Ching-Wen Chiu[4]

National Taipei University of Education[134], National Chushan Senior High School[2]

In recent years, mathematical literacy has become a focal point in global landscape for education. This trend has also influenced mathematics education in Taiwan, which has demonstrated strong performance in mathematics on the PISA. Taiwanese mathematics teachers are required to possess the ability to construct Mathematical Literacy Questions (MLQs) to assess students' mathematical literacy. However, the criteria for valid MLQs remain uncertain, and global variations in mathematical literacy definitions impact assessments. This study aims to identify ***MLQ characteristics*** and ***indicators*** suitable for Taiwan, establishing evaluation standards for MLQ design from the perspective of primary and secondary school teachers?

Our study using a questionnaire survey and grounded theory to examine MLQ features and indicators. The questionnaire used in this study include 42 MLQs, which from the Literacy-oriented Assessment samples declared by Taiwan's education authorities and the PISA released mathematics items. To collect our data, we purposefully selected 187 participants, including in-service primary and secondary school teachers, as well as pre-service teachers. For each item, participants were asked to determine the level of mathematical literacy according to their own criteria. Additionally, participants were asked to articulate the essential characteristics they believed MLQs should possess.

This study analyzed 1486 teacher responses using grounded theory, resulting in 29 open codes. Among these, 15 were mentioned by teachers in over 10% of cases, thus considered the 15 specific characteristics of MLQs in this research. The top 5 characteristics mentioned by Taiwanese teachers are as follows: Utilizes Everyday Life Contexts, Real Situations, Applies Mathematical Tools for Problem-Solving, Reasonable Contexts, and Problem-Solving with Motivation/Needs, with percentages of 79.4%, 38.3%, 37.71%, 34.29%, and 30.86%, respectively. This study integrates these 15 specific characteristics into 5 dimensions: Contextually Relevant to Real Life, Concise and Understandable Descriptions, Problem Conditions Aligned with Requirements, Effective Use of Mathematics for Problem-Solving, and Assessing Different Abilities Compared to Traditional Questions. These five dimensions serve as the MLQ indicators in this study. Furthermore, based on these 15 specific characteristics, we formulate a comprehensive checklist and establish a comprehensive MLQ scoring system. According to the criteria we have established for grading literacy questions, the results align with the evaluations made by the teachers. This system will serve as a definitive set of assessment criteria, providing valuable guidance to educators in designing effective MLQs in the future.

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Oral communication

DIFFERENTIAL INSTRUCTIONAL DESIGNS TRIGGER THE EPISTEMIC EMOTION AND COGNITIVE LOAD INFLUENCE ON GEOMETRIC LEARNING

Shang-Kuei Hsu[1] and Yuling Hsu[2]

School of Computer Science and Software, Zhaoqing University, Guangdong, China[1]
Institution of Education, Tzu Chi University, Taiwan[2]

When learners engage in mathematics learning, the learning process will include both cognitive and emotional processes. The epistemic emotion is a specific type of emotion that refers to the individual's emotions toward knowledge and the process of generating knowledge (Muis et al., 2015). The learning process also produces cognitive load. Sweller (2019) pointed out that instructional design needs to consider intrinsic cognitive load, reduce external cognitive load, and stimulate an appropriate amount of germane cognitive load. This study investigated 206 learners (84 indigenous and 122 non-indigenous) learning geometric area concepts on a computer-based simulation platform. We investigate the effects of epistemic emotions and cognitive load experienced by individuals with different backgrounds in four different experimental conditions (2×2). Four conditions distinguished as representations (dynamic vs. static) and strategies (solution concept imagination or solution procedure imagination). Preliminary research results show that girls' positive epistemic emotion reactions (i.e., surprise, curiosity, enjoyment) and confusion emotions are generally higher than boys. In the condition 4 (dynamic representation & procedure imagination), there is a reversal (i.e., girls are lower than boys); as for negative emotions (i.e., frustration, boredom), girls are lower than boys. For cognitive load, the pattern of boys and girls is entirely consistent: the highest intrinsic load in condition 1 (static representations & concept imagination); those with the lowest extraneous cognitive load and moderate germane load in condition 4. For learners with different socio-cultural backgrounds, indigenous students generally have higher emotional reactions than non-indigenous students. Non-indigenous children have lower negative emotions in condition 4. Indigenous students have lower negative emotions in condition 2. This study preliminarily shows different learning effects in different experimental conditions. These factors influencing performance show a preliminary trend (not yet reaching a statistically significant level). In the future, we will clarify the differential effects of instructional design based on evidence-based research.

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COLORFUL CIRCLES AND ARROWS: WRITING GESTURES IN ONLINE TUTORING SESSIONS

Nicole Infante[1], Keith Gallagher[1], Deborah Moore-Russo[2], Michelle Friend[1], Shang-Kuei Hsu[1], and Yuling Hsu[2]

[1]University of Nebraska-Omaha, [2]University of Oklahoma

Access to free, high quality tutoring services is critical to helping students be successful in mathematics courses. The pandemic forced many institutions to move their tutoring operations online, creating more equitable access to tutoring. Tutors working in the online environment needed additional training to help them adapt their in-person skills for online interactions. Our research question is: How do tutors use a tablet and stylus to provide high-quality, student-centred online interactions? This project builds on Lepper and Woolverton's (2002) INSPIRE model of effective tutoring practices. Our efforts expand on this work to describe effective practices in online tutoring. Here, we focus on how tutors use writing gestures to enhance online communication. Writing gestures, as defined by Alibali et al. (2014), are "writing or drawing actions that were integrated with speech in the way that hand gestures are typically integrated with speech" but produced with a writing tool, in this case a stylus. Examples of writing gestures in our data include underlining or circling parts of a problem, drawing arrows to make connections, and using multiple colours to highlight mathematical moves. Data come from hundreds of hours of video-recorded online tutoring sessions at two U.S. institutions that each employed 4-5 online tutors each semester from August 2021 through May 2023. Tutors were given a tablet and stylus to facilitate their writing in online tutoring sessions and received training aimed at improving online interactions. Most tutors used a laptop and the tablet together; this combination yielded higher quality interactions and the most writing gestures. We will share exemplars of how tutors' writing gestures evolved as well as the link to our training materials should others want to use them.

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MATHEMATICS TEACHER EDUCATORS AND PRACTICUM PARTNERSHIPS DURING THE PANDEMIC

Jyoti Jhagroo[1], Zanele Ngcobo[2], Msebenzi Rabaza[2], and Justice Enu[4]

Auckland University of Technology[1], University of KwaZulu Natal[2], University of Free state[3], Kwame Nkrumah University of Science and Technology[4]

In 2020, the Covid 19 pandemic brought the world to a standstill. Travel had ceased, social distancing became a norm, teaching and learning had to be reimagined and places of work had changed. Students and teachers were thrust into an environment of emergency online teaching and learning. Within days higher education shifted to a new mode of delivery from in person face-to-face to an online platform in an attempt at connecting students, teachers and content via learning management systems. A vital component of teacher education comprises school-based practicum experience in partnership with schools. With the lockdown of schools, preservice teachers faced were confronted with the possibility of not completing their programmes of study. Through an autoethnographic study, this presentation offers insight into what the sudden shift meant for us as mathematics teacher educators and how our respective institutions in New Zealand, South Africa, and Ghana, responded to this situation in supporting our preservice teachers. Data is drawn from our individual documented narrative reflections.

As mathematics teacher educators, not only were we grappling with the idea of how to teach in a changed platform, but we were also figuring out a changed mode of partnership with our practicum schools to support our preservice teachers. Through our reflections, we share our individual thoughts on the practicum opportunities that our preservice teachers engaged in to meet the practicum requirements. Having created these opportunities in a relatively short period of time, presented challenges such as reshaping our role as mathematics teacher educators and evaluation of the preservice teachers, working in new contexts, managing relationships, and its associated tensions. We use the three models of effective school-university partnerships developed by Bernay et al. (2020) to analyse the practicum programmes provided for the preservice teachers. A few years on since the pandemic has allowed us to embrace the potential of online teacher education, not for the purpose of replication of mathematics teaching, but reimagining authentic partnerships between the schools and the universities.

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FACILITATING ARITHMETIC-ALGEBRA TRANSITION WITH PROGRESSIVE LINKING STRATEGY

Huansen Jian[1,2] and Yanxing Qi[1]

[1]East China Normal University [2]Escola Sun Wah

Mastering arithmetic and non-arithmetic operations is vital for grasping equations and inequalities. Compared to high achievement in arithmetic to elementary students, succeeding in complex operations such as term transposition and sign changes in algebra is quite challenging. The Progressive Linking Strategy (PLS) innovatively highlights algebraic problem-solving steps to address this challenge.

As shown on the left side of Figure 1, PLS effectively decomposes arithmetic operations and non-arithmetic operations through a visual representation approach, highlighting the subtle and crucial process of the concept of inequality reversal—a non-arithmetic operation that is often neglected in early math education. This significantly reduces the cognitive load on students, helping them differentiate between arithmetic reasoning and algebraic reasoning, understand and master these complex concepts, particularly in terms of changes in operator symbols and the selection of operation sequences, as discussed by Stephens et al. (2020).

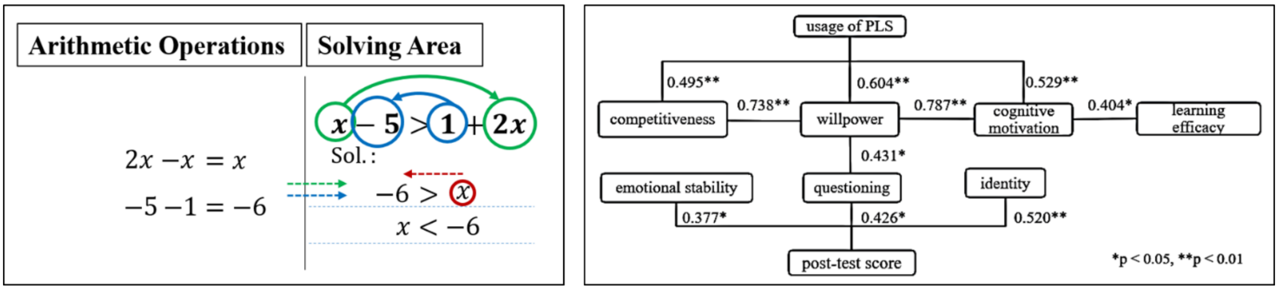

Figure 1: Illustrations of PLS (Left: Operation of PLS, Right: The effect of PLS)

To evaluate the impact of PLS on algebra skills and its interaction with non-cognitive factors, a Macao study on 7th-graders used a month-long PLS intervention. Utilizing t-tests, no initial difference in pre-test scores ($p = 0.43$) was found; however, post-test scores significantly improved to 68 compared to the control's 57.94 ($p = 0.04$). Additionally, as depicted on the right side of Figure 1, PLS exhibited varying degrees of influence on students' non-cognitive factors, with the most substantial impacts observed in willpower ($r = 0.604$) and cognitive motivation ($r = 0.529$).

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SUPPORTING MATHEMATICS TEACHERS' PLANNING OF MULTIMODAL TEACHING

Helena Johansson[1], Malin Norberg[2], and Magnus Österholm[1,2]

[1]Mid Sweden University, Sweden, [2]Umeå University, Sweden

Multimodality, that is, the use of combinations of different modes (e.g., symbols, images, words), is central to mathematics and the teaching of mathematics. A combination of different modes implies that aspects of cohesion (e.g., similarity, continuity) and tension (e.g., contrast, distance) between modes arise. To promote learning, there must be a balance between cohesion and tension, and tension between modes must be sharp enough to be challenging, though not sharper than what is possible to bridge (Engebretsen, 2012). Empirical results show that the balance between cohesion and tension in mathematics textbooks is not always productive (Johansson et al., in press). Thus, there is a need to support teachers in their use of textbooks and other resources for planning mathematics teaching. Although there is much research on mathematics textbooks, little is still known about teacher guides (Jukić Matić & Glasnović Gracin, 2021). This ongoing study is part of a larger project and focuses on developing principles to guide teachers when using teaching resources in multimodal settings. Through an iterative process, a guide with the purpose to highlight aspects of cohesion and tension in relation to the planning of teaching of mathematical concepts (subtraction in this case) has been developed by analysing four lower grades teachers' experiences of using different versions of the guide. By, for example, changing focus in the guide from *expected challenges for the students* in the tasks, with respect to using different modes, to *contrasts between present modes* in the tasks, preliminary results show that the guide helped the teachers to reflect on aspects of cohesion and tension and consciously create a contrast between different modes, with respect to the concept in focus. Teachers also expressed that they had changed their way of thinking, from seeing a contrast between different ways to present the mathematics in focus as something bad, to seeing that it could be beneficial for students' learning. Further analysis will be conducted to identify particular features in the guide that influenced the teachers.

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EXPLORING ALGEBRAIC TASK DYNAMICS, PRE-SERVICE TEACHERS' REASONING AND SENSE-MAKING

Constanta Olteanu[1] and Monica Jonsson [2]

[1]Linnaeus University, [2]Skogsborgsskolan

Researchers highlight the critical role of algebraic reasoning and sense-making in developing students' conceptual understanding of mathematics. Consequently, understanding and developing pre-service teachers' own reasoning and sense-making abilities are essential for developing students' algebraic skills. In this presentation, we present the results of a study in which we explore pre-service teachers' reasoning and sense-making in algebra across two distinct types of tasks: problem-solving concerning growing patterns and problem-posing tasks involving first-degree equations. The research question guiding the analysis is: Which opportunities do different tasks provide for pre-service teachers' reasoning and sense-making, and how do pre-service teachers utilize these opportunities in their reasoning and sense-making? The study involved 79 pre-service primary teachers taking a mandatory written exam as part of a mathematics teacher education course. The analysis applied a qualitative/interpretative methodology, utilizing variation theory and assemblage theory as analytical frameworks (Deleuze & Guattari, 1987; Marton, 2015; Olteanu, 2022). Key analytical tools - lines of flight, segmentarity, and rupture - inform diverse connections. The study results indicate that problem-solving and problem-posing tasks offer dynamic opportunities for connecting reasoning and sense-making among pre-service teachers. This connection is characterized by essential actions in reasoning like selecting, exploring, abstracting, encoding, and connecting information, while sense-making involves recognition, relationship identification, comparison, explanation, and verification. Pre-service teachers utilize these opportunities through the interplay of lines of flight, which provide new ways for reasoning and sense-making that enable the distinction of critical aspects. While pre-service teachers connect reasoning and sense-making by segmentarity and rupture in problem-solving tasks, they do this by employing segmentarity in problem-posing tasks. Segmentarity restricts changes in pre-service teachers' reasoning and sense-making, thus limiting the distinction of critical aspects, while rupture leads to transformation or introduces new opportunities for distinguishing critical aspects.

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ANALYZING THE RELATIONSHIP BETWEEN UNDERGRADUATE STUDENTS' COMBINATORIAL THINKING AND COMPUTATIONAL THINKING

Won Jung and Oh Nam Kwon

Seoul National University

This study aims to analyze the relationship between undergraduate students' combinatorial and computational thinking through combination tasks solved in programming settings. Tasks in combinatorics have generally involved understanding the context of problems accurately. However, undergraduate students often find it difficult to come up with ideas when solving contextualized combinatorics problems. Lockwood et al. (2020) showed the potential of teaching and learning of combinatorics by analyzing undergraduate students' combinatorial problem solving in programming settings. Exploring undergraduate students' thinking during combinatorial problem solving in programming settings can contribute to the discussion of a deeper understanding of concepts in combinatorics.

Seven undergraduate students participated in this study. Students were given tasks that can be solved by permutation with repetition or combination and asked to solve them by both handwork and coding in Python. We applied Lockwood's (2013) model, which frames students' combinatorial thinking in terms of formulas/expressions, counting processes, and sets of outcomes. In the programming context, we questioned students about the patterns of the code regarding variables, conditionals, and loops. We further questioned students about the relationship between the handwork and corresponding programming code to analyze the relationship between combinatorial and computational thinking.

Students first enumerated all possible cases by hand. Considering all the cases, students formed a set of outcomes and formulated a formula for permutation with repetition. Then, by reorganizing and recounting each element of the set of outcomes, they derived a combination-based formula that led to the same solution. Students constructed the programming code based on the set of outcomes that they had formed by hand. The final codes were completed based on students' ideas using combinations rather than permutations with repetition. The programming settings helped students to understand the connection between permutation with repetition and combination.

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TRANSFORMATIVE TEACHING: ENHANCING EDUCATORS AND PRACTICES THROUGH TASK-ORIENTED LEARNING

Natalia Karlsson and Constanta Olteanu

Södertörn University, Linnaeus University, Sweden

In recent decades, approaches have evolved to define the subject matter knowledge essential for teaching, based on Shulman's framework (1986) and additional insights provided by Ball et al. (2005). At the same time, scholarly researchers agree that teachers' subject matter knowledge has a strong impact on the quality of instruction, something that plays a central role in the students' subsequent ability to learn mathematical concepts and acquire mathematical skills (Adler & Ronda, 2015). In line with this, it is relevant to extend research and further investigate by addressing the question of teaching that is linked clearly to students' solid and deep learning of mathematics, both by theoretical and empirical studies. This presentation introduces analytical findings from our research that examined a transformative teaching approach as an extension of teachers' reflective practice with the starting point being different learning processes in the theoretical context of transformative pedagogy (TP) and transformative learning (TL) (Mezirow, 2003). The TP and TL theoretical framework proposed in this study prompts discussions on the structural aspects of task-oriented teaching, offering a conceptually grounded model for transformative instruction (TI). The focus in the TI- model is on various types of task-oriented learning, encompassing instrumental, dialogic, and self-reflective learning and conceptual overlap from prior knowledge to profound knowledge of mathematical concepts. The analytical result is also illustrated in the analytical application of the TI model by a practical example involving the conceptual overlap in students' learning, from rational numbers to rational equations. The evaluation of these analytical findings will be improved by empirical implementation related to transformative teaching and the students learning specific-algebra concepts.

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AN INTEGRATION BETWEEN ART AND MATHEMATICS IN LEARNING GEOMETRICAL ATTRIBUTES OF OBJECTS

Khem Khenkhok, Chitipat Prasertsang, Maitree Inprasitha, Narumon Changsri, and Piyasak Pukcothanung

Khon Kaen University

Math and Art have much in common, such as being creative and driven by the search for beauty (Budd, 2020). It is easy to see areas of overlap between math and art. Geometry, including shape dynamics, is just an area of math that uses shapes. Art uses shapes to create complex drawings, paintings, and so on (Haggerty Museum of Art, 2024). Inserting fine arts into mathematics classes makes the learning experience more inwardly active and the subject matter more comprehensible (Brezovnik, 2017). This study aimed to explore 1) what geometrical shapes students could trace from given objects, 2) what objects students could create from the traced shapes, and 3) students' imagination through their stories from created objects. The target group was 17 second-grade students. The three-weekly cycle of the TLSOA model (Inprasitha, 2022) was employed in the research processes. In the collaborative plan, the lesson study team designed problem situations by integrating aspects of shapes in Mathematics and Arts. Household objects such as boxes, cans, colored papers, colored pencils, and a zoo map were used as teaching materials. Data were collected from classroom observations, reflections, and students' works. The results revealed that students realized that some faces of a box could create a rectangle and that the cylinder's top or bottom could create a cycle. Furthermore, students visualized animals from the traced shapes. For example, students create a worm by forming a curve or a line using cycles and rectangles. They also created a crocodile, a dinosaur, and a tiger. Lastly, we observed that students located their objects (animals) on the zoo map based on their stories, such as a worm eating leaves on a tree and a dinosaur eating grasses and leaves on a field. These experiences suggest that learning mathematics could become more personalized to each student's narrative through arts.

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DEVELOPING PRE-SERVICE MATHEMATICS TEACHERS' COMPETENCY IN DESIGNING MODELLING TASKS

Hee-jeong Kim, Gima Lee, Byoung-rag Soh, Yun Hwa Noh, and Jinfa Cai

Korea University

Mathematical modelling has its unique place in school mathematics because it is both a mathematical topic and a process and its pedagogical value for students' learning (e.g., CCSSM, 2010; Freudenthal, 1973; MoE, 2022). Despite this recognition, the complexity and unfamiliarity of teaching mathematical modelling pose significant challenges for teachers to implement in classrooms (Greefrath et al., 2022). In order to successfully teaching mathematical modelling, teachers need to be supported and professionally developed. One of the important areas is to support teachers to develop modelling tasks. The purpose of this study is to examine pre-service secondary mathematics teachers (PSTs)' development of designing modelling tasks as they engage in a 16-week course on mathematical modelling. The study is to answer the following two research questions: (1) What are the features of the PSTs' three versions of their modelling tasks? and (2) How do the PSTs' design competencies develop throughout the course? Data for the analysis includes the three versions of their modelling tasks (initial sketch, an earlier version of the modelling tasks, and modified modelling tasks) designed by 46 PSTs along with two distinct types of individual reflective essays. Preliminary analysis indicated a progression towards more 'open' tasks that incorporate multiple stages of the modelling cycle, and more realistic contexts. Reflective essays revealed PSTs' challenges in designing "good mathematical modelling tasks" and in assessing actual student works, alongside their evolving views on the nature of modelling tasks. The issues of PSTs' challenge in crafting a task involving a full cycle of mathematical modelling will be also discussed. This study deepens our understanding on the development and challenges of PSTs' competency in modelling task design by examining multiple versions of their modelling tasks, in addition to the theories of improvement science.

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EXAMINING HOW TEACHERS ADAPT AND IMPLEMENT A HIGHLY PRESCRIBED MATHEMATICS CURRICULUM: THE SWEDISH CASE

Tuula Koljonen

Department of Behavioural Sciences and Learning, Linköping University

Implementing new curriculum material is a complex, multifaceted process influenced by factors like the curriculum, teacher interpretation, and classroom dynamics. As Remillard and Bryans (2004) highlighted, there's a gap in our understanding of how teachers interpret and apply new materials, emphasizing the need to study teachers' interpretations and methods to improve math education, particularly in Sweden, where curriculum materials are typically not prescribed and not mandatory.

This research, grounded in curriculum material and teacher cognition theories, examines the application of three lessons from the unfamiliar, innovative TRR (Thinking, Reasoning, and Reckoning) mathematics curriculum by two Grade 1 teachers in Sweden. The aim is to comprehend how teachers interpret this structured material, identify factors causing instructional variations, and understand these variations' impact on student engagement and learning. The central question is: "How do teachers adapt and implement lessons from a new, prescribed curriculum material?"

This ongoing study uses video recordings, teacher interviews, and student work as data sources. Initial findings suggest teachers' instructional methods are heavily influenced by their styles and interpretations of the curriculum material, considerably influencing students' learning opportunities. Established mental routines, like ingrained teaching habits, are observed to influence teachers' actions, creating a gap between intentions and behavior (Sfard, 2023).

The study's findings aim to improve our understanding of curriculum implementation practices, potentially informing future teacher training and curriculum development efforts. The results, shared at the conference, strive to shed light on the relationship between teacher interpretation, curriculum implementation, and student learning outcomes within Swedish education, where prescribed curriculum materials are uncommon.

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STUDENTS' MEANINGS FOR THE INTEGRAL CONCEPTS

Anatoli Kouropatov[1], Lia Noah-Sella[2], Tommy Dreyfus[2], and Dafna Elias[2]

[1]Levinsky-Wingate Academic College, [2]Tel-Aviv University

The understandings of calculus developed by students in high school are likely to influence their tertiary studies. Combining qualitative and quantitative methods, including 725 questionnaires and 207 detailed interviews, our project methodically explores students' meanings for derivatives and integrals based on a theoretical framework inspired by Thompson (2016). This framework highlights the importance of personal schemas and the implicative nature of meanings in the learning process. Our research investigates meanings in intra- and extra-mathematical contexts.

The first phase of our project involved a qualitative exploration through an analysis of 92 in-depth interviews. This initial exploration laid the groundwork for a subsequent quantitative phase, which sought to chart the prevalence and variation of these meanings among a wider student population. The project is characterized by its iterative methodology, which allows for continuous refinement of research tools and techniques to capture fine-grained aspects of students' meanings.

The results of our research highlight the varied interpretations students have of the integral concept, as exemplified by the experiences of three students. Nathan uses the 'collapse metaphor' to interpret an integral as the sum of infinitesimally thin lines or y-values. Mika's understanding fluctuates between physical and mathematical contexts, transforming as she moves between these two domains. Oren distinguishes between two types of integral, 'regular' and 'accumulating'.

Our analysis indicates that Nathan maintains a stable, well-defined, consistent, and adaptable meaning of integral. In contrast, Mika appears to hold at least two distinct, discipline-specific meanings, which seem somewhat disjointed. Meanwhile, Oren's meaning of integral demonstrates a complex and not entirely clear relationship with its symbolic representations. These results illuminate the personalized nature of mathematical meanings and emphasize the complex interaction between symbolic representations and individual meanings. They call for a nuanced method of teaching calculus that respects and supports the varied interpretative lenses through which students view the subject.

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Oral communication

A STUDY OF ELEMENTARY SCHOOL STUDENTS' CORE LEARNING OUTCOME REGARDING DESIRABLE ATTRIBUTES

Anucha Koyata[1], Dhanachat Anuniwat[3], Kanita Pamuta[1], Narumon Changsri[1,2], and Maitree Inprasitha[1,2]

[1]Mathematics Education Program, Khon Kaen University, Thailand
[2]Center for Research in Mathematics Education, Khon Kaen University, Thailand
[3]Graduate School of Humanities and Social Sciences, Hiroshima University, Japan

The desirable attributes influence students' perceptions of their learning throughout the mathematics learning trajectory. It also influences students' perceptions of their learning, prepares them for success in the 21st century and helps them progress toward achieving a common goal (OECD, 2019). Fostering desirable attributes is crucial for quality education beyond academic achievement (Zurqoni, Apino & Anazifa, 2018). Since its introduction in 2002, the Thailand Lesson Study incorporated with Open Approach (TLSOA) (Inprasitha, 2022), has been widely adopted and is spreading nationwide. This study explores the core learning outcomes regarding desirable attributes among elementary students in schools that have implemented TLSOA, thereby providing empirical evidence of the model's impact on a broad scale. The study's methodological design involved data collection from 1,796 elementary students in 24 schools nationwide at the beginning of the 2021 academic year after the project had been implemented for a year. This was done using online 5-Likert scale questionnaires and interviews with clinical interview techniques, ensuring a comprehensive understanding of the students' perspectives. The quantitative data analysis, which included mean scores and standard deviations, revealed that students reported high levels of overall desirable attributes ($\bar{x}$=4.19), with collaborative work at highest score ($\bar{x}$=4.25) and attitude towards learning at lowest score ($\bar{x}$=4.11). The interview's qualitative data further supported these findings, highlighting students' positive intentions toward collaboration when working in groups. These findings underscore the importance of fostering collaborative skills and positive attitudes toward learning among elementary students within the TLSOA model.

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A MOVE ANALYSIS OF MATHEMATICAL PROOF

Valentin A. B. Küchle[1] and Paul C. Dawkins[2]

[1]Auburn University, U.S.A.; [2]Texas State University, U.S.A.

Although there are multiple traditions of genre theory (e.g., English for Specific Purposes [ESP], Systemic Functional Linguistics, Rhetorical Genre Studies, Bakhtin), mathematics education researchers have rarely drawn on these to study genres. In particular, researchers have hardly drawn on genre theories to study the genre of proof. Since past research results about proof (e.g., proof has multiple functions, mathematicians do not solely rely on empirical evidence for validating proof and may be swayed by authority) are natural consequences of proof being a genre—genres are socially situated rhetorical actions which reflect and enforce existing power structures—we posit that much can be learned from applying genre theories and its tools to mathematical genres like proof.

Here, we report on a move analysis of proof. Move analyses stem from the ESP tradition and are conducted to understand the moves (i.e., functional units) that make up a text. Although proofs will typically have non-optional moves that further the proof function of verifying a mathematical result (e.g., indicating deduction), proofs may also contain optional moves that may serve the reader's understanding (e.g., *simplifying notation* ["write $g = \gcd(a,m)$"]). We call these pedagogical proof moves.

To identify such moves, we used a move analysis in line with the ESP tradition and analysed 40 proofs from four abstract algebra (AA) texts. The four texts were: (a) a set of AA lecture notes identified for their exceptional pedagogical merit, (b) the most-used AA textbook in the U.S.A. (presumably for its pedagogical merit), (c) a popular AA book known for its terseness, and (d) an AA book with a visual approach. The four were selected to cover roughly similar content while having distinct proof styles. The 10 proofs from each text were randomly selected with a random number generator.

Our preliminary analysis led to the identification of 16 pedagogical proof moves. Examples include: (a) *emphasizing a (significant) detail* (e.g., "it is the **least positive** [*sic*] integer"), (b) *concluding a (sub)proof by stating what has been shown* (e.g., "This last equation shows what I set out to prove, that $\bar{r}$ is a root of the same polynomial"), and (c) *making connections between (sub)proofs* (e.g., "This is similar to the proof of Proposition 25").

Although we suspect that most of the identified moves facilitate proof-reading and -understanding, we acknowledge that some optional moves, such as *indicating the difficulty of an argument* (e.g., "it is easy to verify that"), may have harmful effects. Thus, we hope that the identified moves can raise awareness for authors' choices and spark discussion of which choices can help authors create easier-to-read and more illuminating proofs.

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ASSESSING THE MATHEMATICAL CAPABILITIES OF CHATGPT ON KOREAN NATIONAL EXAMS

Oh Nam Kwon[1], Se Jun Oh[2], Jungeun Yoon[1], Kyungwon Lee[1], Byoung Chul Shin[1], and Won Jung[1]

Seoul National University[1], Hongik University[2]

ChatGPT is a generative artificial intelligence tool that is continuously undergoing functional improvements. Its emergence and widespread use worldwide have led to discussions about how ChatGPT can be utilized in mathematics education. This study aims to evaluate the mathematical capabilities of ChatGPT by analyzing its responses to questions from mathematics assessments in Korea.

Mathematics problems from the National Assessment of Educational Achievement (NAEA) and the College Scholastic Ability Test (CSAT) in Korea were utilized for this study. After excluding problems that require diagrams for solving, 116 items from NAEA and 144 problems from CSAT spanning three years of Korean national exams were selected. Each mathematics problem was converted into LaTeX format for input into ChatGPT 3.5, and solutions were obtained by entering these problems as prompts. The correctness and the accuracy of ChatGPT's solutions were measured, and error types were classified. The accuracy was rated using a scoring system based on the criteria proposed by Freider et al. (2023), resulting in partial scores out of 5 and evaluation criteria.

The findings revealed the rates of correctness of 37.1% for the NAEA and 15.97% for the CSAT, with the accuracy of the solutions being 3.44 and 2.49, respectively. Mathematics problems related to numbers and operations yielded the highest rates of correctness. The mathematical capabilities of ChatGPT cannot be solely attributed to the school level, content area, or problem weighting. Errors in ChatGPT's solutions were categorized into procedural errors, which could be improved through further mathematical learning, and functional errors, which might be mitigated as ChatGPT's function advances. Procedural errors included mistakes in formulating and solving equations or computational errors, while functional errors involved failures in recognizing question requirements or mathematical symbols. These results provide foundational data for utilizing ChatGPT in mathematics education. As the functions of ChatGPT continue to improve, there is a need to expand the set of mathematics problems for research at an international level. Such efforts will significantly contribute to the ongoing discussions about the mathematical capabilities of ChatGPT.

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MATHEMATICIANS' ORAL COMMUNICATION OF THEIR RESEARCH TO A MATHEMATICS EDUCATION RESEARCHER

Rox-Anne L'Italien-Bruneau

University of Auckland

In recent years, attention towards oral communication in mathematics education has grown, and characterising mathematicians' oral communication could yield significant insights into this mathematical practice. In the field of scientific communication, researchers identified several mechanisms specialists use when orally explaining advanced concepts or processes to different audiences (e.g., Gülich & Kotschi, 1995). In the specific case of oral communication about research, Ciapuscio (2003) suggests that researchers typically use *illustration* and *reformulation* mechanisms when explaining their work and results. In mathematics education, little is known about mathematicians' oral communication (Barwell, 2013), let alone communication about their research.

The study presented here aims to characterise the mechanisms mathematicians use to make the objects and processes of the research project accessible to a non-specialist audience. As part of my doctoral research, I conducted one-on-one semi-structured interviews with ten mathematicians. During the interviews, I prompted participants to describe their research and day-to-day activities, among other aspects. The results reported here concern the case of Charles, a first-year doctoral student in algebra. The transcripts and recordings of the interaction between Charles' and I, a mathematics education researcher, were then analysed through the lens of the mechanisms described in scientific communication research (Gülich & Kotschi, 1995; Ciapuscio, 2003).

Findings show that besides *illustration* and *reformulation* mechanisms, Charles uses mechanisms that appear specific to mathematical oral communication. Aligned with results from Barwell (2013) and Ciapuscio (2003), the investigation of mechanisms in Charles' communication of his research shows that he connects everyday experiences and mathematics to support his explanation. However, data suggest that the connections are sometimes opaque and hinder Charles' communication of his research. In conclusion, the case of Charles is a first step in characterising mathematicians' practice of oral communication and describing their use of various mechanisms to make the objects and processes of their research accessible to a non-specialist audience.

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A STUDY OF BILINGUAL MATHEMATICS TEACHING ON SECOND-GRADERS' LEARNING EFFECT – TAKING THE TOPIC OF LENGTH AS AN EXAMPLE

Yen-Ting Lai and Erh-Tsung Chin

National Changhua University of Education, Taiwan

In Taiwan, in response to the national bilingual policy set for 2030, efforts are currently underway to actively integrate bilingual teaching into the curricula of primary and secondary schools, which highlight the importance of bilingual education. In our mathematics bilingual classrooms, the predominant approach is the CLIL (Content and Language Integrated Learning) method, which integrates subject knowledge with language learning (Coyle, 2005). This method also facilitates the development of critical thinking skills and prepares students for internationalisation. Therefore, the integration of bilingual education may enhance national competitiveness (Chostelidoua & Griva, 2014).

This study focuses on implementing CLIL bilingual mathematics teaching in a 2nd-grade class of only eight pupils on the topic of "length". The objective is to explore the students' current learning outcomes in terms of their conceptions of length and proficiency in English, as well as to investigate their perceptions of bilingual mathematics teaching. The case study approach is adopted in the study, whilst the data collection and analysis include the test papers of mathematical concepts and corresponding English with descriptive statistics and non-parametric analysis, and interviews which are coded and analysed for understanding students' perceptions of bilingual mathematics teaching.

The main results of this study are as follows: (a) The case students were more engaged in practical lessons than in other lessons; (b) The case students performed better in "Measurement" than in "Estimation"; (c) The case students were able to recognize the English words for "centimeters" and "meters" and write them down.

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DEVELOPMENT OF A FOUR-INDICATOR MATHEMATICAL CREATIVITY TEST FOR GRADE FIVE STUDENTS

Rhett Anthony C. Latonio and Catherine P. Vistro-Yu

Ateneo de Manila University

Development of a valid and reliable instrument to measure mathematical creativity, widely considered as one of the vital 21st-century skills, is essential in identifying components which foster mathematical creativity in the classroom.

The goal of this presentation is to develop a Mathematical Creativity Test for Grade Five Students (MCT5) anchored on a theoretical framework which associates mathematical creativity with divergent thinking (based on Bal-Sezerel & Sak, 2022) and utilizes the design principles of Model-Eliciting Activities (MEA). MCT5 uses problem-solving, problem-posing and reasoning items to measure divergent thinking indicators of fluency, flexibility, originality and elaboration. Preliminary results from the pre-pilot test on the reasoning items showed a low percentage of subjects' outputs exhibiting the four indicators, with some subjects showing hesitance in making independent decisions based on their own analyses of the given data. Feedback from teachers reflected their difficulty in terms of time management.

A mixture of ten problem-solving, problem-posing and reasoning items, based on extant literature on mathematical creativity assessment and MEAs, were content-validated by a trio of subject matter experts (SME). Content validation results, SMEs' feedback and pre-pilot test results were used as basis for selecting six out of the ten items for the MCT5 pre-test test and post-test pilot study, with both tests having a content validity index (CVI) of 0.78. The MCT5 pre- and post-tests will be pilot-tested on two grade five sections with at least forty students each, with a time limit of fifty minutes. Data generated by the pilot tests, highly likely to reflect the pre-pilot test results due to the subjects' lack of exposure to divergent thinking tasks, will be utilized to develop rubrics for each item and will be subjected to inter-rater reliability tests. MCT5's final version will be deployed in a larger study aimed at identifying components which promote mathematical creativity in the classroom setting.

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INTENTIONAL ACTS OF TEACHING: SUPPORTING STUDENTS TO CO-CONSTRUCT MATHEMATICAL MEANING

Generosa Leach

Massey University

Supporting students to engage in productive mathematical discourse practices require specific intentional acts of teaching. These actions can include those such as talk moves (e.g., Chapin & O'Connor, 2007), wait time (Ingram & Elliott, 2016), and teacher invitation moves (e.g., Franke et al., 2015). However, as Franke et al. (2015) found, developing productive discourse where students are supported to critically sense-make with mathematics requires teacher actions beyond initial moves. The aim of this presentation is to provide examples of specific intentional teacher actions that extend beyond initial moves and demonstrate how students can be supported to engage in productive discourse to co-construct mathematical meaning.

This presentation will draw on data from recorded classroom observations in one mathematics lesson with students aged 9-10 years old in an urban school in Aotearoa New Zealand. The focus is on illustrating the intentional acts of teaching that supported students to co-construct mathematical meaning through productive discourse. Observation data were analysed thematically.

The findings highlight how the teacher facilitated productive discourse using several distinct approaches. Specific examples are provided of intentional in-the-moment teacher actions, such as, the teacher using both silence and wait-time as non-verbal and non-evaluative actions and extending invitations for the co-construction of mathematical explanation and argumentation. These deliberate acts of teaching established clear processes and expectations for students to engage capably and work towards reaching consensus about important mathematical concepts. It is acknowledged that while the teacher actions presented here were consistent throughout the lesson, further investigation is needed on how these findings can be generalized.

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MESSAGING FOR SUCCESS: STUDENT PERCEPTIONS OF NUDGES IN FIRST-YEAR TERTIARY MATHEMATICS

Alex C. Y. Lee, Antony Sowards, Paul Hernandez-Martinez, and Jason Skues

Swinburne University of Technology

Through the lens of student engagement and achievement emotions, this study delved into how first-year tertiary mathematics students perceive the effectiveness of messaging as an educational intervention. Secondly, the essential features of such educational messaging interventions are discussed.

To address student disengagement in tertiary mathematics, educational interventions, or "nudges", serve as a promising tool for influencing student learning behaviours and attitudes, potentially leading to improved engagement and academic performance (Kahu, 2013; Weijers, De Koning & Paas, 2020). Pekrun's control value theory also suggested that students' achievement emotions would influence learning behaviours and outcomes (Pekrun & Linnenbrink-Garcia, 2012). Therefore, nudge interventions are expected to influence attributes of achievement emotions and student engagement. This paper explores the potential of messaging communication and how to design tailored messaging nudges that effectively promote and sustain high levels of student engagement.

Employing a mixed-method approach of surveys and interviews, 51 Australian first-year (mathematics and non-mathematics majors) students completed the survey, and nine students were interviewed. The survey results supported the use of messaging interventions in tertiary mathematics units. Through a thematic analysis on the interviews, the study revealed a strong preference for personalised messages among students (even if it is computer-generated). These messages were viewed as more likely to be read and influence future actions. Valued content such as assessment information, deadline reminders, and individual study progress updates were considered crucial for enhancing engagement and academic outcomes. Furthermore, the study results offered directions to develop a conceptual model of the complex interactions of educational nudges (personalised email messages), motivations, achievement emotions, academic engagement, and performance.

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STUDENTS' USES OF THE CHATGPT IN SOLVING A MATHEMATICAL MODELLING TASK

Kyungwon Lee and Oh Nam Kwon

Seoul National University

This study explores how students use ChatGPT in solving a mathematical modelling task. Recently, there has been a significant increase in the number of users engaging with OpenAI's generative AI, ChatGPT. With the advancement of generative AI such as ChatGPT, there's a growing need in mathematics education to discuss how to utilize these new tools. Research on the use of digital tools in teaching and learning of mathematical modelling has been conducted (Greefrath et al., 2018). Thus, the analysis of students' uses of generative AI as a digital tool can provide information on the potential and limitations of using this tool in the context of mathematical modelling.

Four middle school students in Korea participated voluntarily in the individual interviews, which involved solving a mathematical modelling task. The data collected included students' utterances, solutions to a task, and the records of individual uses of ChatGPT. The data analysis was carried out in two stages. First, the analysis focused on students' cycles of mathematical modelling (Blum & Borromeo Ferri, 2009) when they used ChatGPT. Second, the students' works at those cycles were analyzed, along with the use of resources. This analysis includes how students completed their solutions to a mathematical modelling task based on the responses from ChatGPT.

The uses of ChatGPT by students in solving a mathematical modelling task can be divided into two categories. First, ChatGPT was used to assist students in assuming numerical estimations and measurements in solving a task. Students used ChatGPT to obtain information and then set approximate values as assumptions to construct solutions. Although there were differences in how and when students used ChatGPT, they commonly used it to set assumptions for solving a task. Second, ChatGPT was used as a tool for mathematical works. Mathematical modelling tasks often involve calculations with fractions and decimals, requiring calculators. One student's use of ChatGPT suggests that it can potentially serve as a tool like a calculator. These findings show students' uses of ChatGPT in solving a mathematical modelling task based on empirical data. It will serve as foundational work for discussions regarding the use of generative AI tools in solving mathematical modelling tasks.

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CREATING A POSITIVE LEARNING ENVIRONMENT: PRELIMINARY RESULTS

Kien H Lim[1] and Hilda Sotelo[2]

University of Texas at El Paso[1], University of Texas at El Paso[2]

This action research aimed to create a positive learning environment for 27 middle-grade preservice teachers, promoting active learning, conceptual depth, mathematical habits of mind, and a growth mindset. Grounded in a self-based methodology (Suazo-Flores et al., 2021), the first author navigated challenges in becoming a more compassionate mathematics instructor. Key research questions include: (a) What actions were taken to establish a positive learning environment? (b) What challenges did the first author encounter, and how were they addressed? (c) How did students perceive the learning environment?

This study adopts a mixed-methods approach, incorporating traditional journal entries from varied perspectives—teacher, action researcher, and spiritual being. A distinctive element comprises 18 reflective dialogue sessions via Zoom with a distant spiritual friend, recorded to enrich reflection and provide external feedback. Student-focused data collection includes 10 weekly questionnaires, a pre-post belief survey, and a post-exam focus group facilitated by the second author with five students. Critical organic writing (Sotelo, 2024) is used for analysing video transcripts and journal entries.

Preliminary findings indicate students' generally positive reception of the learning environment. Yet, they also highlight the author's transition from transactional to empathetic care (Maloney & Matthews, 2020), underscoring the significance of genuinely accepting and nurturing students. This reflection catalyses a broader contemplation on the essence of teaching, the value of understanding students holistically, and the transformative potential of incorporating critical organic writing into educational research and practice.

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Oral communication

MATHEMATICS PERFORMANCE OF TAIWANESE STUDENTS WITH DIFFERENT MINDSETS

Su-Wei Lin

National University of Tainan, Taiwan

Mathematics is a challenging subject, often perceived as difficult by children. Some students even believe mathematical ability is innate and unchangeable. This mindset, combined with rigid math education, poses challenges for teachers aiming to improve students' math skills. Why do some people bounce back after setbacks while others remain disheartened? Dweck's theory differentiates growth mindset (believing abilities can change with effort) from fixed mindset (viewing abilities as unchangeable). These mindsets profoundly shape life development (Dweck, 2006).

This study analysed 5,768 Taiwanese 15-year-old students who participated in PISA 2022 to explore the students' response status to the Growth Mindset Scale by using MPLUS software to conduct latent class analysis, then examined the relationships of gender and mathematics performance with mindsets. The items were: "Your intelligence is something about you that you cannot change very much." and "Some people are just not good at mathematics/ Chinese, no matter how hard they study." The results indicated that a two-class model provided the best fit. 42.37% of students fall into the first class. The conditional probabilities of agreeing with the three scale items for these students were 0.186, 0.052, and 0.043. These responses suggested that this group of students believes in the value of effort and development. On the other hand, the second group of students (57.63%) exhibited different probabilities of agreement: 0.639, 0.974, and 0.922. They seemed to lack an understanding of the value of effort and appeared to believe that abilities are predetermined. These two latent classes corresponded to what the literature refers to as the growth mindset (for the first class) and the fixed mindset (for the second class). Examining the association between gender and different mindsets, 41.16% of females and 43.51% of males belonged to the growth mindset group. In comparison, 58.84% of females and 56.49% of males belonged to the fixed mindset group. Using the growth mindset group as the reference, the odds ratio for the fixed mindset group was 0.91. The proportion of males was slightly higher than that of females but did not reach statistical significance. Further, the results revealed that students in the growth mindset group exhibited higher mathematics performance (*M*=559.34, *SD*=108.17), whereas students in the fixed mindset group had lower mathematics performance (*M*=540.28, *SD*=111.79). It suggested that students with a growth mindset have better learning performance than a fixed mindset. Mathematics educators could consider applying this theory in classrooms, hoping to inspire students to become growth thinkers and promote development.

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A CROSS-CULTURAL COMPARISON OF MATHEMATICS TEACHERS' NOTICING ON TASK POTENTIAL AND ITS USE IN DEVELOPING FLEXIBLE PROBLEM-SOLVING STRATEGIES

Yu-Ting Lin[1], Josephine Paul[2], Anke Lindmeier[2], Anika Dreher[3], Feng-Jui Hsieh[1], and Ting-Ying Wang[1]

[1]NTNU, Taiwan, [2]FSU Jena, Germany, [3]PH Freiburg, Germany

High-quality use of task potential significantly impacts students' learning opportunities. Thus, teachers' noticing of task use is understood to be indicative of their teaching competence. However, what constitutes high-quality use of task potential may vary across cultures (Dreher et al., 2021). The TaiGer project, a collaborative study between Taiwan and Germany, investigates the impact of cultural norms on teacher noticing. This study, as part of the project, explores the noticing of Taiwanese and German teachers regarding the use of task potential to develop flexible problem-solving.

The research instrument included a text vignette depicting a teaching scenario in which the teacher presents a problem concerning the relationship between body weight and fever medicine dosage (5 kg to 15 ml). The students are asked to propose at least two solutions to develop their flexible problem-solving strategies. In the scenario, the teacher collects students' strategies involving quantitative reasoning on numbers or the table in a whole class discussion and finally focuses on a solution using proportional relationships ($y=3x$). As the teacher does not emphasize the connection between the different solution strategies, the task potential for developing flexible problem-solving is not used in a high-quality manner. The participating teachers were prompted to "evaluate the teacher's use of the task in this situation and give reasons for the answer." 115 secondary mathematics teachers from Taiwan and 113 from Germany took part in the study. Qualitative content analysis was performed to identify what teachers noticed.

The answers allowed us to identify different perspectives on use of the task potential beyond the focal aspect of not connecting different strategies to foster flexible problem solving. For example, mainly Taiwanese teachers praised the task for giving students sufficient autonomy in their approaches to problem-solving and participation in class discussions, while this aspect was not salient for German teachers. The presentation will show the breadth of teacher perspectives and relate differences between countries with known differences in respect to the signatures of mathematics education to provide possible explanations.

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ON DEVELOPING TEACHING MATERIALS IN USING CONTINUED FRACTIONS TO EXTRACT SQUARE ROOTS

Yung-Fa Lin and Hak Ping Tam

National Taiwan Normal University

Extracting square roots is not easy to learn as some students may find it rather abstract, technical, complicated, and not realistic in real life (Ozkan & Ozkan, 2012). In Taiwan, taking square root of a whole number is introduced in the eighth grade via the empirical method of "approaching by the division of tenths" that created iteratively closer upper and lower bounds by estimating an extra decimal digit each time. The process is tedious, and students question why not push the square root button directly. Since continued fractions used to be an important topic, conceptually connected to the Euclidean algorithm, reciprocals, recursive relations, irrational numbers, and convergency, this study explored how to develop learning materials on taking square roots via continued fractions. In addition, it examined what difficulties students might encounter as well as their attitudes towards learning continued fractions to extract square roots. The learning materials were developed iteratively in three rounds following a design research approach and content analysis. They were taught to seven seventh-graders, eight eighth-graders, and another seven ninth-graders during February, June, and August of 2023, respectively. They all attended an international school in Taipei. The teacher has thirty years of teaching experience. The final set of materials had six lessons, totaling 270 minutes. Pretests and posttests were delivered, and a questionnaire was administered regarding their attitudes towards this unit and preference between extracting square roots using continued fractions with the official approach. We will briefly report the design of the final set of teaching materials below. We motivated the existence of square root numbers by finding the length of a square given its area. In the first three lessons, we motivated the concept of continued fractions through the Euclidean algorithm, explored how to interchange between fractions and continued fractions, and explored how to find a series of approximates. The fourth lesson covered the nature of square root numbers and introduced the concept of irrational numbers. The fifth lesson covered the textbook approach to extracting square roots. The last lesson focused on hand computation and using Excel to take square roots and visualization. Most students indicated their acceptance of this unit despite being easy to err in computation. We will discuss the value of introducing continued fractions at the secondary school level in the full paper.

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LEVELS OF NOTICING IN EXPERT MATHEMATICS TEACHERS WHEN TEACHING THE PYTHAGOREAN THEOREM

Ledher M. López and Diana Zakaryan

Pontifical Catholic University of Valparaiso

One of the current perspectives in the study of noticing is the *expertise-related perspective of teacher noticing*, which focuses on the differences in noticing between expert and novice teachers. New research is needed to delve deeper into the topic (Köning et al., 2022). In this context, and as part of a doctoral thesis, the following research question arises: What levels of noticing do mathematics teachers with different years of teaching experience demonstrate when teaching the Pythagorean Theorem?

This research is based on the study of three Chilean mathematics teachers: P1, with 9 years of experience; P2, with 22 years of experience, both holding master's degrees; and P3, with 8 years of experience. The data collection process took place immediately after the first class on the Pythagorean Theorem in 8th grade (students aged between 12 and 14 years). This was done through a structured interview, posing the question: "What caught your attention in the class?" The answers of the three teachers were analyzed considering the levels (baseline, mixed, focused, extended) constructed by van Es (2011) for the selective attention and interpretation dimension.

The preliminary results show significant differences in the levels of noticing among the teachers. Concerning the selective attention dimension, P1 exhibits an extended level, P2 has a focused level, and P3 is at the baseline level. Regarding the interpretation dimension, P1 is in the focused level, P2 in the mixed level, and P3 in the baseline level.

These findings suggest notable disparities in the noticing levels among teachers with different years of teaching experience, and even among those with similar experiences. This highlights the need to explore other factors associated with the development of noticing in mathematics teachers, such as the teacher's knowledge or beliefs, which will be the focus of our future studies.

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EXPERIENTIAL LEARNING FOR MATHEMATICAL THINKING & 21ST CENTURY COMPETENCIES

Low Leng and Teh Wan

Academy of Singapore Teachers

Engaging in mathematical thinking is essential for fostering meaningful mathematical concepts as opposed to merely memorizing procedures. Kolb's Experiential Learning Cycle (2015) is a versatile framework aligning with 21st-century competencies (21CC) (MOE) that empower students to apply reasoning, utilize metacognitive abilities, generate innovative solutions, and manage complexities. The research question seeks to explore how teachers can enhance students' mathematical thinking while nurturing critical, adaptive, and inventive thinking skills within the framework of 21CC. This exploratory case study focuses on a primary and a secondary school classroom. Specifically, the study delves into finding unknown angles (primary school) and differentiation of power functions (secondary school). In the primary classroom, the focus was on the process of finding unknown angles in a composite figure through a hands-on exploration of geometric concepts. In the secondary classroom, students utilize Desmos to visually engage with various power functions and observe the variations in the behaviours of the tangent lines on the graph. Qualitative insights for this study are gathered through classroom observations, interviews, student samples, and reflections. Results indicated that primary school students demonstrated enhanced understanding of the relationships between angles in a composite figure. Through the application of mathematical thinking, students were able to systematically analyse the given figure, identify relevant properties and apply them to determine unknown angles. In the secondary classroom case study, the findings revealed that Desmos provided an effective platform for students to visually understand the relationship between power functions and their derivatives. Students gained a deeper understanding on the concept of differentiation when the parameters of power functions are varied. The findings suggest that the integration of Kolb's experiential learning model and mathematical thinking provides a framework for engaging learners in the exploration of mathematical concepts. This approach equips students the ability to draw conclusions in different contexts, reason and communicate in a classroom community, which are important skills to navigate in the rapidly evolving 21st-century landscape. The implications of our finding for both educators and learners suggest opportunities for improved student engagement and pedagogical innovations, leading to a deeper appreciation for mathematical concepts.

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LECTURER'S TOOLS FOR TEACHING PROOFS AND PROVING

Angeliki Mali

University of Crete, Department of Mathematics & Applied Mathematics, Greece

The relationship between lecturers' teaching practice in proof-based undergraduate mathematics courses and students' mathematical meaning making (understanding) has been reported to have a dearth of research in mathematics education (Melhuish et al., 2022). This area of study nevertheless holds significant value for the community of lecturers as it provides valuable insights into the craft of teaching proofs and proving, as well as the opportunities for students to make mathematical meaning.

The distinctive nature of this study is concerned with the conceptualisation of undergraduate mathematics teaching practice within a Vygotskian perspective. In particular, I use for the characterisation of teaching practice the notions 'tool-mediation' and 'dialectic' (Wertsch, 1998). So, the lecturer acts with tools for teaching, which mediate the student's development of mathematical meaning making, as well as the lecturer's own development of teaching and mathematical meaning making. In the study, I ask: What is the nature of lecturer's undergraduate teaching practice with proofs and proving at a university in the UK? What is the relation between lecturer's teaching practice and her consideration of students' mathematical meaning making?

Over three academic semesters, I observed and audio-recorded the teaching of twenty-six lecturers and engaged in discussions with them about their underlying teaching considerations. The focus of this study is the characterisation of one lecturer's teaching practice, which I observed for more than one semester. I took a grounded analytical approach to observational data (Glaser, 1998), constantly comparing excerpts coded with the same open code or the same concept found in the research literature.

I found a variety of lecturer's tools (e.g., heuristics, such as 'sketch a graph' and 'consider special cases') drawn from the context of mathematics, and tools for designing the teaching in order to translate the mathematical content into the context of students. In the oral communication, I will explain these sets of tools, contributing to the research literature with a framework for researcher analysis of undergraduate teaching practice with proofs and proving for students' mathematical meaning making. The knowledge produced can be valuable for research and professional development.

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ADAPTATION OF THE MONTY HALL PROBLEM AS AN ACTIVITY FOR HIGH SCHOOL PROBABILITY

Harold V. Mangubat[1], Angel Mae O. Ombid[2], and Dennis Lee Jarvis B. Ybañez[1]

University of the Philippines Open University[1], Ateneo de Manila University[2]

In today's data-driven society, one can never fail to encounter situations requiring them to make informed decisions. Important aspects of probability and statistics such as chances, risk, outcomes, and uncertainty are encountered more frequently in our day-to-day lives. However, due to the highly abstract nature of probability, learning this concept becomes challenging for students. In most cases, they solely rely on procedural approaches resulting in a difficulty to use probabilistic approach in decision-making (Ybañez and Vistro-Yu, 2022). To emphasize the importance of probabilistic thinking in decision-making, one activity used in teaching probability is the Monty Hall Problem, where students act as players of a game show. In this study, 32 in-service Filipino mathematics teachers are tasked to develop a lesson plan in probability, incorporating an adapted version of the Monty Hall Problem, to explore the kinds of conceptualizations they will present in the lesson.

In the adapted version of the problem, the students are still presented with three doors (behind one of which is a prize, the other two are goats), but the teacher as the host has control over what is behind the doors during the activity. Students are initially asked to pick a door (say door A), and the teacher opens another door to reveal a goat. The students are then offered a choice to stick with their initial pick (door A) or switch to the remaining unopened door (door B), but before letting them decide, the teacher will discuss the probability of these two doors (i.e., door A has a probability of 1/3, and door B has a probability of 2/3). In the lesson plan, the teachers are tasked to indicate the following: (1) after the students pick a door, where will the teacher place the prize? Behind door A or door B? (2) how will the teacher proceed with the activity, and what concepts of probability will they discuss after? Results show that majority of the teachers chose to place the prize behind door B as they wanted to discuss the straightforward idea that "the greater the probability, the higher the chances of winning". Consequently, their lesson plans did not discuss probability based on frequency and failed to state that 1/3 is not an impossible event. On the other hand, the teachers who chose to place the prize behind door A were able to include more conceptualizations in the lesson plan, such as: (a) 1/3 is not an impossible event, (b) 2/3 is not a certain event, (c) probabilities involve frequency, i.e., on average, the prize is behind door A 1/3 of the time, (d) if the game is to happen several times, door B is desirable; but if not, then luck or intuition is another concept to be considered.

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STUDENTS' GRAPHING ACTIVITY & DIGITAL TASK DESIGN: THE SKETCH-TO-ANIMATION BOTTLE PROBLEM

Claudine Margolis

University of Michigan

Researchers have argued that covariational reasoning—attending to the relationship between quantities that vary simultaneously—is a productive way of reasoning about concepts from middle school through undergraduate mathematics (Thompson & Carlson, 2017). Digital tasks that provide students with opportunities to reason about simultaneously varying quantities from dynamic situations can promote students' covariational reasoning. Some prior research has done this in the context of graph construction tasks in which students view or interact with an animated situation embedded in a digital environment and then construct a graph to represent covarying quantities from the situation (e.g., Moore, 2024). Other research has focused on students' covariational reasoning through paper/pencil tasks that are intended to elicit a mental image of change. The Bottle Problem, in which students graph the relationship between Height and Volume of water in a bottle as it is filled, has appeared in a number of these types of studies. In this oral communication, I analyze students' activity on a digital version of the Bottle Problem which I designed to study the role of sketch-to-animation (StA) tasks on students' graphing activity and covariational reasoning.

In StA tasks, students sketch a graph and then view the interpretation of their sketch in the animated situation. The StA Bottle Problem differs from the paper/pencil version in three important ways: (1) students view an animation of the bottle filling prior to sketching the relationship between quantities, (2) students receive visual feedback from the digital environment related to their sketch, and (3) students are provided with a series of animations in which the same bottle is filled with water in different ways such that the quantities vary with respect to time differently across animations. I utilize two frameworks for characterizing students' covariational reasoning with respect to students' mental images of how quantities change (Thompson & Carlson, 2017) and the role of time in coordinating those images of change (Moore, 2024). By analyzing students' in-the-moment covariational reasoning on the StA Bottle Problem, I draw conclusions about how StA tasks can be designed to support covariational reasoning. I conclude with implications for digital math task design that utilizes dynamic situations as the context for students' covariational reasoning.

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BULDING AN INSTRUMENT TO CHARACTERIZE TEACHER NOTICING ABOUT ARGUMENT AND MATHEMATICAL MODELING COMPETENCIES

María Victoria Martínez[1], Victoria Arriagada[2], and Horacio Solar[2]

[1]Universidad de O´Higgins, [2]Pontificia Universidad Católica de Chile

Noticing is a teaching competence characterized by what teachers identify, interpret and decide about student reasoning (Jacobs et al., 2010). In a review of research on the development of noticing in teachers, there is a predominant focus on students' thinking in the context of mathematical objects (Santagata., 2021). Our study of teacher noticing focuses on an element that is lacking in research: the development of reasoning and modeling competencies. The components of noticing that we will study are identifying and interpreting (van Es & Sherin, 2002). Within the framework of a larger project whose objective is to characterize the development of noticing by promoting modeling and argumentation in teachers, we have developed an instrument to characterize the noticing of mathematics teachers during their participation over two years in a program professional development to promote argumentation and modeling. This instrument is made up of videos, questions and an analysis rubric. The process of construction and validation of the instrument included a series of steps. A bibliographic review, in relation to the characterization of argumentation noticing and mathematical modeling. Based on the literature, identification of elements of noticing with a focus on arguing and modeling would be incorporated into the instrument. Development of questions that would elicit teachers' noticing regarding arguing and modeling, considering the stages of noticing that we need to evaluate (identify and interpret). Editing of 12 videos based on available resources from previous projects. Interviews with 9 teachers to test videos, questions from the instrument and raise categories in the development of the rubric. Validation of videos and questions by 8 experts to select the final videos. Design of the rubric to correct the answers based on what the teachers answered in the interviews and validation of the rubric through expert judgment. This instrument will contribute to characterizing teacher noticing, incorporating a focus by promoting mathematical competencies.

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NUMBER TALKS IN SECONDARY MATH CLASSROOMS

Richelle Marynowski and Sandy Bakos

University of Lethbridge

Number Talks have been shown to benefit student understanding of numeracy concepts by shifting the focus of attention from the solution (product) towards making sense of the strategies used (process) to answer computation problems (Sun et al., 2018). A deeper understanding of mathematical concepts is required for students to articulate their metacognition, communicate their strategies and defend their solutions (Parrish, 2011). This paper reports on a collaborative research project with three rural school divisions in Alberta, Canada involving Number Talks. We wanted to know: Does the implementation of Number Talks in grades four to eight mathematics classrooms increase student confidence and improve student competence in numeracy?

There were 30 teachers who volunteered to participate in the study and more than 300 of their students experienced Number Talks 2–3 times per week in their mathematics classes. The teachers participated in five professional development sessions that were co-lead by the two university professors and the mathematics coaches from each school district. These sessions were designed to support teachers and their implementation of Number Talks, which took place from September through December, 2023.

Numeracy assessments were developed as part of the project and students completed pre-tests (September 2023) and post-tests (January 2024) to determine if there was a change in competence in numeracy. Students also self-rated their confidence level for each question, which were compared between the pre- and post-test. Additionally, interviews will be conducted in March 2024 to learn more about teacher and student experiences and perceptions of Number Talks. A convergent, one-phase mixed-methods approach (Creswell & Creswell, 2018) that uses a quantitative comparison of numeracy test scores and confidence ratings along with a qualitative analysis of student and teacher interviews will used to respond to the research question.

Initial analysis of student responses on the pre- and post-tests show that there was an increase in both student competence and ratings of their confidence. As one teacher shared: “I have a higher percentage of kids ready to defend their answers, take risks, chances and work through a problem even if they're not quite sure how they got there.”

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A PHENOMENOLOGICAL STUDY OF THE FORMATION PROCESS OF VALUES IN MATHEMATICS LEARNING

Mitsuru Matsushima

Kagawa University, Japan

In mathematics education research, values are seen as the internalisation, recognition and decontextualisation of beliefs and attitudes by individuals in a socio-cultural context (Seah & Wong, 2012). International comparative 'Third Wave Project' research on values has been multifaceted, but the process of value formation remains a research question (Seah & Wong, 2012). This study addresses the exploration of the value formation process that is emphasised within mathematics learning classrooms.

The method of interpretive phenomenology is used to articulate the process of value formation (Eddles-Hirsch, 2015). Specifically, the researcher is first immersed in the phenomenon in order to understand it. Then, the researcher interprets the phenomenon based on inferences of its meaning from the participants' writings, words and actions. Specifically, the author, who has 20 years of teaching experience at elementary, junior high and special-needs schools, enters one fourth-grade class at a primary school attached to a national university in Japan for one hour every week for six months to conduct observations. During the observation, the author writes in the Fieldnotes about his findings on values related to mathematics education. The author then reconstructs the observations into episode descriptions using phenomenology. These episode descriptions are compared chronologically and the formation process of values emphasised in the classroom is discussed.

As a result of the observations, 11 episode descriptions were collected in six months. For example, children contrasted and discussed their own ideas with the teacher's statements based on their absolute trust in the teacher; they felt an increased satisfaction in creating their own learning because their opinions were valued even at the end of the class; they proceeded with the class in a child-centred manner while speaking freely; they had heated discussions when they created their own volume unit; and they murmured, "It's interesting that everyone's opinions are different. The descriptions of these episodes show that in this classroom, the values of satisfaction with learning with the teacher, valuing different ideas in the classroom and valuing independent learning have emerged.

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Oral communication

FOSTERING GENDER EQUALITY IN FINANCIAL MATHEMATICS UNIVERSITY PATHWAYS AND PROFESSIONS

Amedeo Matteucci[1], Adamaria Perrotta[2], F. Saverio Tortoriello[1], and Ilaria Veronesi[1]

[1] University of Salerno, Italy, [2]University College Dublin, Ireland

Both nationally and internationally the need for STEM graduates is constantly growing and this has been echoed by EU governments' STEM Education Implementation Plan. While there has been an increase in students enrolling in STEM programmes, there is a stark gender disparity in students' choosing degrees with a core Mathematics focus, with implications at wider society level in terms of gender gap in STEM careers. This is particularly evident in Applied Mathematics (AM) subjects, where a sense of mathematical identity and sense of belonging are the strongest predictors of female students continuing with a mathematics-based degree (Good et al., 2012). This gender gap is a concerning issue for Actuarial (Act) and Financial Mathematics (FM) programmes, where only 30% of students are female; accordingly, the gender gap is reflected in financial services, that are still male dominated. This study aims to foster the diversity and inclusion of students enrolling in AM programmes, with a particular focus on reducing gender gap in Act and FM pathways. Research has demonstrated that learning environments which provide spaces to develop students' interest, understanding outside of the formal learning environment and reflection can reduce the lack of gender and socioeconomic diversity. Moreover, teaching mathematics through more open-ended, collaborative, problem-solving approaches in a computational learning environment contributes to achieving more equitable results and reducing the gender gap (Ardito et al. 2020). Inspired by these research studies and referring to a similar study conducted in Ireland at university level, we have designed student-led computational FM lab practices for students participating to the Mathematical High School (MHS), a research project involving 29 universities and 160 high schools in Italy. The activities will be delivered on pilot basis in May 2024 to 40 students attending two MHS classes (10^{th} and 11^{th} grade). Students will be driven to transfer newfound mathematical and computational knowledge to FM situations. Pre and post activities survey and field notes will be collected to address the following RQ:

Do the proposed lab practices contribute to foster diversity and inclusion in AM, Act and FM pathways? A blended-framework of discipline-based identity that incorporates perception of the self will be used as an additional lens for data analysis.

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A NEW AGORA: THE ROLE OF AI-SHAPED DEBATE IN TRANSFORMING MATHEMATICS EDUCATION

Amedeo Matteucci[1], Ilaria Veronesi[1], and Francesco Saverio Tortoriello[1]

[1]University of Salerno, Department of Mathematics, Salerno, Italy

The use of AI chatbots among students is a widespread reality. Concerns exist that AI chatbots risk making presentation and organisation of students' ideas uniform (Chan, 2023). We face a contradiction: the emphasis of our current educational systems on performance will force students to increasingly use AI tools, this will in turn equalise students' outputs, nullifying the said emphasis. Collaboratively identifying and resolving contradictions within an educational system is the trademark of the theory of Expansive Learning (Engeström, 2016) that informed our approach.

We single out two problems that emerge from a perspective integration of AI tools into current educational routines: the homogenisation of students' knowledge; the lack of viable verification tools for teachers. Our research question is: can debate be used to address these issues by problematizing AI use without banning it?

We already studied the adaptation of the so-called competitive debate to mathematics teaching devising a taxonomy that classified debatable metamathematical motions into the categories: 'worst error', 'comparative analysis', 'best explanation'. To address the above issues, our ongoing experimentation revolves around three main phases: 1) giving the students a problem, along with the solution devised by an AI chatbot, to critically review; 2) asking them whether they found the solution to be: correct, complete, effective, original, clear, understandable; 3) If most students deem the solution incorrect or incomplete, the teacher initiates a 'worst error' debate; significant disagreement on its effectiveness or originality prompts a 'comparative analysis' debate with an alternative solution; likewise, differing views on clarity or understandability lead to a 'best explanation' debate, also with an alternative solution.

We expect the students to enhance their critical thinking and cooperation skills—literature shows debate activities already achieve this—along with their mathematical knowledge, since analysing and debating solutions, they are likely to gain a deeper understanding. This addresses both the problem of homogenisation of knowledge—the proposed activities foster an environment where multiple perspectives are not only accepted but required—and the problem of viable verification tool—the teacher will be able to validate the correctness and depth of understanding required to engage with complex problems without fear of examining a copied solution.

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ASSESSMENT OF BIG IDEAS OF EQUIVALENCE: INVESTIGATING AN AGGREGATED APPROACH

Mohamed Jahabar Jahangeer, Tin Lam Toh, Eng Guan Tay, and Cherng Luen Tong

National Institute of Education, Singapore

Charles (2005) defined a “Big Idea as a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole” (p.10). The essence of Big Ideas is connection - of concepts and understanding across topics, strands or levels. To date, there has been little research on assessing Big Ideas, perhaps because it is difficult to create suitable items.

We constructed items to assess the ability to make connections using Big Ideas. Each item consists of four tasks. The first three tasks are usually from the same mathematics topic while the final task, from another topic, seeks to assess the ability to make connections. We have piloted two of the items with results reported in Jahangeer et al. (2023). An important observation from that analysis was that Task 1 to Task 3 violate the item independence requirement of a Rasch scale. Hence, only Task 4 can be used as a reliable measure of Big Idea ‘ability’. This would require us to have at least eight items for the instrument.

To reduce assessment time and fatigue (Ackerman & Kanfer, 2009), the instrument, comprising eight items, is divided into four different forms, each containing two items. We developed the concept of a ‘Composite-Student’ (CS). Each CS is made up of four students of similar ability in Mathematics. The four students are then given the four different forms which make up the instrument. We validated the items and the CS structure and the results will be presented during the oral communication.

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TEACHERS NOTICING IN THE MATHEMATICS CLASSROOM: INSIGHTS FROM TRU MATH FRAMEWORK

Jaclyn Murawska, Jing Guo, and Qiaoping Zhang

Skokie-Morton Grove School District 69, The Education University of Hong Kong

With the increasing interest in research on teacher noticing, the development of a domain-specific noticing framework is essential. The Teaching for Robust Understanding of Mathematics (TRU Math) Framework (Schoenfeld, 2014), functions as a diagnostic tool intended to deepen teachers' insights into the interplay between pedagogical practices in mathematics classrooms and student learning outcomes. This framework encompasses five dimensions of observation: Mathematics; Cognitive Demand; Access to Mathematical Content; Agency, Authority, and Identity; and Uses of Assessment. This study aims to examine the efficacy of the TRU Math framework in enhancing teacher noticing by involving mathematics teachers in classroom observations. Seven mathematics teachers enrolled in a Master of Art in Mathematics Education program were enlisted to observe a classroom session segmented into three distinct phases: whole-class activities, small group work, and student presentations. Participants were instructed to document their observations, specifically what and how they noticed phenomena, in relation to each of the five TRU Math dimensions during each phase. The observational data were subsequently analysed and coded according to the categorization established by van Es and Sherin (2006). Analysis of the observational data revealed a shift in focus from Teacher and Pedagogy during whole-class activities to student participation and interactions during student presentations. Teachers predominantly employed descriptive language in articulating their observations. Evaluative (Evaluation) and explanatory comments (Explanation) were more prevalent during whole-class activities, often relating to pedagogical methods. Overall, teacher noticing tended to concentrate on Teacher and Pedagogy, likely reflecting an intrinsic motivation to enhance their teaching competencies. Noticing patterns regarding both the Agent and the Topic evolved through the different classroom episodes, reflecting increased attention to children's mathematical thinking. These patterns highlight the TRU Math framework's influence in directing teachers' noticing, suggesting its potential as a valuable tool for professional development in mathematics education.

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Oral communication

RETRIEVAL PRACTICE - A TOOL TO NARROW THE KNOWLEDGE GAP IN LEARNING HIGHER MATHEMATICS

Anna Muzsnay[1] and Csaba Szabó[2]

[1]University of Debrecen, [2]Eötvös Loránd University

One of the most important factors that have an impact on academic success is prior academic achievement. A strong indicator of academic success is grades (Alyahyan, & Düştegör, 2020). Aiming to reduce the gap between students' first-year academic achievement in mathematics courses, we applied a special retrieval practice. The positive effects of retrieval practice – the strategic use of retrieval to enhance memory – have been shown in several cases. Still, to what extent it can help students learn higher mathematics of different input levels has been a question (Agarwal et al., 2021).

In this research, we investigated first-year pre-service mathematics teachers' performance in two mathematics courses, Number Theory and Abstract Algebra, each of which lasted 13 weeks. The sample comprises 43 and 44 students who attended these courses. Within the two courses, we divided the students into two groups. At the end of the practice sessions, the experimental group wrote a 5-10 minute test on the material learned on the given day. They had to solve two problems individually without external help. In the control group, the teacher presented the solutions to these problems. Students' input level was assessed at the beginning of their studies, and their topic-related problem-solving skills were measured twice during the semester. Also, students wrote a post-test 3 and 5 months after finishing the course.

We examined their test and post-test results in the two courses relative to their input level using linear regression. The results show that in Number Theory the knowledge gap narrowed over time in the experimental group; on the post-test students with lower input scores could catch up ($y=0{,}07x+69{,}24$; $R^2=0{,}018$). In the control group, the knowledge gap remained ($y=0{,}95x-4{,}30$; $R^2=0{,}399$). Similar results emerged in Algebra. In the presentation, results will be discussed in detail. Our findings suggest the applied retrieval practice can be an effective way of reducing the knowledge gap in learning higher mathematics that teachers can easily incorporate into their lessons.

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HOMEWORK AT UNIVERSITY – WHY DO STUDENTS (NOT) DO ASSIGNMENTS?

Silke Neuhaus-Eckhardt and Hans-Stefan Siller

Julius-Maximilians-University Würzburg

It is often assumed, that doing additional exercises out-of-class benefits the students' learning outcomes at university. But many students don't use these learning opportunities, especially if the exercises are voluntary. However, if homework is mandatory, students also tend to engage in inappropriate learning behaviour, such as copying solutions. Thus, before demanding mandatory homework, one should identify the students' reasons for (not) doing homework. According to Eccles and Wigfields' situated expectancy-value theory (2020, SEVT), students' performance related decisions, like doing homework, could depend on their expectancies for success or their task values, like intrinsic value, utility value, attainment value and costs. As such our research questions for this contribution are:

What reasons do students give for or against doing homework in courses with mandatory homework, homework with benefit and voluntary homework? Do the reasons fit into the categories of SEVT?

Our sample consists of $N = 115$ students of one public university in Germany but different mathematics (education) courses and different submission formats. All the students were at least in their second semester and already familiar with the new learning situation at university. Embedded in a bigger paper-based questionnaire we asked them why they did or did not do their homework. The written answers were coded using qualitative content analysis with SEVT as basis for the coding manual. Overall, most of the reasons for or against doing homework could be categorized using SEVT, only a category of social aspects was added. Most reasons for doing homework referred to utility values, esp. preparing for the final exam or getting the admission to or a benefit for the final exam. As such, most students stated an extrinsic motivation to do homework. The reasons why students do not do assignments mostly referred to costs or expectations of success, especially that the assignments were too time consuming, or that they were not able to solve them, which is in line with the expectancy-value theory (Eccles & Wigfield, 2020). Additional results as well as limitations and practical implementations will be discussed in the presentation. Overall, the results show differentiated reasons why students do or not do homework at university, which should be kept in mind, especially when choosing homework assignments.

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MATHEMATICS TEACHER EDUCATORS' PERSPECTIVES ON THEIR PREPAREDNESS FOR ONLINE TEACHING

Zanele Ngcobo[1], Jyoti Jhagroo[2], Msebenzi Rabaza[3], and Justice Enu[4]

[1]University of KwaZulu Natal, [2]Auckland University of Technology, [3]University of Free state, [4]Kwame Nkrumah University of Science and Technology

This article offers insight into Mathematics teacher educators' (MTE) preparedness for a sudden shift to online teaching and the professional training offered by their institutions for online teaching. While online teaching is not a new phenomenon, the unexpected COVID-19 pandemic forced the shutdown of institutions that offered face-to-face-only mode of study. In an attempt to continue the study programmes while not endangering lives, institutions resorted to emergency remote teaching and learning. Decisions of this nature are often made at senior levels and teacher educators were expected to cope with the changed platform. In this article, we present perspectives of MTE from initial teacher education institutions in South Africa, Ghana and New Zealand. The focus on MTE emerged from the notion that teaching STEM subjects are anchored on collaborative teaching. Drawing on the socio-cultural work of Sfard (2007) and others, premised on teaching and learning environments as discursive spaces with a great deal of communication, we set out to gain a preliminary understanding of how the shift to emergency online teaching enabled such an environment of teaching and learning.

The sample consisted of eleven mathematics teacher educators across the four institutions who responded to the online questionnaire after the first two waves of the pandemic. The findings show that mathematics teacher educators in all four ITE institutions had to cope with emergency online teaching at very short notice. In terms of professional training offered, this mainly focused on operational matters and the use of learning management systems. Not all mathematics teacher educators felt that the training added value to their online teaching nor to their motivation for work nor did it help them cope with the challenges. There was an overwhelming agreement that the sudden shift to online teaching had a negative impact on the preservice teachers. For example, lack of peer interaction, student-lecturer interaction course delivery not being able to meet their individual needs.

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ANIMATING CLASSROOM REALITIES FOR ENHANCED TEACHER NOTICING

Jihwa Noh, Byeongsoo Kim, Ho Kyoung Ko, and Nan Huh

Pusan National University, Ajou University, Kyonggi University

Teacher noticing has been a recognized component of expert practice that allows teachers to develop sensitivities toward certain aspects of their work relevant to their practice (e.g., Larison et al., 2022). Different resources—videos, written cases and comics—have been used to engage teachers in productive noticing. As different representations possess different kinds of advantages and limitations, other representations are thinkable. The purpose of this study was to develop another useful resource, *animated-documentary classroom video* (hereafter referred to as ' ani-docu video') offered with cartoon characters using real classroom stories. This study was guided by the following research questions: (1) What are design principles that help create ani-docu videos as effective resources for teacher noticing? (2) What are possible impacts of ani-docu videos on the nature of teacher noticing?

To help guide the development of videos, this study applied design-based research in which different methods are used to perform a series of investigations (Brown, 1992). In order to examine impacts of ani-docu videos on teacher noticing, two comparable groups of secondary mathematics teachers were invited to view and discuss videos: one group viewing ani-docu videos and the other group viewing real classroom videos that were the basis of the ani-docu videos developed in this study. Teachers' written notes taken while viewing videos were collected and after-discussion were transcribed for analysis.

Micro-learning approach was found to be a highly important principle that makes ani-docu videos more effective for noticing. Teachers viewing ani-docu videos were better attentive to the attributes relevant to the practice highlighted and were more willing to try unfamiliar and/or challenging practices in their own classrooms after they knew the videos were based on real classroom stories.

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PRIMARY TEACHERS' CONCEPTIONS OF PROBLEM SOLVING

Mary Ochieng[1] and Penina Kamina[2]

[1]Strathmore University, [2]University of New York Oneonta

This study is part of a larger study on implementation of the competency-based mathematics curriculum of a select county in Kenya. Analysis of the pilot classroom observation data findings showed that teachers meaning making of problem- solving skill differed. Various studies (e.g., Sikoyo, 2010) have highlighted the challenges of implementing problem solving in mathematics classrooms and the need for support for teachers in this area. This study reports on primary teachers' learning process and articulation of problem-solving.

The purpose of this study is to examine primary mathematics teachers' conceptions of mathematical problem solving with the view to support them to implement problem solving in mathematics classrooms. The research question is: *What are primary mathematics teachers' conceptions of problem solving?*

This is a case study of a learning community of nine primary mathematics teachers and two researchers. The study applied an iterative process of unpacking and sense making of the problem-solving process via rich tasks and use of Frayer's model. Focus group discussions were audio recorded, transcribed and coded. Teachers completed the Frayer template after each session, recording their conceptions of the definition, characteristics, examples and non-examples of problem solving.

We report here brief preliminary results since the study is ongoing. At the point of reporting these results the teachers saw problem solving as finding an answer to a problem which they sometimes classified as challenging or real life. They were able to articulate certain characteristics of problem solving i.e., can be solved using more than one approach. Some of the teachers were able to cite examples and non-examples of problem solving. The teachers' conceptions that aligned most with the curriculum were on the characteristics of problem solving.

Supporting teachers' development of a common understanding of problem solving that aligns with the curriculum, may require a differentiated approach with regard to specific teacher needs and aspects of problem solving.

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UNIVERSITY STUDENTS' READING OF MATHEMATICAL PROOFS VARIES BY CONTEXT AND PROFICIENCY LEVEL

Takuo Oguro[1], Masahiko Okamoto[1], and Mitsuru Kawazoe[1]

[1]Osaka Metropolitan University, Japan

Using an eye-tracking technique, Inglis and Alcock (2012) discovered that the process of reading mathematical proofs varies between mathematicians and students, e.g., the latter pay more attention to equations than the former. Panse et al. (2018) reported a minor difference between the reading processes for understanding and verifying a proof. The purpose of this study was to replicate (1) whether the reading process differs between the reading contexts of proofs and (2) whether the reading process differs between different levels of math students. Forty-eight Japanese undergraduate and graduate students participated in this study. Their proficiency in mathematical proofs was measured using two proof problems. Participants were asked to read proofs in three contexts: to judge whether mathematical induction and/or "reductio ad absurdum" were used in the proof (comprehension); to confirm whether the proof was valid (validation); and to learn the proof to prove a similar theorem after reading it (modelling). The reading process was recorded using an eye camera. A practice trial was conducted prior to each proofreading context, and the participants were asked to judge their subjective difficulty after reading the proofs. Eye tracking data, including fixation duration, pupil size, and eye movements between lines, were analysed using a linear mixed model. Our results revealed a significant effect of context on between-line move frequency, equation fixation rate, total reading time, pupil dilation, and mean fixation duration. In particular, pupil dilation was greater in the modelling than in the comprehension or validation contexts. This suggests that modelling requires a high cognitive load. We also found significant interaction between the proficiency level and context on the standard deviation of the between-line movement. The slope analysis showed that pupil dilation in modelling was greater for low-proficiency than for high-proficiency students, after controlling for the subjective difficulty of the proof. This suggests that a modelling context is difficult for low-proficiency students and requires high cognitive loads, but higher-proficiency students can read a proof in modelling without high cognitive load. These results provide an insight into the differences between novice and expert learners, and guide interventions in mathematical proofs.

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VALUING IN MATHEMATICS CURRICULUM AND TEXTBOOKS FOR GRADES 1 AND 2

MinYoung Oh

Graduate School of Korea National University of Education

Through learning mathematics, students not only understand mathematical knowledge, but also recognize the values of mathematics. Values and valuing are related but distinct concepts. “*Valuing* is clearly a behavior but with no specification of what is to be valued. *Values* on the other hand represent what is to be valued (Bishop, 2014).”

Given that mathematics textbooks are one of the resources to support mathematics learning, they can be analyzed not only in terms of how they help students understand mathematical concepts, but also in terms of how they intend students to value mathematics or mathematics learning. However, prior research analyzing mathematics textbooks has been focused on the former, with little research on the latter. Research on the valuing of elementary school mathematics textbooks is still very limited.

Recently, there have been several changes in mathematics education in Korea. The national mathematics curriculum was revised for the first time in seven years, and the revised curriculum explicitly emphasizes the values of mathematics by introducing ‘values and attitudes’ as a category of mathematics learning. In addition, textbooks based on the revised mathematics curriculum are being developed in sequence, and new textbooks for grades 1 and 2 will be applied in the field starting this year.

In this context, this study aimed to is to explore the valuing in mathematics curriculum and textbooks for grades 1 and 2 of South Korea. This study analyzes the valuing in curriculum and textbooks based on Seah and Bishop’s (2000) mathematical values and mathematics educational values. Mathematical values include the pairs of rationalism-objectivism, control-progress and openness-mystery. Mathematics educational values include the pairs of formalistic-activist, instrumental-relational, relevance-theoretical, accessibility-specialism, evaluating-reasoning.

This study can contribute to discussion on the issue of what mathematical and mathematics educational values are emphasized in Korea, and the issue of how to align the valuing in curriculum with the valuing in textbooks.

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VOCABULARY AS AN INDICATOR OF NUMBER SENSE? A CASE STUDY WITH PRE-SCHOOL PUPILS

Oduor Olande

Linnéuniversitetet

Subject-specific vocabulary, particularly in mathematics is acknowledged as essential for learning. “Without an understanding of the vocabulary that is used routinely in mathematics instruction, textbooks, and word problems, students are handicapped in their efforts to learn mathematics” (Miller, 1999, p. 312). While Miller’s observation is largely uncontested there is need for research on how pupils deal with potential *ambiguities* that may arise with subject-specific vocabulary attached to number sense. This is especially relevant given some educators' emphasis on definitions as a measure of cognition (cf. Vinner, 1991). It has been established that definitions, particularly concerning number sense, are not without difficulties; Griffin (2004) argues that while aspects of number sense are easily recognizable, defining it precisely is challenging.

The present research is a case study of pre-school pupils endeavour to explain the mathematics concepts associated with number sense namely: *numerals* and *digits* (in Swedish *tal* & *siffror*). Unlike studies focusing solely on mathematics vocabulary as a prerequisite for engaging in mathematical activities (e.g., Miller, 1999, Griffin, 2004) the present study seeks to explore how pupils connects previous knowledge and available recourses in a sense making process. The guiding questions are: *i)* how do the pupils explain the concepts in question? *ii)* what are some of the emerging thoughts in their sense making process?

Data was collected through an interview based on aspects related to the pupils written answer to a task. Using a semiotic framework (Olande, 2014), the pupils’ sense making process and emerging ideas connected to number sense were identified. Preliminary results highlight the fluid nature of the concepts and also demonstrates the pupils’ use of what is at hand - the *apparent* to approach some profound aspects of number sense.

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THE FUNCTIONS OF ARGUMENTATION: A LITERATURE REVIEW FOR MATHEMATICS EDUCATION

Jorge Olivares-Aguilera and Manuel Goizueta

Pontificia Universidad Católica de Valparaíso

It is common to find in the specialized literature that argumentation has the main function of justifying an assertion (Mohammed, 2016) in order to convince an audience. However, since argumentation presents specificities related to the context in which it is developed, more nuanced functions have been suggested. For instance, De Villiers (1990) points out that within mathematics demonstration –a particular form of argumentation– can also have the functions of explaining, discovering, systematizing and communicating. Other functions of argumentation have been suggested in relation to mathematical practice and its learning but the field lacks a systematic approach. The need arises to investigate argumentation from a functional perspective, starting with a systematic review of the literature. For this purpose, we ask ourselves the following question: what are the functions of argumentation that have been identified in Argumentation Theory and Mathematics Education literature?

We conducted a grounded theory literature review encompassing both fields. We retrieved articles indexed in WoS and SCOPUS that included the terms "function* of argument*" or "function* of proof*" in the title, abstract or keywords, and considered the semantically related terms use, role, aim, purpose and goal. After excluding off-topic texts the search yielded 297 articles; 74 of which focus on the functional aspect of argumentation as its main research object and 223 address the topic indirectly.

The initial results show that: (i) there is still no consensus regarding what the main function of argumentation is and whether there are other functions of argumentation beyond justification; ii) the justifying function of argumentation plays a predominant role within argumentation theory, but in the school context it does not have such a predominance, suggesting the need to generate a link between practice and theory; and iii) there is scant evidence regarding the study of the functional aspect of argumentation in mathematics teachers education. The result is part of an ongoing PhD project aims at presenting a comprehensive and systematic picture of the various approaches to the topic as a base both to inform the ongoing discussion within mathematics education and to conduct theoretically based empirical research in the mathematics classroom.

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DIFFERENT WAYS STUDENTS INTERPRET AXES ON GRAPHS

A. Olshefke[1], T. Paoletti[1], C. Margolis[2], A. L. Gantt[1], H.Y. Lee[3], and H. Hardison[3]

[1]University of Delaware, [2]University of Michigan, [3]Texas State University

Graphs are a powerful way to communicate quantitative relationships, but students experience difficulties with graph interpretation and construction (Leinhardt et al., 1990). We characterize two novel types of *reference frames* (RFs) and describe how a student used these RFs to reason about the quantities represented via graphs.

Lee et al. (2020) described how coordinate systems (CSs) are mentally constructed by coordinating RFs. RFs are constructed to gauge relative extents of attributes in phenomena and consist of a reference object, direction, and some anticipation of a measurement process. We distinguish between *continuous* RFs and *ordered-discrete* RFs. A *continuous* RF involves understanding a RF as consisting of a continuum of magnitudes. An *ordered-discrete* RF involves delineating a quantity into distinct, bounded regions that are arranged in some sequence.

To address the research question: *"How might students' construction of continuous and ordered-discrete RFs impact their interpretation of graphs*?" we report on data from a 10-session teaching-experiment (Steffe & Thompson, 2000) with a 12-year-old student, Nina. Consistent with the teaching experiment methodology, we analysed the data by building models of Nina's mathematics. We observed Nina independently construct both types of RFs. Further, we provide several examples highlighting how Nina's RFs impacted her interpretation of relationships between quantities in a graph. In one example, Nina's use of ordered-discrete RFs led to her interpreting a graphed horizontal line as a static representation of a situation (the line represented "nothing" changing). However, with prompting, Nina quickly transitioned to reasoning about the horizontal axis as representing a continuous RF, leading to a more normative interpretation of the line (the graph shows the x-quantity increases while the y-quantity remains constant). We conclude with nuance in students' use of RFs, implications different RFs have for teaching and curriculum, and areas for future research.

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UNIVERSITY STUDENTS' PERSPECTIVES ON MATHEMATICS AND THEIR CONCEPTUALIZATION OF FUNCTIONS

Angel Mae O. Ombid[1], Dennis Lee Jarvis B. Ybañez[2], and Catherine P. Vistro-Yu[1]

Ateneo de Manila University[1], University of the Philippines Open University[2]

Function, being a concept that plays a role "at the very center of teaching" mathematics (Klein et al., 2016), has been firmly embedded in the school mathematics curriculum. Throughout the basic education curriculum in the Philippines, function or functional thinking is either explicitly defined as a concept or implicitly presented through patterns, relationships, and changes among shapes and quantities. This significant role of function is intended to familiarize students with functional thinking, which will help them seamlessly transition from school mathematics to university mathematics (Krüger, 2019). Given this, one should ask: What kinds of function conceptualizations do students develop and/or retain when they reach the university level? This study explores the different conceptualizations of functions among Filipino university students, considering their perspectives on mathematics. There were 210 participants who are students from three different non-mathematics undergraduate programs at a Higher Education Institution in the Philippines. Online questionnaires were administered to investigate how the participants define mathematics and examine how they present functions based on their understanding of its definition.

Three categories of students' perspectives on the definition of mathematics were identified as follows: 1) content perspective, 2) process perspective, and 3) content-process perspectives. Among these three categories, only those with the content and content-process perspectives were able to state the fundamental property of functions (i.e., every input corresponds to one output) in their presentations (e.g., table, graph, equation, or diagram). On the other hand, all students with the process perspective and majority of those with content and content-process perspectives presented functions as either a mere relation among two quantities, only a one-to-one relation, a literal non-mathematical definition of a function, or a processing entity that produces an output given an input without any described relation at all. These results imply that most of the participants have an inaccurate conceptualization of functions, and that content perspective might play a role in developing an accurate conceptualization of functions.

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TEACHER ACTIONS TO SET UP FIVE-YEAR-OLD STUDENTS TO ENGAGE IN MATHEMATICAL PRACTICES.

Emily Pearce

Massey University

Mathematical practices support learners to become doers of mathematics. These practices encouraging thinking process in a dialogic heavy environment. For educators around the world, these practices are widely recognised within school curriculum documents (Grootenboer et al., 2023). For this study the following practices: explanations, justification, argumentation, generalising and representation were examined. In New Zealand, students start formal education at aged five years old. Given the difference in their five years of experiences leading up to starting school, new entrant students enter school with a vast difference in mathematical knowledge (Clements & Sarama, 2018). This study aimed to answer the following question: How does a teacher support five-year-olds to learn, engage with and demonstrate mathematical practices?

This study was conducted in classroom with ten students aged five. Out of these ten students, five students started school the week prior to the research beginning, and the other five had started within ten weeks to the research beginning. Over the course of five weeks, seven mathematics lessons where videoed and observed in the hope to capture the specific teacher actions used to develop mathematical practices during their beginning formal mathematics sessions at school.

The preliminary findings demonstrate the capabilities of these young students in their use of mathematical practices with the scaffolded support of their teacher. In the initial analysis of the data, it is evident how inconsistent the level of support is for these young learners suggesting that the teacher has to be responsive and increase or reduce the amount of scaffolding required in the moment. These emerging findings suggest that the enactment of mathematical practices with young students in the classroom may be challenging as teachers need to be adaptive and responsive.

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GENDER AND SOCIOECONOMIC BIASES IN ADULTS' JUDGMENTS OF CHILDREN'S MATHEMATICAL ARGUMENTS

Jorge Peña Araya[12], María Guerrero[1], Claudia Cornejo Morales[12], and David M Gómez[123]

[1]Universidad de O'Higgins [2]Millennium Nucleus for the Study of the Development of Early Math Skills (MEMAT) [3]Center for Advanced Research in Education (CIAE)

Hypothetical cases have been previously used to study people's perceptions and biases in an empirical manner. Auwarter and Aruguete (2008) used them to study gender and socioeconomic biases in in-service teachers' perceptions of students. Later, Del Río and Balladares (2010) replicated this work with pre-service teachers. However, these efforts have so far centered on general achievement and development expectations about children. We used this approach to study adults' perceptions and biases about hypothetical children's mathematical arguments. To approach this issue, we adapted a questionnaire used in previous works to assess adults' perceptions of a hypothetical child in four dimensions: future expectations, credibility, personal characteristics, and math performance. Here, we present data from 137 participants (general population) who answered the questionnaire online. Each one of them read a randomly assigned hypothetical case drawn from a set of 8 options. Each option corresponded to a 5-year-old child who could be a boy/girl, coming from a high/low SES family, who answered a counting task by providing a high/low quality argument.

An analysis of main effects revealed that perceptions about the credibility of the case were affected by SES, whereas perceptions about personal characteristics and math performance were affected by argument quality. An analysis of interactions revealed that the cases about boys were more credible when they gave a high-quality argument, while the opposite held for girls (more credible when they gave a low-quality argument). We also observed that, when contrasting cases with high- and low-quality arguments, the change in perception of the cases was larger for boys than for girls. Results from an additional open question suggested that participants focused more strongly on internal characteristics in relation to boys' performance, but on external (school) characteristics in relation to girls' performance. Altogether, these results show nuanced gender biases in people's perceptions of mathematical arguments.

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WHAT MATH DO I NEED TO KNOW? PERSPECTIVES ON SPECIAL EDUCATION TEACHERS' KNOWLEDGE

Juan Luis Piñeiro[1] and Eder Pinto[2]

[1]Universidad Metropolitana de Ciencias de la Educación, [2]Universidad de O'Higgins

The teaching of mathematics in special education is an area that has received scarce attention. This paper aims to describe the ideas held by six special education teachers and six in-service elementary school teachers regarding the knowledge they need to co-teach using MTSK model (Carrillo et al., 2018). Two types of teachers were invited to participate: six special education teachers (SET) who co-teach in elementary education (1st to 6th grade) and six elementary teachers (ET) who teach mathematics and who co-teach with SET. A focus group was carried out with each one and then, in a third data collection, the two previous groups were combined. For the content analysis of the data, we used the concept-driven and data-driven approach, and it was carried out sequentially. For the analytical categories, we started from the model provided by Carrillo et al. (2018), particularly, the dimensions involving mathematical knowledge (MK) and pedagogical content knowledge (PCK). Second, this initial analysis was completed with a data-driven analysis within each knowledge dimension.

The results show that there is awareness of these limitations concerning MK. In particular, the participants' perspectives about MK can be grouped into three ideas: a) lack of knowledge of the needs of the school system; b) need for specialized training; and c) self-training as a tool for professional development. Regarding PCK, there is a relevan given by ET's in showing students the everyday sense that the contents of this subject may have, i.e., the phenomenology of content. A second way in which this PCK is approached and understood, both for SET and ET, is concerning the teaching strategies for certain specific content. Finally, as a third sense, the use of manipulatives is a theme that emerges especially among SET. We conclude with a call for attention to mathematics educators and special education educators to discuss the mathematics training of special education teachers.

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EXPLORING THE IMPACT OF BAR MODEL VIRTUAL MANIPULATIVES IN ALGEBRA LEARNING WITHIN TECHNOLOGY-ENHANCED SETTINGS

Poh Hwee Sim, Claire

Charles University

The bar model manipulative is a web-based version of the physical one, allowing students to digitally construct the models representing known and unknown quantities and their relations, as well as to visualise the problem and devise strategies to solve it. Studies suggest that it supports a deeper comprehension of algebra by allowing learners to explore, manipulate, and visualise algebraic expressions before working with symbolic notation (Kho et al., 2009). The research project aims to examine the impact of bar model virtual manipulatives in algebra learning within technology-enhanced classroom settings. The research questions guiding the study are as follows:

- How did the students incorporate app scaffolds with mathematical problem-solving?
- What technical attributes of classroom aggregation technology and the app potentially support conceptual learning through collaborative work and peer assessment?

A project was piloted with an elementary school in the Czech Republic involving a cohort of nine Grade 8 (age 14) participants, during which they engaged in algebraic word-problem solving, utilising bar model virtual manipulatives apps provided within their tablet devices. The study leveraged classroom connectivity through digitally accessing and analysing students' mathematical thinking via screencast, i.e., recordings of their on-screen interactions while using bar model manipulatives. The students were given a test during which on-screen activities were recorded. 5 out of 9 students utilised the app, with the remaining 4 answering the questions without using the manipulatives. A follow-up discussion involving a side-by-side comparison of students' bar model constructions were screen-mirrored onto the class display. This presentation triggered considerable discussions, with noticeable sharing and exchanging of information among the students. This was followed by another test. The results were inconclusive in terms of absolute post-test improvement. However, based on students' digital data, all the students used the app for test 2. It can be hypothesized that the digital format inspires participation and classroom discourse.

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DEVELOPING EFFECTIVE SCHOOL LEADERSHIP FOR CULTURALLY SUSTAINING MATHEMATICS PEDAGOGY PROFESSIONAL LEARNING AND DEVELOPMENT

Mary Rahiti

Massey University

Effective school leadership is an important element of implementing professional learning and development (PLD) to develop more equitable outcomes in mathematics. In school settings with culturally diverse groups of students and a focus on culturally sustaining mathematics pedagogy (CSMP), leadership practices rooted in indigenous perspectives are crucial for supporting educators to consider how to foster students' cultural identities while teaching mathematics. As Campbell (2004) argues a failure to understand indigenous and diverse learners results in the absence of culturally appropriate practices. In the case of Aotearoa New Zealand, Hunter and Hunter (2018) highlight the structural inequities Pacific learners in mathematics classrooms have encountered and the impact that this has on their achievement, mathematical disposition, and cultural identity. Investigating indigenous perspectives of leadership approaches creates an opportunity to bridge the gap between knowledge systems and develop an inclusive environment where students can see themselves in mathematics.

This presentation will draw on interview data collected from four indigenous school leaders from Aotearoa New Zealand whose schools were participating in PLD focused on CSMP. The focus of the presentation is on indigenous school leaders' perceptions of the barriers and enablers for non-indigenous school leaders in leading PLD focused on CSMP. Interview data was analysed thematically.

The preliminary findings suggest that indigenous school leaders highlight the importance of taking a CSMP approach, however, they view a key barrier for non-indigenous school leaders in leading such PLD as a lack of deep understanding of students' cultural knowledge and experiences. It was noted that current school leadership mentor programmes have a narrow focus on administrative tasks instead of cultural understanding. Indigenous school leaders in this study suggested a need for non-indigenous leaders to experience situations which relate to student lives and cultural mentoring so they can develop a deeper understanding of what would be CSMP for diverse and migrant students.

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FOLLOWING A CONVERSATION SCRIPT: ADDRESSING THE DISCORD OF THE IMPACT OF LANGUAGE SUPPORTS IN PEER INTERACTIONS FOR MULTILINGUAL STUDENTS

Restani, R., Jimenez-Silva, M., Albano, T., Abdelrahim, S., Ambrose, R., and Martin, R.

University of California, Davis

Learners deepen their mathematical understanding when they have opportunities to share and compare their ideas with peers. What is less known is how to effectively support emergent multilingual learners when publicly discussing their mathematical ideas. The research team questioned the role of language supports, grappling with the notions of authenticity of the peer conversations, deep mathematical learning, and strengthening primary students' English language development.

We employ Moschkovich's (2015) Academic Literacy in Mathematics (ALM) framework to examine where there are places for negotiation between language supports and mathematical learning. ALM includes mathematical discourse, mathematical practices, and mathematical proficiency. Moschkovich claimed that when teachers are aware of the three facets of academic literacy in mathematics they are inclined to choose tasks that support academic literacy in mathematics, provide opportunities for emergent multilingual students to practice academic literacy in mathematics, and recognize academic literacy in mathematics in student activity.

During a design-based research project, a group of researchers and primary teachers in the U. S. met monthly using a cognitively guided instruction (CGI) approach to share and analyse student work and language samples. Each participant brought in audio and video language samples, including transcripts, of partner conversations between three pairs of emergent multilinguals in grades 3-5 (7-11-year-olds). The primary teachers supported student conversations using instructional scaffolds.

Using a case from the language samples, we found that the "Partner Interview Protocol" was sometimes a helpful tool when it gave learners a starting place to initiate the discussion. On the other hand, an organic discussion without the use of a protocol sometimes deepened mathematical proficiency because students asked genuine questions about the other's reasoning, helped each other clarify their explanations, and used precise fraction terminology. Our findings demonstrate how complex the use of language supports can be during peer-to-peer discussions and lead to further investigation of teachers' instructional decisions.

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Oral communication

THE USE OF PERSON-ORIENTED RESEARCH METHODOLOGY IN IDENTIFYING STUDENT AND TEACHER VALUES IN MATHEMATICS EDUCATION

Justine Sakurai , Cath Pearn, and Wee Tiong Seah

Faculty of Education, The University of Melbourne, Australia

This paper argues for the use of a phenomenological Person Oriented Research (POR) methodology when studying student and teacher values in mathematics education. In POR, participants are considered as a ‘system’, a product of the inter-relationships that exist between and around them (Bergman & Wangby, 2014). POR offers a means to overcome the paradigm of research seeking generalisations which may not consider the broader environment and sociocultural factors by exploring individuals within their encompassing systems. The insights from Bergman and Wangby (2014) emphasise the uniqueness of individuals, the prevalence of non-linear models in personality traits, and the importance of identifying patterns and emergent themes in students' ascribed values. Studies of student and teacher values in relation to learning mathematics employ thematic analysis for coding and analysis (Hill et al., 2021). However, the results of these similar studies show that there has been little consensus on the values that are identified. POR methodology may address this limitation.

In the proposed study, POR classification methods as outlined by Bergman & Wangby (2014) would be employed to qualitatively examine how five secondary mathematics teachers interact with up to 20 students, focussing on the values demonstrated by students and teachers. The data to be collected in this study includes surveys, observations and semi-structured interviews. In this context, a POR methodological approach could provide a nuanced understanding of values. Whilst thematic analysis provides a basis for observing quantitatively collected data in real world, the rigidity in qualifying and categorisation does not fit the needs of this mixed methods study. On the other hand, a POR approach to coding and analysis may provide a fresh lens to observing student and teacher values in secondary mathematics classrooms.

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PARENT-CHILD INTERACTIONS: MOTHERS DOING MATHEMATICS WITH THEIR CHILDREN

Fany Salazar and Marta Civil

University of Arizona

This study explores the nature of the bilingual (English / Spanish) interactions between Mexican origin mothers and their children throughout three informal mathematics workshops designed using the funds of knowledge framework (González, et al. 2005). The participants were seven mothers and their children (ages 9-15). Spanish was their home language, but the children had mostly been schooled in English. Mothers and children worked together on open ended, culturally based mathematics activities aimed at promoting dialogue. Each workshop lasted 90 minutes and was videotaped. To analyze the interactions, we draw on the mathematics discourse strategies (MDS) developed by Tzuriel and Mandel (2020). To reflect our sociocultural approach to research, our analysis also integrates principles from bilingualism and the concept of sophisticated collaborations (Rogoff et al., 2017).

Findings indicate the mothers' use of culturally based strategies such as storytelling, particularly with the younger children and their drawing on their funds of knowledge to make sense of the mathematics problems and to support their children's work. Storytelling led to questions like "could there be different combinations?" which is an example of MDS "math extension." With the older children, the collaboration between mother and child showed a two-way learning experience. While Tzuriel and Mandel (2020) focus on how the parents used the MDS to guide their children, in our study we found that the children also used MDS such as "visualization" to teach their mothers. The mathematical interactions reflected a fluid collaboration where mothers and children shared their problem-solving strategies in both languages, which adds a different dimension to the MDS of "use of math language."

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MATHEMATICAL COMMUNICATION: THE FIRST-YEAR UNDERGRADUATE PERSPECTIVE

Alba Santin Garcia and TriThang Tran

University of Melbourne

An important part of learning mathematics is learning to *communicate* it. University students often struggle with communicating mathematics well. For example, written solutions often lack cohesion and clarity. Moreover, students often struggle with discussing mathematical ideas more broadly. The literature itself is also sparse on this issue – while many studies on aspects of mathematical communication exist (e.g. Pierce & Begg, 2017), few of them look at mathematical communication as a whole. To tackle this issue, we undertook an exploratory study to answer the question: *What are the different ways in which first-year mathematics students conceptualise mathematical communication?* The study uses a phenomenographic research design (Marton, 1981) with the data analysis carried out following the principles of grounded theory (Glaser & Strauss, 1967). We interviewed 24 students with differing mathematical backgrounds across a range of 7 first-year mathematics subjects and identified six themes under which all interviewed students' experiences and conceptualisations could be grouped: *maths* (conflation of mathematics and mathematical communication), *form* (what mathematical communication looks like), *intent* (why mathematical communication might be used), *levels* (communication depends on mathematical experience), *philosophy* (philosophical or semantic definitions of mathematical communication) and *parties* (who or what can communicate mathematically). Of particular interest is the identification of themes that have not been well studied in the literature such as *intent, levels, philosophy and parties*. The *maths* category is also intriguing in showing that many students did not have any real conception of communicating mathematics. In the presentation, we discuss these results and how they form a useful framework for considering mathematical communication in the classroom context. We identify how these themes open up different avenues for future studies and how they might be used in teaching mathematics.

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THE RELATIONSHIP BETWEEN UNDERSTANDING EQUIVALENCE AND RELATIONAL THINKING

Atsushi Sawada

Ashiya City Yamate Elementary School

This study is intended to clarify the relationship between understanding equivalence and relational thinking, which is important in algebraic reasoning. To achieve this research objective, survey questions on both equivalence and relational thinking were administered to 151 public elementary school students. The results of the survey questions on equivalence were categorized into four levels (L4: Comparative Relational, L3: Basic Relational, L2: Flexible Operational, L1: Rigid Operational) using a map created by Matthews et al. (2012). The results of the survey questions on relational thinking were also categorized into four levels (EsR: Established Relational, ConR: Consolidating Relational, EmR: Emerging Relational, NonR: Non-Relational), using the criteria of Stephens and Xu (2009). The distribution of answers is shown in Table 1. The results of this study revealed the existence of developmental sequencing with respect to understanding equivalence and relational thinking (see Table 1).

	NonR	EmR	ConR	EsR	
L4	1	31	➡7	➡7	46
L3	9	➡15	1		25
L2	10	3			13
L1	13	3			16
	33	52	8	7	100

Table 1: Distribution of understanding equivalence and relational thinking (%)

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APPLYING MULTILEVEL MODELLING TO ANALYSE FACTORS AFFECTING MATHEMATICS PERFORMANCE IN NEW ZEALAND SCHOOLS: EVIDENCE FROM TIMSS DATA

Tanya Saxena, Tanya Evans, and Stephanie Budgett

University of Auckland

The state of mathematics education in New Zealand (NZ) schools appears to be considerably compromised, as indicated by the results of various international assessments such as the Trends in Mathematics and Science Study (TIMSS), the Programme for International Student Assessment (PISA), and domestic evaluations like the National Monitoring Survey of Student Achievement (NMSSA). A below benchmark performance is noted for NZ in TIMSS and PISA for the past few assessment cycles, while outcomes of the recent NMSSA report show that, only 45% of Year 8 students were at the expected level. The concern about raising students' maths performance is hypothesised to be linked to New Zealand education systems' contextual features, including a generic and competency-based curriculum, schools' concern regarding insufficient qualified academic staff, teachers' classroom instructional practices and their autonomy over the selection of curricular areas in the classroom (Morrow et al., 2022). The situation demands extensive research that goes beyond just case studies and theoretical explorations. Driven by the above argument, the current study employs multilevel modelling on the TIMSS nested dataset to investigate the relationships between school-level, classroom-level, and student-level variables as predictors of mathematics scores.

The research delves into the ongoing debate within mathematics education regarding the effects of inquiry-based versus explicit teaching methodologies (Kirschner et al., 2006). The causal-comparative research design aggregates TIMSS survey items reported by students and teachers representing inquiry-based and explicit instructional features to investigate the impact and involvement of moderating factors. The mixed models then attempt to provide some information about the factors contributing to maths performance, accounting for the country's educational context. The implication of the analysis is discussed for maths educators, maths teachers and educational policy makers.

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Oral communication

INFLUENCE OF PRIOR KNOWLEDGE ON PRE-SERVICE TEACHERS' PERFORMANCE IN SOLVING FERMI PROBLEMS

Carlos Segura, César Gallart, and Irene Ferrando

Department of Mathematics Education, Universitat de València (Spain)

Fermi problems are tasks suitable for primary education that present a real situation, without data, in which assumptions must be made to obtain the estimate of an unreachable quantity (Segura & Ferrando, 2023). This requires prior mathematical knowledge, especially those related to estimation and measurement, but pre-service teachers make errors related to this knowledge in real situations (Baturo & Nason, 1996; Segura & Ferrando, 2023). On this point, we pose two research questions in this work: (Q1) How is pre-service teachers' prior knowledge of estimation and measurement? (Q2) Are there differences in performance on Fermi problems according to pre-service teachers' prior knowledge? To address them, we designed a test with 14 multiple-choice questions on measurement and estimation skills following previous studies (Huang, 2014; Segura & Ferrando, 2023). A panel of 9 experts reviewed the items in two rounds, assessing the wording and appropriateness of each item. Test validity was justified with Lawshe's Content Validity Coefficient for each item. We then conducted an experiment with 87 pre-service teachers. First, they completed the test. Repeated measures ANOVA allows us to find that there are significant differences between the items, that is, between the different estimation and measurement skills. In addition, Ward's hierarchical method was used to obtain three clusters of levels of prior knowledge (low, medium, high) of prospective teachers (Q1). In another session, they completed a sequence of 7 Fermi problems involving lengths and areas (for example, how many cars can fit in a row along the faculty avenue, or how many people can fit under the faculty's porch). One-way ANOVA allows us to find that there are significant differences in performance on Fermi problems depending on pre-service teachers' levels of prior knowledge (Q2). Findings can help improve prospective teachers' instruction on the application of measurement knowledge in real-world situations.

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PRE-SERVICE PRIMARY TEACHERS´ DECISION-MAKING: ATTENDING TO RELEVANT PEDAGOGICAL CONTENT KNOWLEDGE ASPECTS

Tjorven Lea Seibold, Christin Laschke, Lars Meyer-Jenßen, and Bettina Rösken-Winter

University of Münster, Leibniz-Institute for Science and Mathematics Education, Humboldt-Universität zu Berlin, University of Münster

Making diagnostic judgments is a key qualification of mathematics teachers. The core of diagnostic judgements is diagnostic thinking, which, in addition to perceiving and interpreting, also includes the process of decision making, i.e. deciding about next steps in the learning process (Loibl et. al., 2020). In the context of in-service teachers´ noticing it could be shown that although they attend to content-specific elements, they often not coherently focus on the key mathematics issues (Copur-Gencturk & Rodrigues, 2021). In our study, we explore whether this also applies to decision making in diagnostic judgements by surveying 181 pre-service primary teachers from an urban German University. To explore what prompts teachers select to foster students' mathematics understanding, we used an instrument containing 23 tasks on measuring quantities. First, the pre-service teachers solved the task themselves and shortly explained their solution. They were then shown a student´s solution showing some lack of understanding, and had to choose one out of four prompts to react. One prompt was always accurate from a pedagogical content knowledge (PCK) perspective, fostering students' understanding, one contained PCK aspects not relevant for the situation at hand, one was merely pedagogically oriented, and the last one was senseless from a mathematical viewpoint. The majority of the pre-service teachers opted for the accurate PCK prompt across all tasks (60% of all selections). The prompt containing not relevant PCK aspects was chosen second most frequently (22%), followed by the pedagogically oriented one (14%), and the mathematical senseless prompts (4%). Thus, a PCK focus was predominantly visible in pre-service teacher's decision-making for fostering students' learning, but they struggled to differentiate between relevant and not relevant aspects. Further, we found a moderate positive correlation between selecting the accurate PCK prompt and pre-service teachers' ability to solve the tasks themselves, ($r = .292$, $p < .001$). The results underline that merely strengthening PCK aspects in teacher education is not sufficient.

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A SYSTEMATIC REVIEW OF MATHEMATICAL REASONING

Marte Kristoffersen Senneset, Andreas Pettersen, and Lovisa Sumpter

University of Oslo, University of Oslo & Stockholm University

Mathematical reasoning is a core feature in mathematics education and has gained increasing attention in the last decades. One example of this is the PISA test in 2022 where mathematical reasoning is considered the core of mathematical knowledge and is also a big part of the different tasks in the test (OECD, 2023). In the last years, several theoretical frameworks have been presented by different researchers emphasising different ontological aspects of what mathematical reasoning is. One way is to see mathematical reasoning as a product (e.g., Lithner, 2008), and another way is to define it as a process (e.g., Jeannotte & Kieran, 2017). These theoretical standpoints mean separate epistemological and methodological treatments. Although there have been several attempts to provide literature reviews recently (e.g., Hjelte et al., 2020), we see that no one is fully systematic in that sense it includes the majority of different scientific journals or has a profound ontological or epistemological analysis of the concept. The aim of the present paper is to present the preliminary results of a systematic review of a randomly chosen section (n=30) of a larger set of papers (n=652) published between 2003-2022. The analysis was done with overlapping data (i.e., each author coded 15 papers) to ensure that the coding and the results of the coding was negotiated and jointly agreed on. The results signal that besides seeing mathematical reasoning as a product or a process, some papers define it as an ability. The theoretical operationalisation in the methods, however, did not always follow how the concept is defined. The analysis also showed that several papers did not provide an explicit definition, and they were from a variety of different research areas.

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FROM THE INTENDED TO THE PERCEIVED CURRICULUM: TEACHER'S PERSPECTIVES ON CURRICULM CHANGE

Yoshinori Shimizu

Institute of Human Sciences, University of Tsukuba, Japan

The current study aims to examine the alignment of different aspects of school mathematics curriculum with a particular focus on the relationship between the "intended" and the "implemented", while the new national curriculum standards has just been released an enacted. As curriculum reform and its implementation in mathematics classrooms represent a dynamic and complex process to be examined in detail, we explore how teachers interpret the standards and enact it in their classroom.

A theoretical framework of curriculum alignment in the earlier TIMSS curriculum analysis (Valverde, et al., 2002) was used to conceptualize the role of teachers across three different aspects of curriculum. Also, referring to the Education 2030 ecosystem approach to curriculum (OECD, 2020), we explore the "perceived curriculum", namely, how teachers perceive the curriculum and what they interpret or understand from it.

Data were collected at a public elementary school in an urban area in Japan. An experienced teacher was invited to conduct a research lesson in the form of lesson study in 5th grade classroom with the topic of "geometrical figure and angles", that is intended to be treated differently between the previous and the new curriculum standards. The research lesson and the post-lesson discussion were video-taped and then analysed qualitatively. Data sources included a detailed lesson plan, verbatim transcriptions of video-recorded lesson and the post-lesson discussion, and the fieldnotes of researchers.

The results of analysis reveal that the teacher interpreted the curriculum standards in the way that his own perspectives on school mathematics can be manifested and "customized" the instructional materials into his classroom. The importance of textbooks for examining a "potentially implemented curriculum" was also identified.

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MATHEMATICS PD ONLINE AND IN PERSON: DO STRUCTURE AND FACILITATION MOVES VARY?

Victoria Shure[1], Vanessa Bialy[2], Birte Pöhler[3], and Bettina Roesken-Winter[1]

[1]Universität Münster, [2]Humboldt-Universität zu Berlin, [3]Universität Potsdam

In recent years, positive effects of online professional development (OPD) courses have been identified, but little empirical evidence exists on how OPD and in-person PD vary, with findings indicating few differences between the two (Heck et al., 2019). However, such studies mostly did not consider the structuring of the PD in terms of effective PD features (Cohen-Nissan & Kohen, 2023) and whether facilitators employed different moves to support teacher learning (van Es et al., 2014). Our study examines how teachers' learning was supported in a mathematics PD series as well as if differences between the in-person PD and OPD can be found. Particularly, we examined, for both variants, the structure and features of effective PD as well as the employed facilitation moves while considering how these aspects support teachers in promoting students' conceptual understanding of arithmetic. The data consists of videos of three different PD sessions, each conducted in person and as OPD, from two facilitator tandems. Analyses indicate that the tandems allotted greater PD proportions to discussions and support of teacher learning in person, while allocating time online for technical issues. Opportunities for participants to collaborate chiefly occurred in person. The facilitators similarly used facilitation moves in both versions, with an emphasis on requesting teachers' contributions. Thereby, we found evidence for OPD offering an opportunity for scaling up PD programs as both formats allow for facilitators to similarly incorporate subject-specific features and moderate PD in a comparable way. However, for both formats, the facilitation move to help teachers to make their own contributions should be further strengthened.

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TEACHER LEADERS PROGRAM TO SUPPORT TEACHER NOTICING IN PROMOTING MATHEMATICAL MODELING AND ARGUMENTATION COMPETENCIES

Horacio Solar[1], Florencia Gómez[1], María Victoria Martínez[2], and Andrés Ortiz[3]

[1]Pontificia Universidad Católica de Chile, [2]Universidad de O'Higgins. [3]Universidad Católica de la Santísima Concepción

This study aims to evaluate a professional learning program for teacher leaders (TL) to support teacher noticing in promoting mathematical modeling and argumentation competencies. For teacher leaders to productively coach other teachers in a specific disciplinary context, we have identified three essential skills: monitoring, providing feedback, and modelling teaching practices (Shapira-Lishchinsky and Levy-Gazenfrantz, 2015). Recent studies focus on TL development about pedagogical content knowledge (PCK). Then, these studies center their explicit attention on TL decisions and practices to work on PCK with other teachers they coach (Borko et al., 2021). TL's role is meaningful in coaching peer teachers' noticing. Developing TL professional skills such as modelling, monitoring, and providing feedback is highly necessary. However, given the recent research about TL's role in accompanying other teachers in teaching and learning mathematics, understanding the features of coaching provided by TL is relevant, especially in developing mathematical competencies such as argumentation and modeling.

This study comprises a training program for primary and secondary mathematics TL. We selected 13 teachers from two regions in Chile to participate in the one-year training program. The program for TL has three dimensions: developing (1) mathematical modelling and argumentation competencies, (2) teacher noticing, and (3) coaching skills for monitoring, providing feedback, and modelling teaching practices. The evaluation of the TL training program includes collecting data about TL learning in each of the program's three dimensions and data about applying this knowledge and skills to support other teachers. The expected results have relevant implications for teacher professional learning programs since the aim is to gather evidence on a program's contribution to developing TL skills in coaching peer teachers to develop noticing in promoting mathematical competencies.

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WHAT DOES INTERPRETIVE DESCRIPTION HAVE TO OFFER MATHEMATICS EDCUTAION AS A RESEARCH METHODOLOGY

Kerri Spooner

Auckland University of Technology

Interpretive description is a robust qualitative methodology used in applied practice that seeks to understand participants experiences in order to advance and inform practice (Hunt, 2009; Thorne, 2008). While studies influenced by qualitative methodologies have been carried out in mathematics education, this is the first time an interpretive description methodology study has taken place. Qualitative methodologies such as grounded theory, phenomenology and ethnography can be primarily focused on theorising. As an alternative, qualitative interpretive description methodology offers a theoretically flexible way to address complex experiential questions while generating implications for applied practice (Thorne, 2008).

This study uses interpretive description methodology (Thorne, 2008) to explore how students' experience of learning mathematical modelling may differ from learning mathematics and how this might impact teaching practices. The research question established is: What can we learn from tertiary student experiences to inform the teaching of mathematical modelling in the future? Twenty students participated in this study. The use of interpretive description methodology allowed the researcher to reveal shared patterns, themes and differences of participants' experiences and perspectives that sit beneath the surface of the data and use these to inform teaching practice (Hunt, 2009). Findings of the study revealed that all students learning to mathematically model had their boundaries pushed. Students having their boundaries pushed involved either having their boundaries pushed through moving forward independently of the lecturer or being moved forward by the lecturer, resources or their peers.

A key component of an interpretive description methodology is the requirement to generate practitioner implications from the findings (Thorne, 2008). This differs from other qualitative methodologies, making Intrepretive Description a useful methodology for mathematics education. A key teaching implication from the findings recommends lecturers develop a change of culture where struggle and being creative in mathematics is viewed by students as a positive attribute.

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EMOTION GRAPHS: MIDDLE SCHOOL STUDENTS' ENGAGEMENT DURING INFORMAL GEOMETRY ACTIVITIES

V. Rani Satyam, Bailey Bontrager, Adrienne Springer, and Christine Lee Bae

Virginia Commonwealth University

Students' engagement with mathematics in middle school (early teen years) is a known concern in the United States and other countries. Given the need to address this, our study sought to combine experiential learning (Kolb et al., 2014) with contemporary geometry content (knot theory, symmetry, surfaces) for middle school students through a one week informal middle school geometry camp. Our research question is: What patterns of engagement do middle school students report over informal geometry activities? We use Wang et al.'s (2016) dimensions of engagement as a conceptual framework. We held a community-based camp with geometry activities, designed using Kolb et al. (2014)'s experiential cycle. A sample of N=36 middle school students (ages 11-14) reported their engagement by drawing a graph of their emotions after every activity, as a form of experience sampling. This instrument also provided a suggested emotion word bank, for labeling their graph with corresponding emotions. We collected approximately 400 emotion graphs over the one week span. As part of a larger qualitative and quantitative data collection, participants also took a post-survey with items about their engagement and if the emotion graph helped them.

Data analysis is forthcoming; we anticipate insights about the potential of emotion graphing with middle school students as a medium for communicating their feelings about math. Results will include common patterns of engagement and emotion words used, as engagement pathways. In the post-survey, students reported emotion graphing were beneficial in the following ways: "realize where I was confused", "express about the topic", "if I didn't like it I had hope it would change", and "liked you hearing my opinion." Emotion graphing combines the openness of drawing with structure to help students articulate their experience with mathematics to others as well as themselves.

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SECONDARY MATHEMATICS TEACHERS' BELIEFS OF TASKS RELATED TO FUNCTIONAL THINKING

Ute Sproesser and Johannes Cuno

Ludwigsburg University of Education

Functional thinking in the sense of thinking in relationships, dependencies and changes is not only relevant in mathematics education but also in other disciplines and in everyday life (Vollrath, 1986). When developing functional thinking, it is crucial to understand different representations of functions and to change between them – for instance with regard to purely mathematical representations such as equations and graphs, or with regard to situational representations involved in tasks with real-life context. Concerning functional thinking, student difficulties are well documented in the literature (e.g., Sproesser et al., 2022). To counteract these difficulties and to foster functional thinking, teachers play a crucial role. Different models of teachers' professional competence show that not only their knowledge but also their beliefs affect their acting in classroom (e.g., Baumert et al., 2010). This study investigates teachers' beliefs with regard to particular tasks related to functional thinking.

To answer this research desideratum, 49 secondary mathematics teachers (27 female, 22 male; age: M 43.4, SD 11.3) completed an online questionnaire. The questionnaire presented eight student tasks related to dealing with and changing between particular representations of functions and asked the teachers for perceived relevance, frequency to implement comparable tasks and estimated task difficulty.

Results show that the teachers tend to judge tasks involving purely mathematical representations such as equations and graphs to be more relevant than tasks also including situational representations. Moreover, they indicated to use such purely mathematical tasks more often and estimated them easier for students. This is in line with prior research indicating teachers' so-called symbol-precedence view and stresses the need to motivate teachers to implement more situational representations. More detailed results, e.g., with regard to differences according to covariates such as school track will be presented at the conference.

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STUDENTS' CREATIVE MATHEMATICAL REASONING AND DYNAMIC USE OF REPRESENTATION SYSTEMS

Anna Stenkvist

School of Education, Culture and Communication, Division of Mathematics/Natural Sciences with Didactics, Mälardalen University, Sweden

Reasoning is a key factor for mathematical understanding. Today, reasoning is a part of the curricula of mathematics in many countries, but there is not always consensus of the meaning of the term in education. In research the meaning of *reasoning* varies largely. In some studies it means a general ability in mathematics education and in others a topic-specific process in mathematics. In order to clarify possible meanings of *reasoning* in mathematical task solving, researchers have been requested to present more studies which investigate the connection between frameworks and real life situations in mathematical education. In this observational study, five groups of three to four students, were video recorded during task solving in applied probebility, for approximately three and a half hours. The students either followed their last mathematical course in secondary high school or their first course at the university. Data was used to describe the students' reasoning and thereby contribute to the meaning of two frameworks - creative mathematical reasoning (CMR) and dynamic use of mathematical representation systems (DMRS). CMR can broadly be described as a line of thought in mathematical task solving which is (1) new to the reasoner, (2) backed up by the reasoner's arguments and (3) anchored in mathematics (Lithner, 2008). DMRS is either (1) *treatment*, in which mathematical representations (e.g. diagrams or symbolic expressions) are processed algorithmically or (2) *conversion* in which mathematical expressions are interpreted from one representation system, or register, to another (Duval, 2017). The aim of this study is to contribute to rich descriptions of both CMR and DMRS. The research question is: *What characterize students' CMR and DMRS during task solving in applied probability?* Here, students' creative reasoning (CMR) is recognised by a deviation from an expected mathematical reasoning path. CMR is new (or unusual) not only to the reasoner but also to a mathematically experienced observer. DMRS is only analysed in CMR. An example is a table which, in CMR, is *converted* to a new kind of homemade diagram instead of an expected Venn diagram. The students arguing and mathematical anchoring of the reasoning path in CMR is investigated and a sample of their mathematical representations will presented. Further background, results and implications will be presented and discussed in detail.

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WHAT A=2ΠRH TELLS US: A FRAMEWORK FOR MULTIPLICATIVE OBJECTS WITH FORMULAS

Irma E. Stevens

University of Rhode Island

Researchers have identified several student difficulties when reasoning about formulas such as non-quantitative operations stemming from memorized procedures or pseudo-conceptual behaviors. Covariational reasoning – reasoning about two quantities changing together – has been introduced as a productive way of reasoning about quantities students can use to construct a variety of representations, including formulas (Thompson & Carlson, 2017). Here, I share a framework for one of the essential mental constructions involved in covariational reasoning –a multiplicative object.

A multiplicative object involves the cognitive uniting of two quantities with the awareness that as one quantity changes, the other will change with it. For example, a coordinate point on a Cartesian plane simultaneously represents both the magnitudes of the quantities on the axes. When quantities vary and are graphed, the result is an emergent trace. This report characterizes multiplicative objects within formulas.

In this study, I conducted clinical interviews and semester-long teaching experiments (Steffe & Thompson, 2000) with eight total pre-service secondary mathematics teachers at a large public university in the US. During interviews, students reasoned with several dynamic geometric shapes (e.g., cylinder, triangle, sphere). During video analysis, the focus was on understanding the ways in which students reasoned with the dynamic objects to construct formulas via covariational reasoning, and specifically, their constructions of multiplicative objects with dynamic objects and formulas.

The result is a framework of types of multiplicative objects with formulas including: no multiplicative object in formula, one multiplicative object in formula with quantities represented as single letters, one multiplicative object in formula with quantities that can be represented by the uniting of more than one letter, multiple multiplicative objects can be constructed within one formula, and recognition of the same covariational relationships in formulas representing different situations.

In the talk, I describe each type with data examples. I conclude by discussing the need to use dynamic situations to support students' meanings for formulas.

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INVESTIGATING PROBLEM POSING IN MATHEMATICS CLASSROOMS: WHAT MAKES A PROBLEM GOOD?

Anna Krisztina Stirling, Csaba Szabó, and Csilla Zámbó

MTA–ELTE Theory of Learning Mathematics Research Group

Nowadays, there is increased interest in integrating mathematical problem posing into classrooms. A number of international studies have shown that problem posing systematises and deepens the knowledge of students. This activity can promote students' conceptual understanding, foster their ability to reason and communicate mathematically, and capture their interests and curiosity. Because of its personal nature, it better engages students. Also, variating and reformulating problems is an essential part of expert level problem solving – and problem-posing helps to practice it. Additionally, researchers argue that engaging students in problem posing can foster their creativity.

Examining students' problem posing skills during mathematics classes is a rapidly growing, yet fairly new trend in didactics research. Different researchers mean different things when it comes to problem posing (Papadopoulos et. al. 2021). Thus, the concept of problem posing is not yet uniformly defined, and there are no uniform systems either for the way problem posing activities are tested, nor for the assessment of the tasks produced. In our research, we have developed a complex set of criteria for the evaluation of posed tasks and problems.

We examined the problem posing abilities of preservice mathematics teachers $(N = 58)$ at Eötvös Loránd University through two semesters, and the problem posing abilities of high school students in several Hungarian high schools ($N = 84$). Based on the literature (Rosli et. al. 2015; Singer et. al. 2015) and our findings, we developed a complex set of criteria for the evaluation of posed tasks and problems, that could be applied to both students' and prospective teachers' posed problems. In the course of our research, we have therefore compiled and implemented a set of criteria for evaluating mathematical problems. In our talk we will describe this evaluation and illustrate its application with examples.

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AN EMPIRICAL STUDY OF THE EFFECT OF USING LONG TERM GAMIFICATION IN HIGH SCHOOL MATHEMATICS LESSONS

Janka Szeibert and Sára Szörényi

Eötvös Loránd University

Well-adapted gamification in mathematics lessons may promote student mathematical attitudes and enhance academic knowledge (Xiao & Hew, 2024). Several definitions have been proposed to clarify the term of gamification and most of them use the expression "game-elements". A deficiency of these definitions might be that they do not mention the goal of gamification nor do they explain the notion of "game-elements". We use the definition of Huotari and Hamari (2016): "Gamification refers to a process of enhancing a service with affordances for gameful experiences in order to support users' overall value creation." We suggest that if the learning design is based on this definition, long-term gamification will have a positive effect not only on students' behavioural engagement but also on their mathematical knowledge.

In our study, we analyze the effects of long-term gamification on mathematical knowledge, behavioural engagement among 9^{th} and 10^{th} graders. During the 6-weeks long experiment, students were divided into two instructional approaches in both grades: learning without gamification (n=33) and learning with personalized gamification (n=52). Students' mathematical knowledge was measured by input and output tests, while their behavioral engagement was measured by an input and output questionnaire of 25 questions. In addition, teachers registered each student's activity-level with 7-item tables in each lessons.

We found that gamified classes in both grades obtained significantly better results in knowledge: $F(2.40)=15.23$, $p=<0.0001$, $\eta_p^2=0.43$ (Anova-test). Qualitative analyses of the activity-level tables, and the questionnaires about students' behavioral engagement and of the teacher interviews showed that students' changes in engagement were positive in the experimental groups.

Our work can serve as a basis for a more concrete understanding of gamification systems, and their systematic use in secondary schools, and can confirm to researchers and teachers that gamification can be effectively applied in the traditional classroom setting.

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MAXIMIZING PERFORMANCE AND MINIMIZING DROPOUT RATES WITH FLOW: A GAMIFIED MATHEMATICS COURSE

Sára Szörényi, Anna Krisztina Stirling and Evelin Anna Geszler

Eötvös Loránd University

Higher academic success is associated with higher levels of overall engagement (Lei et al., 2018). High engagement is closely related to the experience of flow. According to Csíkszentmihályi, flow is "the state in which people are so involved in an activity that nothing else seems to matter; the experience itself is so enjoyable that people will do it even at great cost, for the sheer sake of doing it (1990)." Research suggests that flow and engagement can be increased by gamification, although there are numerous studies with inconclusive or negative effects (Zainuddin et al., 2020).

In this study, we investigated the impact of gamification on a Number Theory course attended by first-year pre-service mathematics teachers. We gamified half of the course (N=72) by implementing a point-based system, augmented by additional game elements to make the course motivating for all player types defined in Bartle's taxonomy. The other half of the course, the control group (N=83), took the course the regular way. The students of the gamified group could gather points by completing tasks on a weekly exercise list, actively taking part in the lectures and practices and on two written midterms. We measured the two groups' dropout rates, levels of engagement, and performance on the midterm and exam.

The experimental group had an 8% higher score on the first midterm and a 15% higher score on the second midterm compared to the control group. The dropout rate in the experimental group was 17% and it was 45% in the control group. At the end of the course, a qualitative survey was conducted to assess the students' engagement levels based on the SCARF model. It was found that the student's level of engagement was higher towards the gamified Number Theory course compared to other university courses. Based on these findings we believe gamification can be an effective tool to decrease dropout rates and increase students' engagement, while maintaining a high level of knowledge.

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HOW TEACHERS' ACTIONS PROMPT DIALECTIC AND DIALOGIC MATHEMATICAL ARGUMENTATION

Sheena Tan

Simon Fraser University

Mathematical argumentation can function *both* dialectically and dialogically. The former focuses on how we make connections to provide reasons to support or refute our proposed claims, before arriving at an agreement, while the latter focuses on how we interact with others to make meaning through engaging with multiple perspectives (Schwarz, 2009). Most research on teachers' role in mathematical argumentation (e.g., Walshaw & Anthony, 2008) has mainly adopted a dialectic approach, focusing on how teachers can help students develop logical and valid arguments. Less attention is placed on the role that teachers play to support the unfolding of the meaning-making experience during argumentation – corresponding to a more dialogic approach. Thus, this research explores how teachers' actions promote students' engagement with mathematical argumentation that is both dialectical and dialogical.

The data comprises selected episodes from a 12-week university course where students worked on the whiteboard, in random groups of two or three, to solve a variety of problems, with the support of the teacher, in each class. Their weekly problem-solving processes (including the discussion and work done on the whiteboard) were video-taped and analysed. The findings show instances of how the teacher's actions can promote both dialectical and dialogical mathematical argumentation. However, one mode may be more prominent than the other, depending on the stage of the problem-solving process. For example, when students were still trying to understand the problem, the teacher's actions were primarily functioning to engage them in dialogic argumentation to construct an understanding of key aspects of the problem. In contrast, when they were mostly convinced of their solution (and perhaps reached a consensus), the teacher directly introduced another strategy to guide students towards the normative solution – an action that promoted dialectic argumentation instead. Moreover, this research highlights the challenge teachers face in their real time decision making – how do they balance the tension between the push towards dialectic argumentation *and* the pull for dialogic argumentation? In other words, how do they manage the tension between getting to the optimal solution *and* exploring multiple perspectives? In the presentation, examples and findings will be discussed in detail.

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MATH WELLBEING AND MATH VALUE AMONG TAIWANESE UPPER ELEMENTARY SCHOOL STUDENTS IN MATHEMATICS LEARNING

Yu Shan Ting [1] and Yu Liang Liou[2]

National Chengchi University, Taiwan[1]; National Taiwan Normal University, Taiwan[2]

Taiwanese students excel in mathematics tests; nevertheless, it is worth noting that there exists a prevalent negative affection towards mathematics (Kell et al., 2020). Recently, a framework consisting of seven dimensions for mathematical well-being and values has been proposed (Hill et al., 2021). Nonetheless, to date, no similar investigation has been conducted in Taiwan. This study aimed to explore the circumstances in which upper elementary school students experience well-being during the process of learning mathematics and to understand the values that students consider crucial in mathematics learning. The two research questions were proposed in this study: (1) Under what circumstances do students feel happy or confident during the process of learning mathematics? (2) What values do students consider most important in mathematics learning?

Fifty-two upper elementary school students (27 boys and 25 girls) were recruited from a school in the southern region of Taiwan, all in the sixth grade. Two open-ended questions were employed to explore students' well-being and values in the context of mathematics learning. Nvivo 12 was used to conduct the thematic analysis.

The results revealed four key themes in students' mathematical well-being: Competency (41%), Accomplishment (40%), Relationship (13%), and Engagement (6%). In terms of mathematical values, nine themes emerged, including Competency (29%), Performance (18%), Engagement (14%), Attitude (14%), Methods (13%), Relationship (6%), Positive Emotions (3%), Creativity (2%), and Utility (1%). These findings align with the values identified in Hill et al. (2021) and other previous studies while also highlighting cultural differences. The most significant distinction is that Taiwanese students emphasize Competency, differing from Accomplishment, which refers to achieving good marks, completing tasks, etc. Competency emphasizes the understanding of mathematical knowledge and skills. Further discussion will provide suggestions on how these insights can be transferred for use in the math classroom.

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EXPLORING COMPUTATIONAL THINKING IN INQUIRY-BASED MODULES FOR VOCATIONAL HIGH SCHOOLS

Lin-Chieh Tsai [1], Chia-Jui Hsieh[1], and Jeng-Shin Wu [2]

[1]National Taipei University of Education, Taiwan
[2]National Academy for Educational Research, Taiwan

Earlier research indicates that inquiry-based teaching can enhance students' interest in learning, guide them to actively construct knowledge, and cultivate their problem-solving abilities (Yerushalmy, Chazan & Gordon, 1990). The authors of the report focus on vocational high school (VHS) education in Taiwan, where students often lack interest in mathematics and application skills. In recent years, computational thinking has become another educational trend due to rapid information development. Therefore, whether inquiry-based teaching can be applied to computational thinking education in VHS is the main research topic of this study.

This study adopts a design research method and uses the IDEAL problem-solving model (Bransford & Stein, 1993) to design three inquiry-based teaching courses that combine mathematics and computational thinking, including "Bubble Sort," "Shortest Path," and "Clustering." A total of 290 students specializing in Electrical Engineering from eight tenth-grade VHS classes are selected as the research subjects. Each module is taught for 100 minutes, and students' reactions in class are recorded during the process, as well as their performance through post-course assessments. The research tools are developed by the participating researchers and expert scholars to ensure their reliability and validity. The tools were analyzsed with descriptive and inferential statistics, and content analysis.

After completing two rounds of courses (four classes each round), this study found that inquiry-based teaching not only provides students with opportunities to participate but also helps them develop interest through teacher guidance and questioning, compared to the traditional lecture-based teaching method previously used by the researcher. Students can actively seek answers and establish their understanding and learning reflection through teacher questioning. The results from post-course assessments show an average correct rate of 59.4%, and the high scoring group an average correct rate of 79.6%. Therefore, inquiry teaching courses can effectively improve computational thinking abilities in VHS students. In future teaching, this study will further examine the effectiveness of inquiry-based teaching in improving interest, thinking, and exploration abilities among VHS students in Taiwan through assessment tools.

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TASK DESIGN PRINCIPLES FOR PROOFS AND REFUTATIONS FOCUSED ON IMPROPER DIAGRAMS AND CORRESPONDING ARGUMENTS IN PAPER-AND-PENCIL ENVIRONMENTS

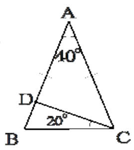
Yosuke Tsujiyama[1] and Kota Kato[2]

[1]Chiba University, [2]Attached Junior High School to Chiba University

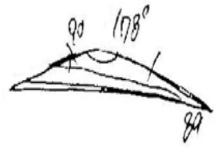
The focus on task design has become increasingly important in mathematics education. Komatsu and Jones (2019) developed task design principles for proofs and refutations in DGEs. However, because their principles initially require proof production, it might be difficult for students who cannot produce proofs to fully experience subsequent activities. Against this background, we focus on improper diagrams and the corresponding arguments that students tend to draw in paper-and-pencil environments.

This study aims to develop task design principles for mathematical activities related to proofs and refutations, which include improper diagrams and corresponding arguments. Based on the theoretical considerations of related concepts (e.g. pseudo-objects), we define an improper diagram (e.g. Figure 1b) as a diagram in which geometrical objects such as points and lines, which should not be drawn, are drawn in the same way as in the previous task (Figure 1a). The argument corresponding to the improper diagram is valid if point D can be taken, and thus can be utilised in a generalised case (Figure 1c).

(a)



(b)



(c) If $\angle A = 178°$, $\angle B = (180 - 178)/2 = 1$

(d)

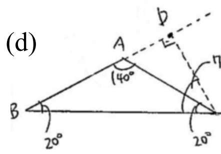


Figure 1: An improper diagram and corresponding arguments (warrants are omitted)

We set up a draft of task design principles to help students utilise improper diagrams and corresponding arguments for proofs and refutations: using initial tasks of finding numerical values in specific cases whose conditions are purposefully ambiguous; using tasks of producing and examining a general conjecture; inducing students' focus on the relationship between improper diagrams and the corresponding arguments. We then empirically tested these principles by designing and implementing tasks in a junior high school mathematics lesson, and qualitatively analysed the data recorded on the whole class and individual student processes. The analysis implied that the task supported 22 out of 24 students' activities in various ways, including 15 who drew improper diagrams. In the presentation, detailed results will be presented and discussed.

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ARE YOU QUITE SURE OF CONSIDERING ETHICS AS OUR RESEARCH ALPHA?

Daiki Urayama

International Pacific University, Japan

According to Radford (2008), a theory consists of three main components, with one of them being P, which represents a system of theoretical principles. As theories are artificial products, P can be hierarchically systematised, being affected by individual philosophical assumptions including axiology, epistemology, and ontology which are organised coherently within a research paradigm (e.g., Scheiner, 2019). Researchers must be aware of the influence of these ideas and their personal philosophical stances. For instance, Marsh et al. (2017) emphasised the significance of a stable research foundation, likening it to a skin that should not be changed frequently, cautioning that unclear personal ontological and epistemological positions can lead to unproductive disagreements among researchers. Therefore, in social science, a profound understanding of logical connections between ontology, epistemology, and methodology serves as a solid basis for reflection on one's own work and also others.

Regarding individual philosophical considerations, there is an argument concerning the "**first**" philosophy in mathematics education as identified by Ernest (2012), and expanded by Dubbs (2020) to highlight the necessity of giving attention to moral philosophy. These considerations work as a starting point for elucidating the relationships between axiology, which encompasses ethics and moral philosophy, and the constituents of P. This study critically discusses the meaning of "**first**", thereby laying the groundwork to comprehend their rational relationships. An inquiry of these ethical and axiological perspectives concludes that "**first**" denotes a comprehensive and recursive approach when researchers reflect on their philosophical stances.

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VISUAL SOLUTIONS: A RESOURCE TO SOLVE CHALLENGING PROBLEMS AND BE CREATIVE

Isabel Vale[1,2], Ana Barbosa[1,2], and Isabel Cabrita[3]

[1]Instituto Politécnico de Viana do Castelo – Portugal; [2]CIEC, Universidade do Minho– Portugal; [3]CIDTFF – Research Centre on Didactics and Technology in the Education of Trainers, University of Aveiro – Portugal

In all areas of knowledge, the need to have more creative people, capable of providing innovative solutions in problem solving has been emphasized. Creativity begins with curiosity and involves students in exploration and experimentation supported by their imagination and originality. Thus, teachers should seek to develop students' mathematical understanding posing challenging tasks that contemplate multiple-solutions, involving a variety of representations, connections and motivate them to work with each other using their personal different thinking processes. Multiple (re)solution tasks allow students to choose the most convenient strategy to achieve the solution, giving them the chance to think outside the box. In this context, we are particularly interested in visual solutions because they play an important role in the construction of mathematical concepts, being also a useful tool in problem solving and usually simpler than other approaches. So, we encourage students, to think visually, *seeing*, because it's yet a not widely accepted strategy in students' school experience.

In this presentation, we report part of a qualitative and interpretative study carried out with 14 preservice teachers (for children aged 6–12), during a learning experience in the context of a didactical course. The aim is to characterize the strategies used by these future teachers when solving multiple-solutions tasks, with at least one visual insightful solution (Leikin & Guberman, 2023), focused on different mathematical themes, in the development of students' creativity and mathematical comprehension, in particular to identify if they privilege visual solutions. Data was analysed in a holistic, descriptive and interpretative way, including classroom observations and written productions of the participants.

Preliminary results allow us to identify: motivated participants who diversified the strategies used; the use of multiple solution tasks provoked their creative potential, mainly flexibility and originality; and all of them successfully solved the proposed tasks, as well as appropriating visual strategies, which they valued.

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SHIFTING DEFICIT PERSPECTIVES OF TEACHER-PARENT PARTNERSHIPS: POST-COVID TEACHER STORIES

Pamela Vale[1] and Mellony Graven[2]

[1]University of Waikato, [2]Rhodes University

It is widely accepted that children whose home environment is supportive of their school learning experience an academic advantage and that parents are influential in whether or not their children experience success in mathematics. In this oral communication we focus on primary school mathematics teacher practices, in the post-Covid context, that enabled (or not) mathematical learning partnerships with parents, and the parent responses to these practices, as storied by the teachers. We address the questions: (1) How are primary school mathematics teachers engaging with and supporting parents in the post-Covid context, particularly regarding supporting learners' mathematical learning? and (2) How are parents (according to teachers) responding to the efforts of teachers to encourage their engagement with the mathematical learning of their child? The research, conducted in October 2022, involved 89 teachers who completed questionnaires that sought narrative responses about their strategies for engaging with parents and sharing resources and knowledge. The theoretical framework that guided the thematic analysis of the data is Barton et al.'s (2004) Ecologies of Parental Engagement (EPE) framework. Parental engagement is situated as a "relational phenomenon that relies on activity networks" (p. 3) and it highlights "the crucial importance that both space and capital play in the relative success parents (and teachers) have in engaging parents" (p. 3) in the mathematical learning of their children.

The research reveals an emerging trend in the relationship between teachers and parents in the post-pandemic context, showing increased teacher-parent communication and parent engagement with children's learning. The study highlighted how teachers have creatively established an intermediate space between school and home, using technology to communicate with parents and share resources. This indicates a shift away from deficit accounts of the home space towards developing partnerships with parents to support student learning. The findings suggest that the pandemic necessitated increased collaboration between teachers and parents to support students' mathematical learning, and that leveraging technology played a significant role in facilitating this partnership. The research lays the groundwork for encouraging ongoing parent engagement as part of teachers' everyday practice.

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ENRICHING MATH TEACHING GUIDES: A COMPETENCY-BASED FRAMEWORK

Albert Vilalta, Jordi Deulofeu, and Laura Morera

Universitat Autònoma de Barcelona & Innovamat Education

Teaching guides play a crucial role in bridging curriculum design and classroom implementation, yet their potential for fostering rich mathematical learning is often overlooked. In our study, we introduce a framework and a tool for enriching teaching guides from a competency-based perspective.

Grounded in the Documentational Approach to Didactics (Trouche et al., 2018), we start by defining guides as documents (i.e. resources with a scheme of use) distinct from textbooks (merely resources). Unlike textbooks, which primarily talk *through* teachers, teaching guides talk *to* them, either in a "directive" or "educative" guidance, as described by Bergqvist et al. (2015), focusing not only on tasks, but also its management. From there, based on Schoenfeld (2016) and Niss & Højgaard (2019), among others, we characterize five key traits of "richness", a commonly used but rather undefined term in math education: Processes (problem-solving, reasoning, connections, communication and representation), Content (contextualized, rigorous, extensible), Cognitive Demand (reflecting over relating and reproducing), Differentiated Instruction (access and challenge for all students), and Environment (structure and rules). From these traits, we developed and validated a tool (the main contribution of the study) that helps assess and enhance the richness of a guide. The application of this tool to a specific case (a 4th grade guide from Innovamat's program) demonstrated its effectiveness in identifying areas for enrichment.

This research aligns with PME's goal of deepening the understanding of teaching and learning mathematics by enhancing the resources that shape these experiences. Our study contributes significantly to the discourse on teaching guides design, providing curriculum developers with a robust tool to enrich their documents.

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TEACHERS' PROFESSIONAL DEVELOPMENT AND MATHEMATICS LSA: FIRST RESULT OF NATIONAL PROJECT

Giada Viola[1], Federica Ferretti[1], Alessandro Gambini[2], Francesca Martignone[3], Carlotta Soldano[4], and Camilla Spagnolo[1]

[1]University of Ferrara, Italy, [2]Sapienza University of Rome, Italy, [3]University of Eastern Piedmont, Italy, [4]University of Turin, Italy

Our study is part of the broad epistemological and didactic debate about how to integrate the results, methods, theoretical frameworks, and findings of large-scale tests into the actions of teachers and the wider context of educational institutions. Large-scale assessments (LSA) becoming the subject of several research in mathematics education (Suurtamm et al., 2016). In Italy, the National Institute for the Evaluation of the Education and Training System (INVALSI) is a public research institution, part of the National Evaluation of the Education and Training System (SNV), that promotes the improvement of educational levels through the deployment of tools to measure the students' learning outcomes and skills, and through the quality assessment of schools. Macro phenomena highlighted in these surveys can provide useful information and become interpretative tools of some aspects of the mathematics teaching-learning processes specific to the Italian context (e.g. Ferretti & Bolondi, 2019). Our study shows outcomes of the National Research Project "Mathematics standardized assessment as a tool for teachers' professional development", which aims to develop theoretical and operative tools for exploiting the results of the mathematics national assessment tests with respect to teachers professional development. The main expected outcome is the development of models for pre-service and in-service mathematics teachers, based on an effective and conscious use of standardized assessments. In particular, a model for the professional development of in-service and pre-service teachers is outlined. Qualitative and quantitative analyses will lead to an increasingly fine-tuned delineation of the model, which will be implemented in numerous in-service teacher professional development courses and degree courses for pre-service teachers. This national project is financed by European Union and Italian Ministry of Education and University (PRIN 2022 - research projects of significant national interest).

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YOUNG STUDENTS FUNCTIONAL THINKING: RECURSIVE MEANS OR AN EMERGING FUNCTIONAL RULE

Bridget Wadham

Massey University

Functional thinking is a key dimension of early algebra and focuses on finding, expressing, and reasoning about relationships with co-varying quantities (Kieran, 2022). Research focussed on young student's capabilities with functional reasoning is still a developing field, and more is needed on how students analyse the visual structure of growing patterns. In this study we examine the types of functional thinking students (aged 7-9 years) demonstrate when reasoning with a functional task.

The study is framed by Stephens et al., (2017) 'levels of sophistication', which describes students' generalisations and representation of functional relationships. This model proposes four overarching types of structural thinking including no evidence of, variational, covariation and correspondence. We report on a data-set of responses from 641 students who completed a written free-response task after receiving five weeks of algebra patterning instruction during 2022. This task required students to reason with a geometric representation of the function $4x + 1 = y$.

Results show that 18.7% of students demonstrated no evidence of structural thinking, 46.7% variational thinking, 2.2% co-variational thinking and 32.5% correspondence thinking. Almost a quarter of students used recursive means (+ 4) to calculate both near and far terms, and 15% developed an emerging functional rule focused on the independent variable. Findings support the proposal within the research community that a larger focus on co-variational thinking in early mathematics instruction will support the development of students functional thinking. Additionally, that professional development for teachers focused on including the dependent variable within a function is likely needed.

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STORYLINES EXPERIENCED BY INDIGENOUS AND NEWLY MIGRATED MATHEMATICS STUDENTS

David Wagner[1], Sacha Dewolfe[2], and Julianne Gerbrandt[1]

[1]University of New Brunswick, Canada; [2]Mount Allison University, Canada

Students are positioned by teachers, classmates, media, community, organizations, policy, legislation, language, and families, through current and historical practices. As researchers and former classroom teachers, we are particularly attentive to the positioning of mathematics students who identify (or are identified) with groups that have often been marginalized, namely students who are Indigenous or are newly arrived migrants. Most research using theories of positioning focuses on the positioning (e.g., Sengupta-Irving, 2021). However, positioning theory sees a triad at work in human interaction, including the three elements of positioning, storyline, and speech/communication act (e.g., Harré, 2012). The people involved in an interaction are associated with positions in a story, which guides their choices about how to interact. This study explores the influence of storylines in the context of learning mathematics. The storylines available (known) to students and their teachers make certain positions possible and exclude other positions from possibility. Storylines are important because they provide repertoires for action for mathematics students and teachers. In our presentation, we will describe how we have been working with students to (a) identify the key storylines that underpin their experiences associated with their mathematics learning, and (b) document their accounts of interactions that they see guided by these storylines. Some storylines that have come up so far include "We have to try hard to learn math", "Teachers respond to students differently based on the students' reputations", "Math is important", "The math we do in school is not interesting", "Math teachers don't understand Indigenous people", and "Math should be done in silence". In our oral report we will share student experiences that connect with these storylines. Our next research step brings these storylines and the associated experiences to the students' mathematics teachers, where we will collaboratively build pedagogies that both respond to students' strengths and are informed by their experiences. (This research is supported by the Social Sciences and Humanities Research Council of Canada, grant entitled "Migration and Indigenous contexts of Mathematics Education (MIME): Changing storylines with strength-based pedagogies", Principal Investigator: David Wagner.)

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CONCEPTIONS OF MATHEMATICAL CREATIVITY OF HIGH SCHOOL STUDENTS IN CHINA

Yuhan Wen[1] and Xinrong Yang[2]

1. College of Teacher Education, Southwest University, China
2. Faculty of Education, Macau University, China

The investigation of mathematical creativity has attracted more and more researchers' interests ever since the beginning of this century within the field of mathematics education. It has been commonly accepted that mathematical creativity enables students to thrive in their current and future lives within our increasingly automated and interconnected high-technology-based societies (OECD, 2014). Because of such importance, in the past years, researchers have investigated practices, tasks and characteristics related to students' mathematical creativity. Moreover, factors such as mathematical expertise, personality and learning environment have also been suggested will significantly influence the development of students' mathematical creativity. However, other researchers have argued that students' own conception of mathematical creativity will also influence their learning attitude and motivation. Moreover, students' conception of mathematical creativity will also be influenced by the social and cultural context within which their mathematical creativity has been shaped and developed. In view of this, this study will systematically investigate the conception of mathematical creativity of high school students in China.

With the use of a semi-structured interview, 30 students at Grade 10-12 were interviewed for 30 minutes to one hour regarding to their understanding of mathematical creativity. Constant comparison method was used for data analysis. It was found that high school students in China mainly understand mathematical creativity from the following seven aspects: 1) overcoming fixation. Students need to be flexible and think independently even under the influence of others and textbooks; 2) connecting mathematical content with other areas; 3) finding multiple solutions to math problems; 4) solving problems efficiently; 5) a skill that can be fostered through hard work; 6) investment and inquisitiveness; and 7) experiencing 'aha' moments. The subconscious mind continues to process the information so that a sudden but appropriate illumination appears. Furthermore, it was found that Chinese traditional culture expecting students to obey rules and work diligently, the mathematics curriculum reform which emphasizes student-centred learning and interdisciplinary learning, the evaluation methods focusing on examinations and the teaching practice limited to problem solving are main factors which influence their conceptions.

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INTEGRATING MATH INTO PLANETARY SCIENCE PBLS FOR THE DIVERSE CLASSROOM UTILIZING EQUITABLE PROFESSIONAL LEARNING

Woolard, T., Novak, M. A., and Heldmann, J.

Morgan State University, Johns Hopkins Applied Physics Laboratory, NASA Ames

Several schools of thought have contributed to describing the growth and development of children as they journey to adulthood. Less emphasis has been placed on how adults continue to mature and develop as they journey towards greater professionalism. With growing consideration of equity and access of those underrepresented, teachers cannot be excluded from the discussion as this has great bearing on the need for equitable professional learning. Project-based learning (PBL) has long been established as a powerful tool for engagement and affective support for student learning. However, a gap still exists between what is most culturally relevant for today's students and how teachers have been trained to deliver instruction. This study explores how a developmentally-differentiated professional development (DDPD) can facilitate the utilization of math PBLs in the diverse classroom by tending to different teacher needs (Woolard & Khalil, 2022). Several teachers of color from a large urban school district participated in a professional learning initiative to integrate planetary science PBLs with a math focus into their current classroom instruction. Teachers were given opportunities to create lessons as well as receive training on teaching existing lessons with the expectation that at least one lesson would be implemented with students inside their classroom. Data was collected via observation notes, training/implementation artifacts, and teacher interviews to explore how the DDPD framework assisted teachers with learning about planetary science and implementing math-based PBLs. Teachers requiring less support participated in constructing new lessons based on their content. Teachers preferring more support, participated by receiving focused training on existing lessons. All teachers self-reported growth in professional learning about planetary science concepts and committed to implement at least one lesson with their students. Findings suggest that providing teachers with access, resources, and ongoing support can increase the utilization of math PBLs in diverse classrooms especially when teachers are provided multiple points of entry to engage with and make meaning of how the strategy can live within their instructional style. This has implications for the design of equitable professional learning of math teachers.

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DOES HIGHLIGHTING KEY INFORMATION HELP OR HINDER MATHEMATICAL READING?

Bethany Woollacott, Lara Alcock, and Matthew Inglis

Loughborough University, United Kingdom

Research on mathematics textbooks is rapidly growing and yet in England, mathematics textbook publishers have traditionally relied on intuition, convention, or market research with teachers to design their textbooks. For example, English mathematics textbooks often highlight key information within expository text to make it visually distinct. However, being able to identify key information is potentially an expert skill (Inglis, 2009) and therefore doing this for the learner might not be helping them. That said, how students read and learn from their textbooks is one of the least researched aspects of mathematics textbooks (Fan et al., 2013) so we have a limited understanding of how students read or interact with such design features. As such, we conducted two studies investigating how students perceived and read highlighted key information in their mathematics textbooks. In Study 1, we used comparative judgement, asking 60 students aged 18 to make pairwise comparisons on a set of 16 expositions from a mathematics textbook, asking “which is the better explanation?”. We investigated whether participants’ judgements were influenced by the proportion of text highlighted in colourful textboxes within the main text. We found that when highlighted key information was present, higher proportions of highlighted key information significantly positively correlated with the comparative judgement scores, $r = .63$, $p = .01$. This suggests that students preferred expositions with higher proportions of highlighted key information when present. In Study 2, 33 students aged 18 participated in an eye-tracking study reading the same expositions as Study 1. We found a significant difference in dwell time per unit area between how students read highlighted key information compared with the main text, $t\ (32) = -5.534$, $p < .001$, $d = -.963$, suggesting that students spent longer looking at the main text than highlighted key information. In our session, we will discuss why students preferred expositions with higher proportions of highlighted key information yet spent more time reading the main text, exploring whether reading highlighted key information requires less effort, and how this should affect future mathematics textbook design.

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LEARNING TWO-SAMPLE T-TESTS WITH EXPERIMENTS AND VISUALIZED TOOLS

Ching-Ching Yang and Jenn-Tsann Lin

Department of Applied Mathematics, Feng-Chia University

From pass experience, we find over 50% blank sheets on written exams of two sample T-test in statistics course for applied mathematics major students in our university. They have difficulty to understand the relation of thresholds and type I error; not to mention the effect of equal and unequal variances on the T-tests. This study aims to design instructing plans based on Learning-by-Design (Kolodner et. Al. 1998) with learning circle of experiment, analysis, and inference steps to help students to realize testing errors and steps of constructing tests.

In variances testing, firstly let students simulate two sets of normal data with equal varices from simulation website designed by this study. Two sample variances are collected by Google form to make sure students are with the class – experiment step. For analysis step, the plot showing the ratio of sample variances are displayed side by side with F distribution with thresholds on both ends. We demonstrate F distribution by Geogebra designed by this study. Students observe the extreme ratios happened, even variances are set to be equal. This help students to understand testing errors. In inference step, Type I error is introduced by teaching videos. Circles of Type II error and T-test are designed similarly with their own simulation and Geogebra tools. These designed learning circles could implement for both in-classroom and online uses, which is helpful especially during pandemic.

We had 64 students in Statistics course 2022 and had around 85% attendance rate even operated online. Since written exams were not preferred by university policy, we made online quizzes instead. In 54 who took quiz of variances test, 60% understood that extreme ratios on both sides indicating unequal variances. In quiz of T-tests, 65% and 58% identified type I and II error correctly, respectively. We had 47 students in 2023 and 42 took written exam, 76% could conduct test of variances completely, but only 31% conducted T tests completely. Adjustments on circles of T tests would be made. We did lower the number of blank sheets on written exam and students' performance on constructing variances and T tests were much better than before.

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THE DIFFERENCES IN INTEGRATED STEM TASK VALUES BETWEEN MATHEMATICS AND OTHER STEM TEACHERS

Kai-Lin Yang and Xiao-Bin He

National Taiwan Normal University, Taipei, Taiwan

Teachers' values of integrated STEM (iSTEM) education strongly influence their manner to engage and implement integrated STEM education (Margot & Kettler, 2019). Nonetheless, few studies focus on teachers' perceived values on iSTEM tasks implemented in two different teaching settings (major teaching course and elective/alternative course). On the other hand, mathematics teachers tended to have lower identity and self-efficacy than science teachers (Polizzi et al., 2021). Thus, it is worth further comparing mathematics and other STEM secondary teachers' iSTEM task values. Based on expectancy-value theory, the research questions include (1) What are the differences in iSTEM task values between mathematics and other STEM teachers? (2) What kinds of patterns can be identified from the classes of STEM teachers' perceived values on iSTEM tasks in two different teaching settings? (3) What is the association between teachers' teaching subject (mathematics or other STEM disciplines) and the class of teachers' iSTEM task values?

143 Taiwanese mathematics teachers and 175 other Taiwanese STEM teachers were surveyed regarding their perceived values of attainment, interest, utility and cost of implementing iSTEM tasks in their own major teaching course and in elective/alternative course. A mixed MANOVA was performed to examine the differences in the eight scales. Latent class analysis was conducted to identify the number of classes, and three-step BCH approach was used to estimate the comparative influence of teaching subjects on classes of iSTEM task values.

The main results showed that (a) mathematics teachers perceived significantly less iSTEM task values than other STEM teachers ($p = .033$) ; (b) four classes were characterized as high task value (HTV, 67%), relatively low cost value (rLCV, 12%), low task value (LTV, 11%) and setting-specific cost value (SCV, 10%); (c) mathematics teachers more likely belonged to the SCV class than other STEM teachers (*OR = 2.197, 95% CI: 1.01 to 4.77, p = .047*). It indicates that mathematics teachers' cost values were lower and more influenced by the teaching setting.

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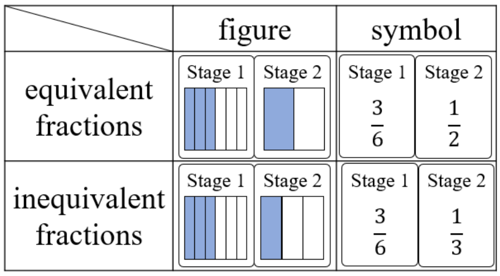
AN ERP STUDY ON MEASURING STUDENTS' DETECTION OF EQUIVALENCE FRACTIONS

Chen-Yu Yao, Hui-Yu Hsu, and Tsu-Jen Ding

[1]National Tsing Hua University, Taiwan R.O.C.



Understanding the concept of equivalent fractions is not only about memory of a fact or application of a procedure, but more importantly about making connections between different representations (Lesh et al., 1983). This study aims to investigate how students detect equivalence in fractions shown in two representation systems: figure and symbol. We designed a test consisting of 180 problems, which requires students to verify whether two given fractions shown by either graph or symbol representations were equivalent. The test was implemented using the E-prime software, and students' response accuracy (Acc) and reaction time to the correct response (RTc) were adopted as indicators for problems' cognitive complexity. Brainwave activities during problem-solving processes were recorded and analyzed using event-related potential (ERP) techniques. The participants were 30 university students.Analysis of behavioral data indicated that students performed more accurately and had a shorter response time on inequivalent fractions than on equivalent ones. This finding suggests lower cognitive complexity when verifying inequivalence in fractions than equivalnece ones. Fractions shown in symbol representation format had higher accuracy and shorter reaction time than those shown in figure representation. The result indicates that verfication by symbol representations is easier than by figural representations. For brain activity, we particulary focused on how students detected the inequivalence by examining the N2 and N400 components. The N2 component refers to error awareness, while the N400 component is involved in error recognition after semantic processing. The study found both N2 and N400 components in students' brainwaves. However, signficant differences was found in the N400 but not the N2 when comparing between inequivalent fractions and equivalent ones. . The finding indicates that no matter inequilavence or equivalence require students similar mental effort in being aware of the error. However, verification on inequivalent fractions ask for higher mental effort in the semantic processing for error recognition than that on equivalent fractions. Additionally, symbol representations elicited a larger N400 than figure representations, indicating greater semantic processing when verifying by symbols than by figures.

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EXPLORING THE INTERSECTION OF VISUALIZATION AND LANGUAGE THROUGH COGNITIVE RESTRUCTURING

Dennis Lee Jarvis B. Ybañez[1] and Lester C. Hao[2]

University of the Philippines Open University[1], Ateneo de Manila University[2]

Cognitive restructuring is described as some form of adaptation that occurs in the mind when acquiring a second language (Pavlenko, 2014). On the other hand, visualization objects are physical objects used as a representation of an underlying concept or mathematical object (Phillips et al., 2010). In this presentation, we illustrate a potential connection between cognitive restructuring and visualization objects by examining results from the lead author's dissertation. To this, we assumed learning the mathematical language is similar to learning a second language such that it possibly induces cognitive restructuring in its learners.

One hundred four Grade 11 Filipino students (ages 16-17) underwent pre-test, class discussion, and post-test on describing the sample space of an event through the use of coins and *teks* (i.e., a Filipino card game wherein the front face is the winning side and the back as the losing side) as visualization objects. The pre-test results show that the students responded differently to the term 'outcome' depending on the visualization object involved. When posed with a coin, they tend to utilize more sophisticated ways, such as tree diagram or formula. On the other hand, when posed with *teks*, the students were mostly descriptive about the possible outcomes (e.g., "all win"). During the class discussion, coins and *teks* were utilized but there was no explicit effort to associate these as parallel visualization objects. The post-test reveals that with respect to the term 'outcome,' most students exhibited a shift in approach when describing the sample space of coin tosses, and some utilized *teks* cards to represent coins.

In this regard, we propose viewing the term 'outcome' in two ways: either from a mathematical or natural language perspective. Initially, 'outcome' is relative to the visualization object; coins induced a mathematical language perspective on 'outcome', thus leading to the use of tree diagrams and formula. On the other hand, *teks* prompted a natural language sense of 'outcome', with students describing the results of playing the game. We surmise that the class discussion could have paved the way for cognitive restructuring, allowing students to realize that they not only can use the natural language perspective when it came to coins, but also can visualize coins as *teks* cards, therefore representing heads with a winning face card.

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AN ANALYSIS OF STUDENTS' PERSPECTIVES ON USING TEKS IN DETERMINING THE SAMPLE SPACE OF AN EVENT

Dennis Lee Jarvis B. Ybañez[1,2] and Catherine P. Vistro-Yu[2]

[1]University of the Philippines Open University, [2]Ateneo de Manila University

The development of meaning of mathematical objects in probability has become one of the major points of interest for mathematics education researchers due to students' difficulties in learning these highly abstract concepts. In fact, it has been noted that the use of conventional visualization objects to teach probability in the Philippines (i.e., coins, dice, and standard deck of cards) seem to not help students construct meaningful learning of probabilistic concepts. In a previous work (Ybañez & Vistro-Yu, 2022), we discussed how a different visualization object with a stronger association to students' cultural background and experiences (i.e., teks cards) could elicit more meaningful subjective perspectives among students. Teks cards are used to play a traditional Filipino game called *teks*; it has two faces similar to the heads and tails of a coin. Its front face (F) is the winning side, its back face (B) is the losing side and winners are decided by flicking the cards. In this paper, we aim to discuss the different perspectives students have formed while solving sample space problems involving teks cards.

Seventy-nine Grade 11 Filipino students (ages 16-17) were tasked to answer the following test items when flicking 3 teks cards: a) determine if the outcome FBF (F-front, B-back, F-front) is possible; b) identify other outcomes aside from FBF; and c) determine the set of all possible outcomes in the experiment. Individual interviews were immediately conducted after administering the test. To capture the different aspects and richness of students' experiences, phenomenography was used. Five different perspectives have emerged from the data gathered: 1) practical – students rely on the actual outcome of flicking 3 teks cards to determine the possibility of an outcome; 2) extrinsic – students believe that outcomes are affected by external factors (e.g., wind), hence they could not determine the possibility of an outcome; 3) representational – students solely rely on the representation of the given outcome to determine its possibility, i.e., FBF means that the second player loses; 4) representational-conceptual – students rely on the representation to list down possible outcomes of flicking 3 teks cards, but are not convinced that the list is exhaustive of all possible outcomes; and 5) conceptual – students form the idea of a sample space by identifying the pattern to determine the list of all possible outcomes, including even the least likely outcomes. Evidently, these perspectives enabled students to meaningfully construct an initial concept of a sample space, but further transformation of these perspectives is needed for students to accurately conceptualize a sample space.

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EXPLORING MATHEMATICS TEACHERS' NOTICING ON RESPONDING TO STUDENT THINKING — A CROSS-CULTURAL COMPARISON BETWEEN TAIWAN AND GERMANY

Fung-Wen Yeh[1], Josephine Paul[2], Anika Dreher[3], Anke Lindmeier[2], Feng-Jui Hsieh[1], and Ting-Ying Wang[1]

[1]NTNU, Taiwan, [2]FSU Jena, Germany, [3]PH Freiburg, Germany

Teacher noticing regarding aspects of high-quality teaching is part of teacher competence (Dreher et al., 2021). Its investigation has increasingly attracted interest in mathematics education research, and prior research indicated that perspectives on high-quality teaching, and hence teacher noticing, might vary significantly across different cultures. The TaiGer project, a collaborative study between Taiwan and Germany, investigates teacher noticing and the impact of cultural norms. The present study explores what teachers notice in these two cultural contexts regarding responding to student thinking.

The sample comprised 115 and 113 secondary mathematics teachers from Taiwan and Germany. The research instrument included a text vignette with a teaching scenario in which the teacher and students worked on a problem with linear functions. It showed distance-time function graphs of four cars, and the students had to determine which car was the fastest. The teaching scenario includes a statement indicative of a misconception that is not sufficiently explored by the teacher, hence violating aspects of high-quality teaching. In detail, Student 1 (S1) incorrectly assumes that the car covering a greater distance is the faster one. S2 challenges this and the teacher asks S2 to explain the solution to S1, what s/he does with reference to the steepness of the graphs. The participating teachers were prompted to "evaluate how the teacher deals with students' thinking in this situation and give reasons for the answer." The written answers were analyzed by qualitative content analysis to identify the aspects the participating teachers noticed.

The answers allowed us to identify different perspectives on the situation beyond the focal aspect of the student's thinking not being sufficiently explored. For example, some German teachers argued that the partially correct thoughts of S1 should be acknowledged, whereas especially Taiwanese teachers expected the teacher to address the misconception directly. The presentation will show the breadth of teacher perspectives and relate differences between countries with known differences in respect to the signatures of mathematics education to provide possible explanations.

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ENGAGING WITH MATHEMATICS AVERSE ADULTS THROUGH STORIES AND REFLECTIONS

Caroline Yoon

University of Auckland

Students who have negative experiences of mathematics tend to become experts at avoiding the subject (Nardi and Steward, 2003). But many will find themselves confronting it again as adults, when changing career, or pursuing a new course of study. They may become parents of young children who bring mathematics back into their lives when they start school. Some may even become primary school teachers who are obliged to teach the subject, despite their reluctance.

Research in mathematics education has brought forth hundreds of valuable insights that could help mathematics-averse adults to reconnect with the subject. But most are unlikely to access them. One barrier is the nature of academia: research reports tend to be written for other academics, filled with technical jargon and kept behind paywalls. Another barrier is emotional: when something has been the source of distress, it is very difficult to reconnect with it.

To help mathematics-averse adults reconnect with mathematics, I have turned to the power of story. Stories offer a safe(r) space for us to make sense of our own difficult experiences. As we read how others deal with challenges, we imagine and simulate how we might too, often with more curiosity and compassion (e.g. Mar and Oatley, 2008). I have written a collection of stories that follow a family as a young girl starts school. Her parents want to support her as she learns mathematics but this isn't easy. Her mother hates the subject and has avoided it until now, and though her father likes mathematics and wants to help, he can't always see what his daughter doesn't understand. The stories are interleaved with reflective essays that discuss insights from research in mathematics education that can help readers make sense of the characters' experiences, as well as their own.

I will discuss the process of writing these stories and essays, read excerpts from them, and share preliminary results of responses from readers.

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DEVELOPMENT OF THE PROSPECTIVE MATHEMATICS TEACHERS NOTICING THROUGH THE MTSK MODEL

Diana Zakaryan and Ledher M. López

Pontificia Universidad Católica de Valparaíso

In the field of mathematics education, noticing is recognized as a fundamental competence for mathematics teachers, as it allows them to identify and understand important aspects of teaching and learning situations in the classroom. Furthermore, there is consensus that noticing is closely related to teachers' knowledge. In this context, this work presents some of the results obtained from a five-session workshop aimed at developing the noticing -selective attention and interpreting dimensions- (van Es, 2011) of prospective mathematics teachers, through the Mathematics Teacher's Specialized Knowledge (MTSK) model (Carrillo-Yañez et al., 2018). The participants in this research were six prospective mathematics teachers who were in their final year of the Mathematics Pedagogy program at a Chilean university. Data collection was carried out in two stages. In the first stage, the preservice teachers were shown an excerpt of a video recorded during a Sequences class and were asked to respond to three questionnaires about the aspects of the video that caught their attention. In subsequent sessions, the basic components of the MTSK model were presented to them. In a second stage, the same video was shown to the preservice teachers, and they were asked to observe it based on the MTSK model. The responses of the preservice teachers were transcribed and analyzed considering the levels proposed by van Es (2011) for selective attention and interpretation (baseline, mixed, focused, and extended levels). The findings of the study show that the level of the selective attention dimension of preservice teachers improved, but the interpretation dimension remained at initial levels. The results invite further exploration into the development of notice of the significant events in the classroom among preservice teachers in a deeper way, and how specialized knowledge can contribute to such development.

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IMPROVING MATHEMATICAL THINKING BY PLAYING BOARD GAMES IN MATHEMATICS LESSONS

Csilla Zámbó[1], Anna Muzsnay[2], and Janka Szeibert[1]

[1]Eötvös Loránd University, [2]University of Debrecen

Board games are examined in mathematics education research from several aspects. Studies using fMRI have shown that the prefrontal and parietal cortex can be developed via board games (Newman, Hansen & Gutierrez, 2016). These two areas have an essential role in mathematical performance and the sensitive period of the development of these areas is at the age of 12-24 years (Dumontheil, 2014). Therefore, it seems plausible that board games can be used as a tool for improving the mathematical thinking of students aged 12-24.

In our investigation, we examine the possible benefits of the implementation of board games in mathematics classroom work by replacing one lesson per week with playing board games. Participants were 10th-grade students (N=170) from six schools. Members of the experimental group (N=79) played board games with either a logical or geometrical nature for 10 weeks during one lesson per week, meanwhile, members of the control group (N=91) had standard mathematics lessons. Each student wrote pre- and post-tests on logical and geometrical skills and uniform topic tests on the learning material covered during the experiment.

Our early results show that the experimental group performed at least as well on the topic tests and also on the logical tests as the control group did. Concerning geometrical skills, students in the experimental group improved significantly, while members of the control group did not improve. The rate of improvement is dependent on the school we examine. (Result obtained using Anova, $F(2,95)=0.1$, $p=0.9$, $\eta^2=0.0002$.) We are continuing our ongoing analysis of the topic, with the expectation of obtaining and presenting further, more detailed results at the conference. Based on our findings, we believe that on the one hand, it is worth implementing adequate board games in mathematics lessons to improve students' thinking; on the other hand, the topic is worth further investigation.

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POSTER PRESENTATIONS

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Poster presentation

AUDIO-PODCAST AS AN EDUCATIONAL TOOL FOR FUTURE ELEMENTARY TEACHERS

Jorge Albella, Teresa F. Blanco, and Antía Fernández-López

Santiago de Compostela University

Representation in mathematics lessons is a central aspect and supports competence across all school levels. This can be carried out in written, graphical, or oral form to enable students to communicate mathematical ideas. The production of mathematical audio-podcasts is an innovative educational practice that focuses on the aspect of orality. Although the process of creating an audio-podcast produces a final oral product, one needs to acknowledge the fact that writing surfaces in one way or another. This is useful for investigating the student's learning process and the final product (Schreiber, & Klose, 2017b). The process has been developed also to reflect on mathematical topics following the phases in figure 1 (Schreiber, 2020). The students undergo the process and develop specific content up to a point where a version of the explanation is ready to be published on the internet (Schreiber & Klose, 2017a).

Figure 1: Process of production

In this work the different phases of the **methodology** will be detailed and exemplified for future teachers learning of the **concept of prism** considering 15 groups of 3-4 **future elementary teachers** in the 3rd year of the degree. In **Phase 1** the topic 'Explain the relation between cube and prism' is provided in an envelope and each group produces an unexpected recording during the opening of the envelope and their first impressions. In **Phases 2 and 3** a first script and its corresponding podcast are produced in a maximum time of 15'. In **Phase 4**, two groups have an "editorial meeting" supervised by the teachers where the first versions of the podcast are reproduced and discussed. Finally in **Phases 5 and 6** a final script and its corresponding podcast are produced in a maximum time of 15'. The results show difficulties to verbally express examples other than a rectangular prism and to acknowledge the relevant characteristics that define the concept of prism. However, a huge improvement is observed between the final version and the unexpected recording.

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MATHEMATICS ON THE RIVER, MATHEMATICS OF THE RIVER

Chiara Andrà, Alberto Doretto, and Cristina Scalvini

Università del Piemonte Orientale, Italy

The research explores how numeracy (Liljedahl, 2015) can be applied in a context of citizen science developing the Sustainable Development Goals 6 and 15, addressing water resource management and biodiversity conservation. Citizen science serves a dual purpose, providing both valuable ecological data and fostering environmental learning and civic engagement (O'Donoghue, Taylor & Venter, 2018). Numeracy, defined as the practical application of mathematical knowledge in various real-world contexts, plays a crucial role in citizens' understanding of river-related issues. By operationalizing numeracy, the study aims to unravel citizens' perceptions and knowledge regarding rivers and sustainable behaviors related to water resources. The research questions is: How does numeracy manifest when individuals are tasked with answering questions about rivers and sustainable behaviour concerning water conservation?

The research methodology involved administering multiple-choice questions and analyzing open-ended responses to 28 undergraduate students enrolled in an Environmental Sciences course at an Italian University. The aim of the questions is to uncover citizens' conceptions about quantities, ratios, and percentages concerning river-related phenomena. The analysis revealed insights into citizens' perceptions of river water availability, water quality, and the impact of human activities on river ecosystems.

Participants demonstrated varying levels of numeracy and knowledge about rivers. While there were misconceptions regarding river water availability and quality, there was evidence of awareness about the importance of river ecosystems and biodiversity. The findings highlight the need for enhanced mathematical education to promote modelling of river-related phenomena and conservation, emphasizing the need for interdisciplinary approaches to address complex environmental challenges.

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CONCEPT OF EQUIVALENCE IN THE CONTEXT OF WEIGHT: A CASE STUDY IN KINDERGARTEN

Nathalie Silvia Anwandter Cuellar[1], Elena Polotskaia[1], Virginie Robert[2], Ildiko Pelczer[3], and Raphaëlle Dufour[4]

[1]Université du Québec en Outaouais, [2]Université Laval, [3]Montreal University, [4]Université du Québec à Rimouski

The early development of algebraic thinking is crucial for a child's success in school, providing a foundation for the understanding of advanced mathematical concepts. Radford's research (2006) emphasizes that young learners can effectively unfold and express their algebraic thinking through oral communication, manipulations, and gestures. According to Devlin et al. (2023), an early understanding of the concept of equivalence is crucial for a child's algebraic thinking development. In our ongoing research, we adopted Davydov's theoretical framework (2008) to study the development of this concept in 4 to 6-year-old learners. Davydov identifies theoretical thinking as the generalization of the essential yet "invisible" characteristics of an object. To facilitate young learners' theoretical understanding of equivalence, Davydov proposes using contexts such as lengthy objects and volumes. In our project, we chose the context of weight, which, unlike length, volume, or the number of objects in a set, is not directly visible. We designed four play-like activities presented successively. The first activity introduces the concept of weight, incorporating relevant manipulations, vocabulary, and gestures. Learners engage in exploring the weights of physical objects both manually and by utilizing a balance. Subsequent activities involve free play with a balance and objects, achieving equilibrium with a set of blocks, and solving equations expressed in a non-symbolic manner. Examining a 5-year-old learner through a case study, we explore her use of gestures, manipulations, and language in solving 63 balance problems. The poster presentation emphasizes how the learner utilizes these semiotic tools to think about and communicate her understanding of equivalence in weights. We aim to sketch a developmental pathway related to the concept of equivalence and deepen our understanding of the child's cognitive processes at the beginning of algebraic and theoretical thinking development.

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USING TEACHING APPLICATIONS IN UNIVERISTY-LEVEL MATHEMATICS COURSES

Elizabeth G. Arnold

Montana State University

The practice of mathematics is a human activity, and the undergraduate preparation of mathematics teachers must attend to future secondary teachers' fluent understanding of the mathematical content they will teach alongside an understanding of how to interact with learners and their mathematical work (Álvarez et al., 2020). University-level mathematics courses provide a venue where these understandings can be simultaneously integrated into coursework. In this study, we examined the use of *teaching applications* in university-level mathematics courses to provide undergraduates an opportunity to encounter ideas about mathematics content and ideas about teaching and learning mathematics. Teaching applications are mathematical tasks that attend to the dual goals of understanding mathematical content and some of the additional complexity involved when human beings learn mathematics.

We employed a qualitative, multiple-case study methodology. Our research is situated within the context of four mathematics courses: abstract algebra, discrete mathematics, introduction to statistics, and single-variable calculus. Eleven instructors from universities across the United States implemented two lessons that incorporated numerous teaching applications. We collected and analysed approximately 200 undergraduates' written work on the teaching applications and interviewed 61 of these undergraduates to identify what kinds of ideas about teaching and learning mathematics they generated after engaging in these applications.

Our findings indicated that undergraduates (1) identified the broad applicability of teaching skills, (2) recognized the value of examining hypothetical learners' mathematical work, and (3) reported empathy for hypothetical learners within the teaching applications. These findings persisted across the four courses we studied, leading us to conclude that our findings can transfer to additional mathematics courses in secondary mathematics teacher preparation. Overall, these teaching applications enriched university-level mathematics courses, and the results of our study add to the existing literature on ways to make university mathematics courses matter for secondary teacher preparation (Wasserman et al., 2023).

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MEASURING AND PROMOTING TEACHER NOTICING FOR INCLUSIVE MATHEMATICS EDUCATION

Anton Bastian[1], Natalie Ross[1], Sarah Strauß[2], Isabelle Klee-Schramm[2], Johannes König[2], and Gabriele Kaiser[1]

[1]University of Hamburg, [2]University of Cologne

Inclusive education requires certain competencies of mathematics teachers to facilitate learning for all students. Teachers need to possess situation-specific skills for inclusive mathematics education (IME), namely teacher noticing. This construct, consisting of perception, interpretation, and decision-making, mediates between dispositions and classroom performance and facilitates students-centred instruction. As few studies measure teachers' noticing skills for IME or develop means to improve them (König et al., 2022), in the Teacher Education and Development Study - Inclusive Mathematics Education (TEDS-IME) project we aim to conceptualize, measure, and promote noticing skills for IME of pre-service and in-service secondary mathematics teachers in algebra education. In this poster, we will present our instrument and professional development (PD) program and focus on its effects on teachers' noticing skills.

To measure noticing skills for IME, we developed a video-based instrument that includes four scripted video vignettes displaying typical classroom situations and 90 rating scale and open response items that test participants' noticing skills for IME from a general pedagogical (P_PID) or mathematics pedagogical (M_PID) perspective. In addition, we created a PD for IME in algebra education together with experts in mathematics pedagogy, general pedagogy, and special education. The 18-hour PD program comprises at its core adapted and newly developed instructional materials and video-based learning opportunities, and focuses on the complementary consideration of mathematics pedagogy, general pedagogy, and special education. For example, the PD explores the scale model using connections of multiple representations to introduce equivalent transformations, and considers respective worksheet construction to support students with learning disabilities. To date, we have implemented the PD with over 450 German pre-service and in-service teachers and tested the participants' noticing skills for IME in a pretest-posttest design. Data collection is in its final stages and analysis using Rasch models and multiple regression analyses is ongoing. Analyses with a subsample from the first wave of data collection ($n = 121$) demonstrated significant increases in participants' noticing skills in M_PID ($d = .25$) and P_PID ($d = .36$) with small effect sizes, providing first insights into the effects of the PD.

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STUDYING THE RELATIONSHIP BETWEEN STUDENTS' FUNCTIONAL THINKING AND ALGEBRA PERFORMANCE

Anneli Blomqvist

Department of Teaching and Learning, Stockholm University, Sweden

Functional thinking is the kind of thinking that is applied, develops, and becomes sophisticated when students are engaged in tasks that deal with functions, relationships, and change (e.g. Pinto & Cañadas, 2021). In students' algebra learning, functional thinking plays an important role (e.g., Kaput, 2008). However, few studies provide insight into the relationship between students' functional thinking and their algebra performance. To address this literature gap, this study aims to answer the following research question: To what extent can students' performance on functional thinking tasks explain their performance on algebra tasks?

The data comes from a test consisting of a total of 121 credits distributed over 63 tasks. By analysing the tasks, it was found that five of them contained the aspects of algebra where functions, variation, and relations are involved. The five tasks can be considered tasks involving functional thinking and thus will be referred to as functional thinking tasks (FTT) in this study. The FTT had a total of 8 credits. From the analysis, it was also determined that 24 tasks involved other aspects of algebra. In this study, those 24 tasks are called algebra tasks (AT) and had a total of 24 credits. Students' (n=1623) results from the five FTT and the 24 AT were analysed in relation to each other using multiple regression analysis. This is to ascertain the extent to which the student's performance on FTT can account for the student's performance on AT.

The results indicate that four of the FTT together explain 52% of the variation in students' performance on the AT (R^2=.517, p<.001) and can explain part of students' algebra performance. Four of the FTT describe different amounts of variation in students' performance on AT, ranging from 32% (R^2=.316, p<.001) to 4% (R^2=.036, p<.001). Additional studies are required to gain more insight into the underlying reasons for the diverse effects of the four functional thinking tasks on students' performance on algebra tasks.

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LINEAR NIM-DIGITAL – PROBLEM-SOLVING DEVELOPMENT OF WINNING STRATEGIES

Birgit Brandt and Andreas Kirsche

Technische Universität Chemnitz, Germany

Playing games in mathematics lessons opens up many productive learning opportunities – the game NIM can be classified as a game that encourages the exploration of winning strategies (Odfield, 1991) and is particularly suitable for promoting problem-solving skills (Krauthausen & Etzold, 2022). NIM is an old strategy game that exists in many variants. The linear variant, in which a maximum number of tiles are placed on a game board with a fixed number of fields, can already be understood by primary school children (ibid.). Etzold (2022) has developed a digital version of this variant. In the App NIM, different settings can be selected and the integrated archive in particular relieves the working memory and is intended to support problemoriented exploration of a winning strategy (Krauthausen & Etzold, 2022).

The mathematical background for the linear variant is a partition of integers into residue classes. Depending on the variant and the number of target fields, different fields can be identified as winning fields. If the number of fields corresponds to 10 and a maximum of 2 tiles can be placed (variant for beginners), then the residue class modulo 3 are considered accordingly. Field 10 is in the remaining class 1mod3. As a winning strategy, fields 1, 4, 7 and finally 10 must therefore be occupied.

In an explorative video study on the use of the APP NIM, we want to describe 1. different problem-solving strategies and 2. the use of the archive in more detail. To this end, we have developed a cooperative, non-competitive learning environment in which the children can explore the winning strategy in tandems. In an initial observation, Onur and Willy perceived different aspects of the winning strategy while exploring the variant for beginners together. Onur recognized the rhythmic pattern of the winning strategy of always adding to 3. Willy recognized the remaining class fields as special fields ("winning fields") by analyzing the archive within the app. According to Krauthausen and Etzold (2022), the knowledge of these two aspects is necessary to generalize the winning strategy for different variants. Further classroom experiments should provide more insight into the context of archive use and strategy development.

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SUCESSIVELY, COLLECTIVELY ANALYTICAL ARGUMENTATION IN PHILOSOPHICO-MATHEMATICAL CLASS DISCUSSIONS ABOUT INFINITY

Julchen Brieger

University of Technology Chemnitz

Fetzer (2011) describes mathematical argumentation in (German) primary schools as following: the students use simple conclusions (without warrants and/or backings), their arguments mostly remain substancial, large parts of the argumentation stay implicit and the students use non-verbal aids while arguing.

In four lessons, designed for philosophical inquiry of the concept of infinity, German primary (3rd grade) and early secondary (5th grade) schoolers discussed about whether one could limit natural numbers qua set braces, how to add another guest to Hilberts Hotel and on Zeno's paradoxes. Said lessons were filmed and the resulting material transcribed and analysed by the means of interaction analysis (Krummheuer & Brandt, 2001), grounded in ethnomethodological conversation analysis, and argumentation analysis (Knipping & Reid, 2019).

Results of said analysis differ from those of Fetzer (2011). First, the students do collectively use a lot of warrants and backings in class discussion while successively extending not only their own, but also the argumentation of their class mates. Also, as the class discussion carries on, their arguments become more and more analytical instead of substancial. Furthermore, the data shows that flat hierarchies lead up to students arguing more explicitly. In the four analysed lessons, students did not have to argue non-verbally at all to express their thoughts.

This begs the question: did the philosophico-mathematical framework or the topic of infinity stimulate these more advanced argumentations? This question could not be answered directly by data analysis and will be discussed in a theoretical framework on the paper. Also, detailed insights into individual reconstructions of argumentation processes as well as examples from the data material will be provided.

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A COMPARATIVE ANALYSIS OF CHINESE AND KOREAN MATHEMATICS TEXTBOOKS FOCUSING ON THE EQUAL SIGN IN GRADE 1 AND 2

Johyeon Chang

Korea National University of Education

Understanding the relational concept of the equal sign is important for success in algebra (Knuth et al., 2006). However, many elementary students still have an inadequate understanding of the equal sign, interpreting it as requiring the answer to an arithmetic operation, rather than as a symbol of mathematical equivalence. In 2022, t양식의 맨 위here have been revisions to the educational curriculum in China and Korea, leading to the release of new mathematics textbooks. This study aims to identify practical approaches for implementing delicate tasks, focusing on analyzing how these textbooks emphasize the relational understanding of the equal sign. This study analyzes the textbooks for 1st and 2nd grades developed by the Ministry of Education in each country. Since the equal sign is introduced in the 1st and 2nd grades, analyzing textbooks from these grades provides crucial insights into fostering algebraic thinking. Three instructional elements used for analysis, based on an integrated assessment of equal-sign knowledge by Matthews et al. (2012), are as follows: 양식의 맨 위(a) activities that emphasize the meaning of the equal sign, (b) tasks that treat equations as objects of reasoning, and (c) tasks involving equations with unknowns.

Every country develops textbooks reflecting its unique culture and characteristics. This ongoing study, by examining each country's distinct approach to the equal sign, provides more effective methods for students to grasp the concept of the equal sign. Identifying components within textbooks from each country that facilitate an effective understanding of the equal sign contributes to development of teaching and learning strategies to enhance algebraic thinking. Consequently, this study offers valuable pedagogical implications for further research in textbook development.

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ENHANCING SELF-REGULATED TEACHING IN MATHEMATICS: A LESSON STUDY APPROACH

Chang-Hua Chen[1] and Chia-Hui Lin[2]

[1]National Changhua University of Education, Taiwan

[2]National Taichung University of Education, Taiwan

Promoting mathematics teachers' proficiency in guiding students' self-regulated learning in the classroom (i.e., self-regulated teaching, SRT) is an emerging issue in mathematics education. Students can benefit from their teachers' SRT and are equipped with 21st-century skills for life-long learning. However, our understanding of developing math teachers' SRT proficiency is limited. Thus, we constructed a teacher-professional learning community and conducted Lesson Study to support participating teachers' SRT. Four teachers regularly attended community meetings and discussed their lesson plans and teaching materials. When a teacher implemented the lesson plans, other peer teachers observed the lessons. They gave feedback to the teacher when the community reflected on the effectiveness of the lesson. In Lesson Study, we applied interviews, video recorders, and lesson observations to collect the data. Subsequently, Ball et al.'s (2008) and Dignath and Büttner's (2018) conceptual frameworks were used to analyze teachers' knowledge and teaching practices, respectively. Research findings suggest the tendency to use more knowledge of content and student (KCS) and specialized content knowledge (SCK) and less knowledge of content and teaching (KCT) revealed in teacher discourse. This finding implied that these teachers focus more on students' mathematics learning and gradually master SRT. The teachers gained more self-efficacy, a critical aspect of teachers' beliefs, in implementing SRT in their classrooms. The participating teachers demonstrated more SRT in the classroom, especially in explicitly teaching metacognitive strategies to their students. This finding is different from Dignath and Büttner's (2018) study. On the other hand, these teachers rarely taught motivational strategies to their students, which corresponded with the two scholars' findings. This study showed the potential of Lesson Study in supporting mathematics teachers' SRT.

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MAKING SENSE(S) OF FUNCTIONS: A DESIGN ENGAGING BLIND LEARNERS THROUGH MOVEMENT AND SOUND

Christina Krause[1], Johanna Gfrerrer[1], Michael Fischer1, and Aaron Pumm[2]

[1]University of Graz, Austria; [2]University of Vienna, Austria

In education, accessibility is often achieved by providing information in alternative formats, which traditional cognitive sciences consider equivalent to original formats. However, theories of embodied cognition suggests that the body and its interaction with the world play a fundamental role in human cognition and conceptual understanding. Special Education Embodied Design (SpEED; Tancredi et al., 2022) is an approach to designing learning environments that builds on this idea, rethinking accessibility in education. It combines two existing approaches, Embodied Design (Abrahamson, 2014)—emphasizing the importance of developing sensorimotor schemas in perceptually guided action as basis for conceptualization when designing interventions—and Universal Design for Learning (Rose & Meyer, 2002) as embracing individual differences in students' learning needs, abilities, styles, and preferences. SpEED thereby aims at providing a proactive, adaptive approach to accessibility grounding design in students' embodied resources. In the design process, the parameters *media* (materials), *modalities* (sensorimotor systems), and *semiotic modes* (systems of meaning-making) are tailored to the sensorimotor abilities and preferences of the learners, as these parameters and the reciprocal relations among them constrain possibilities for perceptually guided action.

This project aims at developing an inclusive learning environment based on the SpEED approach that enables blind and low-vision learners to physically experience functions as a unique mapping. Learners create and explore unique and non-unique mappings through movement and the semiotic modes of haptics and sound, with additional feedback given as colour. The poster will present a prototype version of a touch-based web application and accompanying interactive problems prompting perceptually guided action that grounds conceptual understanding of functions as unique mapping. Also, first results from a usability study with experts by experience will be presented.

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A STUDY OF CHILEAN EARLY CHILDHOOD EDUCATION STUDY PLANS WITH A FOCUS ON MATHEMATICS EDUCATION AND GENDER AND SES INEQUITIES

Claudia Cornejo Morales[12], Mayer Soto Sarmiento[1], and David Maximiliano Gómez[123]

1Universidad de O'Higgins 2Millennium Nucleus for the Study of the Development of Early Math Skills (MEMAT) 3Center for Advanced Research in Education (CIAE)

Many authors and organizations underscore the fundamental role of Early Childhood Education (ECE) in the development of children. Education in the early years is crucial for learning numbers, language, symbols, social skills, and emotional self-regulation (NAEYC, 2022). Teacher preparation programs for this level play a relevant role in achieving these goals. Institutions' educational proposals are made concrete through a study plan. Here, we characterize the preparation that Chilean universities offer to prospective ECE teachers by analyzing their study plans in four areas: (i) mathematics learning, didactics, or development of math skills; (ii) gender perspective and gender gaps and biases; (iii) socioeconomic gaps; (iv) human psychology and development. In the following, we focus on the results related to mathematics. We analyzed 30 study plans from undergraduate, accredited ECE programs. We coded each course (except practicums) for whether it relates to the four aforementioned areas and its location in the study plan. A quantitative analysis revealed that, relative to the total number of courses, each area had 6%, 7%, 12%, and 21% courses, respectively. This translates to numbers of courses per semester of 0.35, 0.40, 0.68, and 1.16. This shows that, across all areas, mathematics is the least represented (about one course per three semesters). Mathematics was the area that co-occurred with the others the least, and the distribution of mathematics courses throughout the study plan concentrated on the second half. A qualitative analysis of the names of all courses related to mathematics showed that they were mainly related to didactics, instruction, and assessment of mathematics (47%); general didactics, instruction, and assessment (19%); integration between different disciplines (13%); specific mathematical content strands (11%); learning environments (6%); and mathematics (3%).Our results suggest that Chilean ECE study plans would benefit from stronger and more pertinent instruction in mathematics, guided by interdisciplinary collaboration between mathematics education and ECE specialists.

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IMPROVING ENGINEERING MATHEMATICS PROBLEM-SOLVING THROUGH INTERACTIVE COMPUTER SIMULATION AND ANIMATION

Ning Fang and Seyed Mohammad Tajvidi

Utah State University, USA

Mathematics is most often involved when solving real-world engineering problems. In teaching and learning college-level engineering courses, a long-standing challenge has been how to improve and reinforce students' mathematical knowledge and skills, so mathematics does not become a significant hurdle for students to learn engineering. To overcome this challenge, various technology-based educational interventions have been developed, such as virtual reality, augmented reality, and computer simulation and animation (CSA) (Leavy et al., 2023).

In this poster presentation, we will demonstrate three interactive, web-based CSA learning modules we recently developed to improve and reinforce undergraduate students' mathematical knowledge and skills when they learn a foundational, second-year Engineering Dynamics course. This calculus-based course requires students to apply what they have learned in prerequisite math courses (Calculus I and II) to solve problems in engineering dynamics. The most important feature of our interactive CSA learning modules is that they integrate mathematical modeling and problem-solving into real-world problems in engineering dynamics.

The well-known revised Bloom's taxonomy of educational objectives is employed as the overall theoretical framework to guide this research to examine the effect of the developed CSA learning modules. Among the cognitive learning levels defined in the revised Bloom's taxonomy, we particularly focused on applying and analyzing as these two levels directly focus on mathematical modeling and problem-solving.

A total of 36 engineering undergraduates were recruited for this research. They were divided into two groups. One group used CSA learning modules to learn relevant course materials in engineering dynamics. The other group learned the same course materials using the traditional textbook style (TTS) instruction. Qualitative data were collected through thinking aloud where students talked aloud to themselves when learning. Quantitative analysis shows that for the three learning modules, the CSA group increased their mental activities over the TTS group by 127.7%, 70%, and 23%, respectively, at the applying level; and by 88.7%, 203.5%, and 531.6%, respectively, at the analyzing level. Thus, the effect of CSA depends on particular learning topics.

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WHAT’S GOOD IN MATH?

Michelle Friend, Gabriel Meints, Kirsten Tetzlaff, Betty Love and Nicole Infante

University of Nebraska at Omaha

In response to high failure rates and negative attitudes in college algebra, our university developed alternative courses to meet university quantitative literacy requirements. In one of these, students learn critical mathematical reasoning and computer programming skills as they design and create digital art. At the end of the course, students’ confidence and interest in math is significantly higher than at the beginning. As a course assignment, at the end of the course, students (n=262) write a structured reflection about their experience in the class (Drake, 2006). Prompts asked students to describe a “peak” moment in the class, a low moment, what they learned and got out of the class, things that were good about the class and things that should be changed.

In this paper, we draw on the framework of the classroom as a figured world of cultural practices (Holland et al., 1998; Nasir & Cooks, 2009). We investigate what practices and affordances of this math class students found supportive, in order to understand the impact of the course on students. Our research question is, “What aspects of this course did students appreciate?”

Reflections were analysed using affect coding to identify positive comments, then inductive and collaborative coding to identify specific aspects of class that students identified as good. Findings include supportive teaching staff, collaboration with peers, open-ended projects, and the utility of skills learned in class as elements students appreciated. The findings suggest that creativity and applicability of content are valuable, and that a supportive classroom environment has a strong impact. We will share examples that can apply to other courses.

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THE PROPOSAL OF EMERGENT HYPOTHESIS MODELLING IN STATISTICS EDUCATION

Hiroto Fukuda

Okayama University of Science

The problem statement is that the issues of what is a problem and what kind of data is necessary in statistical inquiry or statistical inference are not as sufficiently targeted for research as they are for the subsequent problem-solving situation. In our excessively complex society that is facing an indeterministic world, defining problems and determining necessary data are important processes in which students do not necessarily use data. In such a situation, it is ‘context’ that is the key word for the problem statement. Therefore, it is essential to discuss the context to be treated even in statistics education, so the objective of this paper is to develop the framework to conduct statistical inquiry that takes the context into account.

While referring to emergent modelling (Gravemeijer, 1997), backward emergent modelling (Fukuda, 2016) and so on, the author proposes the framework as a new statistical inquiry. The author shows a series of processes for the emergence of two models: the hypothesis model-for by anticipation based on the question, and the hypothesis model-of based on search. It is called ‘emergent hypothesis modelling’ because the hypothesis model concretely emerges at the first stage of the statistical inquiry process. The core of emergent hypothesis modelling is the formation of hypotheses through contextual thinking, which basically constitutes a statistical inquiry process without the use of data.

Additional information

This paper is a partial translation of the author's unpublished doctoral dissertation, with additions and corrections:

Fukuda, H. (2020). Research towards a principle for the statistics curriculum in Japanfrom the perspective of context, Doctoral Dissertation, Hiroshima University. Retrieved from https://ir.lib.hiroshima-u.ac.jp/00049358

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MONOLOGIC CENTERING ANALYSIS FOR GROUP WORK

Allison Gantt

University of Delaware

Group work is the site of crucial learning experiences for students in mathematics classrooms (e.g., Towers & Davis, 2002). Students mutually influence one another's cognitive development and create collective products during group activity (e.g., Towers & Martin, 2015), but students can also have differential learning experiences within these collectives related to status that develops as they interact (e.g., Gresalfi, 2009). In this poster, I will address the methodological challenge of concurrently applying collective and individual lenses on students' mathematics group work data with a technique I have developed called *monologic centering analysis*. I investigate the research question: What can monologic centering analysis support a researcher to observe about students' mathematical learning as they engage in group work?

Monologic centering analysis builds upon a strategy introduced by Towers and Martin (2015) to analyse collective mathematical activity by organizing transcript data from group work into a monologue format. Uniquely, monologic centering analysis 1) incorporates strategic speaker highlighting/greying to present multiple monologues with attention to different individuals' learning experiences and 2) leverages analytic memoing to synthesize conclusions about students' learning across multiple group work observations. In this way, the technique has potential to support analysis of collective and individual mathematical learning with specificity and simultaneity.

In this poster, I will explore what we can learn from monologic centering analysis through an application to transcript data from a case study focused on three U.S. algebra students (age 13) working together around writing linear equations across five lessons. Through monologic centering analysis, I found I was able to draw conclusions about patterns in students' collective activity and learning around the selection of variables (i.e., representational letters like *x* or *t*). Through the same technique, I was also able to describe variations in this learning for individual students related to power and status constructed in the students' interactions. I will discuss implications of monologic centering analysis for future inquiry related to understanding students' mathematical learning and the design of teaching that structures students' interactions.

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STUDENTS' REASONING THROUGH GRAPH CONVENTIONS

B.R. Gaspard[1], H.Y. Lee[1], M. Bui[1], H. Hardison[1] T. Paoletti[2], L. Ford, H. Zolt, and S. Tarigan

[1]Texas State University; [2]University of Delaware

The ability to read and write graphical representations is important for students to progress in STEM coursework and careers (Costa, 2020). Graphical representations commonly draw on conventions. Although students must know these conventions and use them to communicate ideas with others, research has shown that overemphasizing conventions can detract from important student reasoning that could support their graph literacy (e.g., Mamolo & Zazkis, 2012; Thompson, 1992). Collectively, these studies provide insight into complexities students can experience when they have been overexposed to graphing conventions.

Our work examines U.S. middle-grades students (10–13 years old) who have had less exposure to graphing conventions. We document how these students are capable of reconciling cognitive conflict between graphing conventions and their reasoning about quantities; we argue that such reconciliation can result in their understanding of graphing conventions as choices rather than as required rules. In this poster, we describe students' thinking about graphical representations in unconventional Cartesian coordinate systems (e.g., with positive y-values going down the vertical axis), including how these students' attention to graphing conventions, quantitative strategies, and thinking within reference frames interplayed. We discuss implications of students' graph thinking for future research and teaching regarding students' developing meanings for graph conventions.

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SURFACING FAMILY PRACTICES THAT SUPPORT FRENCH IMMERSION STUDENTS IN MATHEMATICS

Julianne Gerbrandt

University of New Brunswick

Research exploring parental involvement in mathematics education has often taken a schoolcentric approach, where parental support is synonymous with supporting school-defined educational goals and pedagogical practices (Jay et al., 2017). This feminist qualitative study involving parent and caregiver participants draws from the view of parents as intellectual resources (e.g., Civil & Bernier, 2006) and highlights family practices that support elementary French immersion (FI) students in their learning of mathematics in French. The research questions are:

1. How are caregivers supporting children's mathematics learning and schooling?
2. How do caregivers adopt the roles of learner, teacher, and leader to support their children in mathematics?
3. How does the relationship between home and school influence the roles that caregivers adopt and the ways that they support mathematics learning and schooling?

This poster draws from my doctoral work examining the tensions of learning mathematics in FI, where both the discipline and the program have been associated with gatekeeping in public education in Canada. Research methods include individual in-depth interviews and journaling with caregivers of elementary FI students learning mathematics in French. This presentation will share preliminary findings from phase two of the study, which began in spring 2024 and involved collection of the first interviews and journal entries. The interweaving of critical discourse analysis and thematic analysis is expected to show that parental involvement in mathematics education extends beyond monitoring homework and attending parent–teacher conferences to also include expressions of parental leadership and curiosity. Findings are also expected to surface tensions between valued and devalued mathematics practices that occur outside of schools. This re-imagining of parental involvement contributes an alternative narrative about learning mathematics in FI and offers a strengths-based orientation toward the relationship between home and school.

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A GRAPH-THEORETIC ANALYSIS OF CALCULUS TEXTBOOK TASKS

Haile Gilroy

Auburn University, Auburn, AL, USA

Despite calls for reforming Calculus instruction to stress conceptual understanding over procedural fluency (e.g. Dreyfus & Eisenberg, 1990), procedural fluency in calculating derivatives remains a heavily emphasised skill in my institution's instantiation of Calculus I. Because textbooks mediate instruction in university mathematics courses (Mesa & Griffiths, 2012), I aim to understand students' exposure to derivative computation skills in Calculus I by conducting a novel type of graph-theoretic analysis of textbook tasks. In particular, I answer the question, What is the structure of students' exposure to derivative computation (DC) skills among textbook tasks? To conduct such an analysis, I developed a novel representation of an elementary function as a directed tree, which I call an Elementary Function Tree, or EFT. The theory of Schemas (Corral et. al., 2020) and Symbolic Forms (Sherin, 2001) informed the design choices of such graphs. The corpus comprised three general Calculus texts, chosen to represent a variety of publishers and publication decades, and for their popularity in my country. To mimic students' realistic exposure to DC skills in a course, only the odd-numbered DC tasks were analysed. After drawing each task's EFT, first, the frequency of each type of elementary function (vertex label) relative to each textbook was calculated. Second, each textbook's EFTs were superimposed into a single edge-weighted directed graph, indicating the frequency of each edge in each textbook. Using standard box plot outlier calculations, it was possible to determine the edges in each textbook with significantly large frequencies. Third, using the outlier edge weight data, the intersection of the textbooks' outlier sets was determined. All three stages of the analysis showed that Calculus textbook tasks emphasise the power rule significantly more than all other DC skills. In my institutional context, this is misaligned with the assumption that students should be equally fluent in all DC skills, which could be problematic for student success. Future work will explore this idea further.

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EXAMINING NAMES IN DIFFERENT LANGUAGES THROUGH MATH

Simon Byeonguk Han, Amanda Sugimoto

Portland State University

This poster will present emerging findings from a case study of how a group of Mathematics Teacher Educators (MTEs) in the U.S. recognized the hegemonic role of English, particularly in the U.S., when we developed a social and/or political mathematics task. Students' names can reflect their identity, language, family, community, and culture (Aldrin, 2016). A group of 9 MTEs with diverse linguistic, cultural, and experiential backgrounds in the U.S. developed a "Name Task" such that students 1) collected, organized, and represented data and 2) explored their own names (serves as a mirror) and connections between culture, language, and names (serves as a window; Bishop, 1990). After students explored their own and peers' names, we planned to explore the length of the 20 most popular names in Korea in 2019, by providing students with a list of Korean names in English-translated versions (e.g. Seo-jun for 서준). However, one MTE pointed out that using English-translated versions can reinforce the hegemonic role of English (Macedo et al, 2015), which is opposed to the social and/or political goal of the task. Therefore, we decided to use Korean names written in Hangul (the Korean written language). However, all 20 names have the same length when written in Hangul because of the unique cultural and linguistic aspects of Korean names. So, we modified the task by providing a list of basic phonemes and asked students to count the phonemes in each name (e.g. Seo is written as 서, and has 2 phonemes, ㅅ and ㅓ). Throughout this process, MTEs learned that language and power may surface in mathematics tasks in ways that reinforce the hegemony of English. The poster will visually show how the data representations differ from each other when Korean names are written in English and Hangul and counted by phonemes.

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CREATING SPACE FOR DATA, ART, AND STORIES: STUDENT-CREATED DATA VISUALIZATIONS FOR COMMUNITY LEARNING

Lynn Liao Hodge and Yilang Zhao

The University of Tennessee

Mathematizing, Visualizing, and Power (MVP) is a three-year project funded by the National Science Foundation that focuses on developing and testing a model for community-centered data exploration catalyzed by youth. The project develops data artistry among young people in East Tennessee communities and provides opportunities for these youth to share their data visualizations with their communities to foster collective reflection and discussion. The creative work generated by the MVP project is completing in two ways, both as statistical art and as powerful statement giving voice to the experience of communities. Critical aspects of the MVP model include (1) youth learning sessions that position youth as owners of data and producers of knowledge and (2) Community Learning Events that support community learning as youth learning occurs.

The MVP project design includes engaging youth with meaningful problems, building a discourse community with possibilities for action, re-positioning youth as knowledge producers within their own communities, leveraging linguistic and cultural resources of the youth participants and their communities, and implementing critical events that support substantial interaction between youth, community members, and the data visualizations. The foundational ideas of MVP are aligned with recent scholars who have argued that good designers of visual graphics for quantitative relationships must be students of mathematics, design, and storytelling (D'Ignazio & Klein, 2020).

The project began Fall of 2022, and since that time the project team has implemented two of three planned cycles of design-based research. Both cycles involved participants between the ages of 11-18 as part of an after-school community center focused on supporting youth programming. The poster presentation will include a discussion of insights gained from the second cycle of design-based research with a focus on student-created visualizations and other artifacts will be shared. Anticipated implications relate to an understanding of community topics that are of interest to students and discussions generated by student-created data visualizations.

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COGNITIVE CONFLICT AS INSTRUCTIONAL STRATEGY FOR TEACHING LOGICAL PRINCIPLES

Alexander Holvoet

KU Leuven

In learning proof, pupils continuously make logical errors when interpreting given theorems. One proposed solution in the literature is integrating logic with proof education (Durand-Guerrier et al., 2012). Though some instructional strategies for integrating logic exist, this study aimed to apply conceptual change theory to teaching logical principles.

The involved ‘concepts’ are pupils’ deductive reasoning strategies. According to the theory of ‘pragmatic reasoning schemes’ (Cheng & Holyoak, 1985), these strategies are “clusters of rules that are highly generalized and abstracted but nonetheless defined with respect to classes of goals and types of relationships” (Stylianides & Stylianides, 2008). In conceptual change theory, instruction is typically organised in three stages: the induction of cognitive conflict through contradictory data, guidance of pupils’ change and cooperative and shared learning to promote collective discussion of ideas (Limón, 2001).

In this theoretical contribution I claim conceptual change theory provides a promising instructional strategy for logical principles in proof education. Three of my arguments are: empirical research shows pupils’ pragmatic reasoning schemes are changeable (Cheng & Holyoak, 1985), and pupils are receptive to pointed-out contradictions in logical arguments (Stylianides & Stylianides, 2008). Thirdly, logical rules were historically constructed to delineate valid from invalid arguments within contradictory data, paralleling conceptual change theory. To support the argument, I have assembled such contradictory sets of arguments as starting point for all logical connectives.

This contribution expands available instructional strategies in proof education and reopens debate in two ways: on the role of logic in proof instruction, and on the use of mathematical proof contexts for deductive reasoning training.

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EFFECTS OF UNIVERSALLY DESIGNED SERIOUS GAMES STUDENTS' FRACTION KNOWLEDGE

Jessica H. Hunt

North Carolina State University

This experimental study investigated the impact of a fraction game created with an equitable designed framework on students' fraction knowledge. Results revealed significantly improved fraction knowledge for students who played the game on two measures. Games that promote multiple means of access, expression, and representation can positively impact student outcomes. Serious games are shown to improve cognitive function and serve as platforms for students to build and internalize knowledge (Tokac et al., 2019). Summaries of gaming research have identified the potential of games to enhance STEM content accessibility and increase collaborative problem-solving and exploration. Despite the positive results, systemic issues of access, such as the inability to have access to and agency over tools or representations students use to learn or express knowledge, is prevalent in many digital mathematical games. In this paper, we report on the impact of a digital fraction game on fourth graders' fraction knowledge. Universal Design for Learning (UDL), a design framework that calls for multiple means of access, representation, and expression, grounded the game's creation. Eighty-six students in the western United States took place in the study. Students were matched in pairs and then randomly assigned to a study or a control group. Fraction knowledge was measured with a curriculum-based measure and a measure of students' fraction schemes (Wilkins et al., 2013). A 2-way multivariate analysis of variance (MANOVA) measured differences between the study and control groups on the two dependant variables over three time points. Game analytics evaluated whether game play predicted changes on the dependant variables.

Results revealed significant differences on the CBM and fraction schemes in favor of the study group. Furthermore, gameplay time significantly predicted improvements in fraction knowledge. Results will be further unpacked in session.

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(MIS)ALIGNMENT OF TEACHER- AND STUDENT- FACING TEXTS IN A GEOMETRY UNIT ACROSS CURRICULA

Soobin Jeon

University of Michigan

Mathematics curricula serve as the primary tool for teachers and students. While existing research often concentrates on the individual roles of the teacher or student, there is a lack of attention to the interactions between the teacher and student when using mathematical texts (Rezat, 2011) and the meanings construed in texts addressed to the teacher and students. Drawing on systemic functional linguistics, we examine what meaning potentials can be associated with the exchanges of knowledge proposed in mathematical text. This study illustrates a way to compare the language written for the teacher and the student in curricula as we consider the following:

1. To what extent do the teacher- and student-facing text draw on processes with similar meanings?
2. What patterns can we observe in the processes used within and across the texts?

Halliday and Matthiessen's (2004) Transitivity system provides a spectrum of options for constructing ideational meanings, expressing experiences construed through processes: *Mental*, *Material*, *Relational*, *Existential*, *Operational*, or *Verbal*. *Mental* (sensing) processes may be realized with verb choices such as consider or think, while *Material* (doing) processes with choices such as draw or project.

Examining a Geometry unit in Illustrative Mathematics, the study reveals distinct patterns between the teacher- and student-facing texts in the learning goals: More use of the *Mental* processes in the teacher-facing and more use of the *Material* and *Verbal* processes in the student-facing goals. In contrast, an analysis of similar geometry units in four more traditional mathematics curricula, originally developed in the US but widely used globally, demonstrated greater alignment, suggesting a less problem-oriented instructional environment. Misalignment of linguistic processes is not necessarily problematic but could be indicative of a characteristic of problem-based instruction. We believe misalignment of processes might suggest a greater need for teacher to operationalize on the instructional exchanges.

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DEVELOPMENT OF MULTIPLICATION LESSONS AIMING AT THE PROGRESS OF PROPORTIONAL REASONING

Hisae Kato [1], Hiraku Ichikawa [2], and Keiko Hino [3]

[1] Hyogo University of Teacher Education, [2] Miyagi University of Education, [3] Utsunomiya University

In this study, we design and practice lessons and discuss the results, while using the concept of learning trajectories to progress proportional reasoning in the lower grades of elementary school. In the Japanese curriculum, multiplication of whole numbers is introduced in the second grade. The meaning of multiplication is treated not only as a simplified form of addition but also as a representation of "one part," "several parts," and "times." In the third grade, children deepen their understanding of the meaning of multiplication through the multiplication of two- or three-digit numbers, the method of calculation using algorithms in column form, and the properties of multiplication (commutative and associative law) (MEXT, 2018). However, the reality is that although many children have sufficient memorization of the multiplication table up to 9 times 9 and calculation skills, they tend to lack understanding of the meaning of “times” and understanding of multiplication with decimals and fractions in the upper grades. To address these issues, we believe that it is important to develop the idea of unitizing (Lamon, 2020). Therefore, in this study, from the perspective of promoting proportional reasoning (especially unitizing), we designed lessons aimed at understanding the meaning of multiplication in the second and third grades of elementary school and analyzed the results. We designed lessons from the following three viewpoints: an introduction to multiplication that emphasizes functional relationships in the second grade (Practice 1), a learning strategy incorporating children's words in learning multiplication in the second grade (Practice 2), and a concrete manipulative activity linking multiplication and division in the third grade (Practice 3). How these practices relate to the framework of learning trajectories is discussed from the perspective of unitizing. These practices were conducted in separate classrooms. Therefore, conducting a longitudinal study is a task for the future.

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EXPLORING THE ROLE OF DATA EXPLORATIONS IN MATHS LEARNING

Jasneet Kaur

University of Auckland, New Zealand

This study explores the use of data visual explorations in teaching and learning of mathematics, addressing the challenge of engaging secondary students during the pandemic. Employing action research and design thinking, the study aims to foster curiosity and meaningful connections with mathematics. The study was conducted on around 60 girl students of grade VIII and IX of a Government school located in the semi-urban area in India. These students, mostly from low-income families, were first-generation learners facing diverse socio-economic challenges.

Students were presented with a data visual depicting handwashing technique from a 1978 study, prompting them to respond to questions regarding their observations and curiosities. The objective of data visual explorations in mathematics education is multifaceted. Research indicates that the exponential growth of data underscores the importance of equipping students with data analysis skills (Messy Data Coalition, 2020). Moreover, studies suggest that engaging students in mathematics with real-world problems fosters deep understanding and connections between concepts (Boaler & Staples, 2008). Additionally, cognitive research highlights the significance of visual processing in mathematics learning, as it activates various brain networks, including visual pathways (Boaler et al., 2016). Data was thematically analysed.

The study revealed that the data visual exploration prompted excitement and curiosity among students, leading to spontaneous mathematical connections. Students questioned the visual representation, debated its authenticity, and sought to validate it through data collection. The exploration sparked informal math discussions, with students estimating and evaluating the area using various mathematical strategies. It underscored the value of connecting real-world data to mathematical concepts, promoting empirical validation and ultimately enhancing students' awareness of hand hygiene practices.

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PRIMARY SCHOOL STUDENTS' UNDERSTANDING OF TIME MEASUREMENT - COMPONENTS OF A CONCEPT OF TIME

Lukas Knorr, Jessica Hoth, and Constanze Schadl

Goethe University Frankfurt & Friedrich Schiller University Jena (Germany)

Individuals are surrounded by temporal structures from birth, but conscious engagement with these structures does not occur in infancy, such as with sleeping and waking phases. As children grow older, time becomes essential for structuring human life. Children's first conscious engagement often begins when they enter school. Time is an abstract variable. It cannot be conveyed through material action, but only through linguistic description or by reference to representatives from the environment. Therefore, directly or indirectly comparing measures of time is challenging, unlike length measurement (Sarama & Clements, 2009). Time has both cyclical and linear aspects. These characteristics make it challenging to define concrete components that must be present for a comprehensive concept of time measurement.

In extensive theoretical analyses, seven components were identified as relevant for the concept of measurement. These components are (1) Benchmark knowledge, (2) measurement skills, (3) quantitative and (4) qualitative estimation skills, and (5) comparison skills (e.g., NCTM, 2000). Additionally, a concept of length shows in students' skills to convert measures (6) and to deal with measures in word problems (7). To empirically investigate the components of the concept of measurement for time, a paper-pencil test was developed. For each of the seven components, we constructed 3-6 items. In the German mathematics education system, time measurement is addressed in the second and following grades. In order to assess the levels of students' concepts, we focus on third grade students as a sample. In this contribution, we present the conceptualization of the test.

The constructed test will be used to assess third-grade students' concept of time to analyze the structure of this specific construct. The resulting understanding will enable better integration and promotion in future curricula, strengthening the development of the concept of measurement.

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PCK RELATED TO FUNCTIONS – A COMPARISON BETWEEN TEACHERS FROM GERMANY AND SLOVAKIA

Sabine Kowalk and Ute Sproesser

Ludwigsburg University of Education

Diagnosing students' task-related strategies and errors is a component of teachers' pedagogical content knowledge (PCK) (e.g., Ball et al., 2008). Concerning linear functions, student errors are frequently caused by difficulties with particular procedures used for representational changes (Hadjidemetriou & Williams, 2001). However, such procedures often are country-specific. Thus, considering PCK in a country-specific way requires looking at differences and similarities between typically used procedures and subsequently expected common errors (Hubeňáková et al., 2022). This study focuses on investigating such country-specific facets of teacher's PCK. Thus, it aims to identify which country-specific procedures for particular representational changes of elementary functions German and Slovak teachers implement in their classrooms and which student errors they correspondingly expect.

The sample comprises 105 in-service mathematics teachers (49 German, 56 Slovak). Data was collected by a paper-pencil test with five open-ended items referring to different representational changes of elementary functions. A codebook indicating procedures and errors typically encountered in Germany and Slovakia was developed in this study and used to code the data.

First findings referring to changes between graph and equation of linear functions indicate that nationally favored procedures are also reflected in teachers` expectations of typical student errors. For example, German teachers most frequently expect difficulties with reading and drawing the slope, whereas most Slovak teachers assume difficulties with reading off or working with coordinates. Hence, this study emphasizes that PCK should be considered against cultural and country-specific particularities and that corresponding comparisons must be interpreted cautiously. More details about this study will be given in the poster presentation.

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DIFFERENCES IN STUDENT PROOFS ACROSS MEDIA

Vasiliki Laina

Johannes Kepler University Linz

Proving is an important, yet hard-to-teach and hard-to-learn practice. One challenge is that the epistemic forms commonly used to mediate proving activity (e.g. paragraph, two-column, or flow-chart proofs) obscure essential characteristics of the explorative proving process, such as its non-linearity, and thus may influence the structure of proofs learners produce. In this proposal, I ask: *how might changing proof media in classroom activities impact the argumentative structure of student-generated proofs?*

Informed by socio-cultural theories that conceptualize learning as participation in form-mediated practices (Saxe, 2015), I posited that a learning environment that (i) did not rely on a linear structure during proof construction, and (ii) employed multiple media, could help diversify students' experiences with and access to proving. To test this hypothesis, I designed and taught a 6-week online summer class on geometric thinking for secondary students (13-15 years old). I video recorded all lessons and collected chat data and student work. To address the research question, I focus on two lessons in which students worked in small groups on a single problem. They engaged in proving activities that were supported by different media: GeoGebra for conjecture generation, Jamboard for proof exploration and construction, and Google Slides for proof writing and presentation. Using Toulmin's (1958) basic argumentative model, I compared the structure of proofs students constructed in Jamboard, Google Slides, and orally presented to peers.

I found that even though the conjecture-to-be-proved for each student group was the same throughout the different proving activities, there were differences in the proofs students authored across media and modalities, with a subtle shift towards supporting their peers' comprehension more as they moved from proof exploration and construction to proof communication. Changes in proof content and structure included differences in the choices students made about what arguments to include, how to organize those arguments, and how to justify them.

Findings suggest that epistemic forms in proving activity can impact structural characteristics of student proofs. As a result, a learning environment that consciously and carefully incorporates different epistemic forms in proving activities can support richer epistemic experiences for students.

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INVESTIGATING THE FACILITATION OF SELF-REGULATED LEARNING BY MATHEMATICS TEACHERS: A PERSPECTIVE ON TEACHER AGENCY

Chia-Hui Lin[1] and Chang-Hua Chen[2]

National Taichung University of Education, Taiwan[1]
National Changhua University of Education, Taiwan[2]

Teacher agency, within the context of educational reform, refers to the capacity of educators to proactively make choices and enact measures to fulfill pedagogical objectives. This capability is a pivotal component for effective instruction and professional development of teachers and has emerged as a focal point of scholarly inquiry in teacher education in recent years (Imants & Van der Wal, 2020). Since implementing a new curriculum reform in Taiwan in 2018, there has been a pronounced advocacy for the significance of self-regulated learning (SRT) in students' educational experiences in mathematics. Nonetheless, realizing students' SRT necessitates teachers' guidance, recognition, and commitment to reform efforts, particularly among those with extensive teaching tenure. Therefore, we aimed to explore the comprehension and implementation of the principles of SRT by a secondary school mathematics teacher with over a decade of instructional experience. This study employed a qualitative research methodology and gathered data through various approaches, including in-depth interviews, document analysis, meeting participation, and observations. It utilized the framework proposed by Imants and Van der Wal (2020) on the model of teacher agency in school reform and professional development for analysis. The findings revealed that the case teacher's agency, in addition to being influenced by personal beliefs, values, learning strategies, and mathematical expertise, is also embedded within the ecological context of the school, organizational classroom dynamics, and the relationships within the teacher community, contributing to a cyclical process of professional growth. The outcomes of this research identify the elements constituting the agency of mathematics teachers, aiming to deepen the educators' understanding in guiding student SRT and make tangible contributions to the advocacy of SRT and teacher agency.

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IS MIDDLE SCHOOL MATH ENOUGH FOR OCCUPATIONS? DATA FROM PRACTITIONERS' RATINGS

Su-Wei Lin[1], Chao-Jung Wu[2], Yuan-Horng Lin[3], and Hong-Wen, Chang[2]

[1]National University of Tainan, [2]National Taiwan Normal University, [3]National Taichung University of Education

"Why do we study mathematics?" Almost all math teachers have met this challenge. The level of mathematics that people suppose is required for various occupations may affect students' beliefs and motivation for learning mathematics. This study collected ratings from working adults on the mathematics level required for various occupations. Twenty-five (15 female) practitioners from various occupations and different cities in Taiwan were invited. Their ages ranged from 25 to 60 years (M = 46, SD = 10), and their occupations included agriculture, construction, education, etc. Their educational attainment was 2 high school, 11 junior college, 5 university, and 7 graduated.

Participants were first presented with a Mathematics Level List. Each level contained 4 to 8 items of mathematics content corresponding to the Taiwan curriculum standards: Level 1_1st~2nd grade, Level 2_3rd~4th grade, Level 3_5th~6th grade, Level 4_7th~9th grade (middle school), Level 5_10th~12th grade (common course of high school), Level 6_11th~12th grade (advanced course of high school) and university mathematics. For example, a Level 4 item is "To present and calculate data (e.g., average family income) using descriptive statistics like mean, median, percentile rank, etc."; and a Level 5 item is "Solve problems using permutations and combinations (e.g., how many kinds of passwords can be made for a 3-digit number lock)". Participants were then asked to rate the required level of mathematics for each of the 127 occupations. If the participant felt hesitant to choose an integer from 1 to 6, he/she could choose 1.5, 2.5, ...5.5. The occupational title followed the Taiwan Standard Classification of Occupations, which consists of 10 major, 39 sub-major, 125 minor, and 380 unit groups. We chose the minor groups as the targets and adjusted them to 127 items with reference to the internal standard (ISCO-08). To help participants understand, each occupation had a short description and some examples.

The results of the ratings were available at https://pse.is/5n8pvd.The average rating of participants was not high (M = 2.85, SD = 0.55), and the number of occupations rated ≤ Level 4 ranged from 79 to 127 (Med = 111). The ratings of female participants (M = 3.10, SD = 0.48) were significantly higher than those of male participants (M = 2.47, SD = 0.41), $t(23) = 3.39$, $p = .003$, but there was no difference among the four educational attainments. Only one occupation, which was [Mathematicians, Actuaries, and Statisticians], was rated > 5 (M = 5.58, SD = 0.84), while 15 occupations were rated between levels 4 and 5. Although as many as 111 (87%) occupations were considered to require only middle school-level mathematics, those higher-rated occupations were of high occupational prestige. This showed that participants assumed that higher-level math was required to pursue highly prestigious occupations.

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LINKING VIRTUAL AND PHYSICAL MANIPULATIVES BY DESIGNING LEARNING ENVIRONMENTS

Jan Löffert and Jessica Hoth

Goethe University Frankfurt (Germany)

Manipulatives certainly play an essential role in learning for primary school students, but their provision alone does not in itself create meaning for them or even help them to develop mathematical concepts (Sarama & Clements, 2016). It stands to reason that not only physical, but also virtual tools can help to develop such an idea if the used learning environments are well planned and give students the opportunity to reflect on their own (mental) actions. (Sarama & Clements, 2016). Furthermore, we see a unique opportunity in the direct connection of physical and digital tools (Soury-Lavergne, 2016). As part of the project *Andi* (linking *an*alog and *di*gital tools in elementary school) learning environments are to be designed for four grades, each containing four virtual tools and their physical counterparts. The purpose is to show the individual potential of each medium in the classroom. In the second grade, learning environments are planned for understanding place value using the example of bundling and unbundling. The selected digital place value tables offer the advantage that when an object is changed to a different place value, only the representation changes due to automatic bundling and unbundling, but not the value of the number (Kortenkamp & Ladel, 2013). The design research focuses on two questions: Firstly, what is the learning effectiveness of the learning environments for the students and secondly, how manageable is the implementation of them on the teacher side?

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STUDENTS' UNDERSTANDING OF THE "GENERAL VALIDITY" OF MATHEMATICAL STATEMENTS AND PROOFS

Sarah Lundt, Milena Damrau, and Stefan Ufer

University of Munich (LMU)

Many students encounter difficulties in understanding a mathematical proof and its purpose to verify universal statements (Fischbein & Kedem, 1982). These proofs are general in the sense that variables detach the argument from individual numerical cases and establish the statements' general validity. Students can assess this generality in different ways: relying on one or more examples (empirical), on content-related explanations, or on the role of the proof (structural). Few studies have investigated the relation between students' understanding of the generality of proofs, the generality of statements, and the role of variables as part of proof comprehension (Mejía Ramos et al., 2012). We asked how students evaluate the generality of statements and proofs, and which role variables play in establishing a connection between these two.

Seven eighth-grade students read one of two number-theoretical statements and corresponding proofs. We applied qualitative content analysis, drawing on responses to questions on the generality of the proofs and statements, the existence of counterexamples, proof comprehension questions, and notes of students' utterances. Only four students answered consistently whether the statement is valid and whether no counterexamples exist. In contrast, one student stated that the statement was valid but also named a concrete (supposed) counterexample. All examined students substantiated their evaluation of the statement's validity through empirical or content-related reasoning. Structural arguments were not applied, even if the proof was identified as valid. Most students understood that variables can be replaced by specific values, but none of them used this to argue for the generality of the proof. Two students believed in the validity of the justification but expressed uncertainty regarding the validity of the statement, not recognizing their intrinsic connection.

Our results illustrate how students' challenges go beyond understanding the generality of proofs (Fischbein & Kedem, 1982), extending to the generality of statements, the relation between these two, and the role of variables in this context. We propose to investigate the effects of brief interventions and scaffolding measures to support students' understanding of generality when engaging with proofs.

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EMERGING PRINCIPLES OF DIGITAL TASK DESIGN TO SUPPORT STUDENTS' DEVELOPING GRAPHING MEANINGS

Claudine Margolis[1], Teo Paoletti, and Allison Olshefke[3]

[1]University of Michigan, [2]University of Delaware & [3]University of Delaware

Despite the importance of graph fluency, students experience challenges with graph construction and interpretation. Digital tasks can support students' development of productive meanings for graphs (e.g., Paoletti et al., 2023). Margolis and Boyce (in press) describe *visual feedback* as visual information provided automatically by the digital environment in response to students' actions. We describe an emerging digital task design principle: Design visual feedback to represent intended ways of thinking about graphs described by Paoletti et al.'s (2023) framework. Paoletti et al. (2023) argue that to understand graphs students need repeated opportunities construct quantities from a situation (i.e., identify measurable attributes in a situation), consider a varying segment length as representing a quantity's magnitude, and bridge those meanings to coordinate the length of a magnitude bar in the graph with the amount of a quantity from the situation. We address the research question: *How does Paoletti et al.'s (2023) framework inform the iterative design of digital tasks intended to support students' developing graphing meanings?* Using data collected from multiple teaching experiments (Steffe & Thompson, 2000), we describe 1) how we use the framework to determine productive ways of thinking about graphs, and 2) how we redesigned tasks to incorporate visual feedback to represent intended ways of thinking. We present examples in which we revised tasks to better support students' activity in intended ways. For example, we incorporated visual feedback allowing students to imitate direct measurement of zoo animals' weight (in pounds) and habitat temperature (in degrees F) within a digital task to support students in conceiving measurable attributes that vary across animals. We provide implications for teachers and researchers engaged in designing digital graphing tasks.

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PRESERVICE MATHEMATICS TEACHERS AS TEACHER-RESEARCHERS

Leah Nillas

Educational Studies, Illinois Wesleyan University, USA

This longitudinal research study represents a mathematics teacher educator's own engagement in a systematic inquiry of teaching practices detailing the challenges and successes of embedding teacher research in preservice teacher education. At a four-year undergraduate liberal arts university, preservice teachers were prepared to design, implement, and write publishable teacher research about their own classroom practices, evidence of students' learning, trends in curriculum and pedagogy, and socio-cultural factors impacting the teaching and learning process. A self-study framework is applied to design this situated inquiry, which is characterized by engaged reflection, programmatic collaboration, and systematic evaluation of teaching practices (Samaras, 2011). This practitioner-research is presented visually to illustrate a paradigm that centers on enhancing instructional practices through an intentional inquiry to improve how K-12 students learn mathematics and how preservice teachers teach mathematics. Throughout this teacher education program committed to developing teachers for social justice, preservice teachers both served as consumers and producers of research. Multiple sources of program and course related qualitative data from more than a decade of implementation were analyzed to describe how preservice teachers consumed research as they learned to question conventional ways of teaching and learning K-12 mathematics. Preservice teachers engaged in critically examining their own practices during student teaching with the aim of improving their practice and conceptually scrutinizing taken for granted assumptions about teaching and learning (Brandenburg, 2009). When engaging in a systematic self-study research, preservice teachers synthesized a variety of research literature to connect research and practice. They systematically analyzed evidence of student learning through multiple forms of classroom data, engaged in collaborative reflections, and addressed challenges in being self-critical and managing tensions between theory and practice. Finally, preservice teachers engaged in analytic writing to produce practice-based classroom research narratives that were relevant to teachers and researchers around the world. Their research experiences challenge conventional ways of conducting classroom research.

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BIBLIOMETRIC ANALYSIS OF RESEARCH ON THE USAGE OF AUGMENTED REALITY (AR) IN MATHEMATICS EDUCATION

O'Brien, D. and Riding, K.

Loughborough University, University College London Institute of Education

AR is an increasingly popular form of visual technology that overlays virtual, digital images onto the physical world as viewed through a device's camera screen. Indeed, this blended environment may become the "real world" of tomorrow. As the technology gains traction, research on AR usage in classroom settings has surged. Concurrently, more researchers are using bibliometric technology to track research trends and make informed decisions about future research foci. Bringing together these technological advancements, this pioneering study aims to uncover central themes and research questions within the under-researched field of AR in mathematics education and identify future research directions recommended by the relevant literature.

We employed a novel, mixed methods' approach to bibliometric research in line with Drivers et al. (2020); a framework which exploits human-machine interplay by triangulating qualitative expert insights with quantitative bibliometric techniques. The first "human-centric" phase was to conduct a content analysis of 13 landmark papers, co-identify and co-interpret key themes, and narrow the focus accordingly. The second "machine-centric" phase used the derived search terms to source relevant bibliometric data from Scopus, Web of Science, IEEE Explore, ERIC and ACM Digital Library – the initial dataset of 2,135 papers was "cleaned" of duplicates and items that fell outside of mathematics education. The whittled 275 papers were then imported as a CSV file into VOSViewer text mapping software for co-word analysis. The resulting text maps were analysed for highly connected and frequent nodal words, which were then compared against themes obtained from the first phase.

We found that the word "effectiveness" was central to all word clusters and frequently connected to "visualization", "interest", "interaction", "group" and "game". However, of these, only "effectiveness" and "visualization" were consistent between both phases – yielding a potential finding that the latter is generally identified as a key cognitive measure of the former and a strong indicator of focus for future as well as current research. This was validated by the researchers' understanding of the related theoretical literature. Given that the second phase contrasts with the first by including only indexed literature, we also conclude that employing such human-machine triangulation may provide a more comprehensive picture of the key themes being addressed as well as highlighting research gaps – particularly within the indexed corpus.

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EFFECTIVE TEACHING IN MATHEMATICS CLASSROOM: FROM THE LOW-ACHIEVERS' PERSPECTIVES

Yun-Hsia Pai

National Tsing Hua University

Most of research findings related to effective teaching in mathematics provide evidences which come from the teachers' perspectives, while from the students' perspectives, there are relatively few studies on effective teaching (e.g. Seah, 2010). Some research reported high, moderate and low academic performance pupils' view for effective teaching (e.g. Tan & Lim, 2010), but there are even still fewer studies focusing on the low-achieving students' perspectives.

This study aims to investigate the perspectives on effective teaching of the low-achievers' in fifth grade. 10 low-achieving fifth-grade students from 2 classes (5 from each class) and their mathematics teachers participated in this study. Low-achieving students are asked to capture the moment of effective teaching, taking pictures immediately by iPad when they thought a good mathematics teaching moment occurred. In addition, Video recording and interview with teachers and students were used to collect data. During the interviews, each photo will be shown to the student who took the photo, and the student will be asked to describe his or her thoughts on the events that occurred at that moment, and then discourse analysis method were used to analyse the transcribed interview data.

The findings of this study showed as follows. What low-achieving students value most is teaching methods and strategies, followed by teaching materials, class management and group discussion and presentation. In the aspect of teaching methods and strategies, they prefer to listening to teachers' lecture, followed by "listening to classmates' opinions" and "visual representations". In the aspect of teaching materials, math task worksheets and interesting or daily-life math problems are their favourites. With regard to classroom management, students pay attention to whether their teacher has established a good class management system to correct classmates' behaviors. In term of group discussion and presentation, they prefer to seek help from classmates or act as assistants rather than express themselves indiviually. However, teaching methods of the two mathematics teachers are slightly different, so students' views on these four aspects are also somewhat different, more details will be presented in the conference.

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ELEMENTARY STUDENTS' UNDERSTANDING OF VARIABLES THROUGH A UNIT ON PATTERNS AND CORRESPONDENCE

Jeong Suk Pang and Ji-Eun Lee

Korea National University of Education

Variables are an important concept for generalizing and symbolizing mathematical ideas. However, it is difficult for students to understand variables because they have different meanings, including an unknown, a generalized number, and a varying quantity in covariational relationships (Blanton et al., 2011). This is especially true for Korean students, who tend to have difficulty representing the indeterminate quantities as symbol variables. Considering this, we reorganized the patterns and correspondence unit for Grade 5 students, who first learn to represent the correspondence relationship between two covarying quantities using symbol variables. Our research questions were as follows: (a) how can the different meanings of variables be taught in the patterns and correspondence unit? and (b) how does students' understanding of variables improve?

Four Grade 5 classes (a total of 83 students) in an elementary school were selected. Two classes (i.e., the intervention group) were taught with our reconstructed materials, and the other two classes (i.e., the non-intervention group) were taught with the regular textbook. Six 40-minute lessons were reorganized, and two types of written assessments were developed to measure students' understanding of the unit and the variables. The intervention lasted for two weeks. All lessons were videotaped, and student performance was compared between the two groups using pre- and post-tests.

The analysis of the lessons showed that representing quantities in a table helped students differentiate between independent and dependent variables. Comparing different meanings of variables within the same problem context was also effective in clarifying their role and meaning. As a result, there was a noticeable increase in understanding the meanings of variables from pre- to post-test. There was a significant difference in understanding variables between the two groups ($t = 5.21$, $p < .001$). Specifically, the intervention group was able to better understand and express the variables for each meaning. However, there was no significant improvement in manipulating variables in equations. The poster will include an instructional sequence of the unit for the intervention group, key activities and representative classroom episodes, and the performance of students in the intervention group compared to their counterparts on sample items of the assessments.

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TASK DESIGN PRINCIPLES TO SUPPORT GRAPH REASONING

Teo Paoletti[1], Allison L. Gantt[1], Srujana Acharya[1], and Claudine Margolis[2]

[1]University of Delaware, [2]University of Michigan

Covariational reasoning, or conceptualizing two quantities changing together, is critical for students' graph interpretation (Thompson & Carlson, 2017). Prior research has argued students should connect their covariational reasoning across situations and graphs and consider the amounts of change of one quantity with respect to a second quantity to describe relationships (e.g., Carlson et al., 2002). In this poster, we address the research question: *How can we support primary school students (age 10-12) to reason covariationally to develop meanings for graphs as representing quantities?* Addressing this RQ, we describe two task design principles: 1. Embed recursive opportunities for students to bridge their reasoning across situations and graphs; 2. Start with relationships that increase by smaller amounts or "grow by less".

We developed our task design principles through four small-group teaching experiments (Steffe & Thompson, 2000) focused on supporting students' graphical reasoning and built models of students' mathematics using conceptual analysis to understand their activity. We present a case study from one teaching experiment where two 11- to 12-year-old students engaged with a particular task to highlight the efficacy of these task design principles. We highlight how the students moved back and forth between their situational and graphical meanings to build connections toward stronger graphical reasoning (Principle 1). Further, we highlight how a "growing by less" relationship presented an opportunity for the students to actively reflect on the relationships between total amount of a quantity and amounts of change of that quantity as they created their graph (Principle 2). We found these principles led to productive shifts in each student's activity and in how they reflected on their own thinking. The poster concludes with implications for task design, teaching, and future research.

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A CASE STUDY OF MGTA GROWTH IN SUPPORTING GROUP WORK

Kelsey Quaisley and Mary Beisiegel

Oregon State University

During their graduate programs, mathematics graduate teaching assistants (MGTAs) teach numerous classes for undergraduate learners and/or impact learners in group learning sessions, yet their teaching practices remain unexamined, especially in terms of growth (Miller et al., 2018; Speer et al., 2010). We implemented a two-year professional development (PD) program to support MGTAs in enacting engaging group work. Our research question is: What growth in supporting engaging group work might an MGTA experience during a two-year PD program?

We explore the growth of one participant, Ayla (pseudonym), in incorporating group work practices. We chose to report on Ayla's growth because of their choice to repeatedly reflect on a salient struggle with engaging students in group work, as well as perseverance in their efforts to work on their struggle. We investigated Ayla's growth through the lens of Theory of Planned Behavior (Azjen, 1991), which focuses on *behavioral intention*, *attitude toward the behavior*, *subjective norms*, and *perceived behavioral control*. In the context of the PD program, *behaviors* refer to the teaching practices that MGTAs use in the classroom and *intention* is an MGTA's plan to use certain teaching practices. While *intention* directly impacts resulting *behavior*, an MGTA's perception of their ability to employ teaching practices influences their pedagogical choices (i.e., *their perceived behavioral control*). We analyzed 5 classroom observations and post-observation debriefs, 2 interviews, and 8 reflections and artifacts from Ayla in the PD program.

We found that Ayla's attitude of discomfort towards group work shifted, as Ayla believed in its benefits for students, and persevered in their intention to enact and ultimate enactment of engaging group work strategies. Perceived behavioral control appeared to influence Ayla to grow in enacting group work strategies in subsequent courses, which has implications on the potential benefits of long-term PD.

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TWO-COLOURED TOWERS AND BEYOND: NAVIGATING AND DEVELOPING PRE-SERVICE TEACHERS' COMBINATORIC AND PROBABILISTIC THINKING

Gale Russell[1] and Egan Chernoff[2]

[1]University of Regina, [2]University of Saskatchewan

This ongoing research study serves to acknowledge and undertake actions toward addressing Lockwood's (2013) concern that "Combinatorial topics have become increasingly prevalent in K-12 and undergraduate curricula, yet research on combinatorics education indicates that students face difficulties when solving counting problems" (p. 251). Further, in order to address these difficulties, this study explores "understanding the nature of pupil's mistakes when solving combinatorial problems and identifying the variables that might influence this difficulty" (Batanero, Navarro-Pelayo, & Godino, 1997). Where this current research diverges from the work of these other scholars is its focus on pre-service teachers (PST), as a subset of undergraduate students in general, and as the future teachers of the K- 12 students.

As the broader research study is one that brings together action research, self-study, and case studies within the theoretical framing of grounded theory, there are many complementary and divergent research questions related to different mathematics content. The broad overarching research question that best serves to define the particular focus for this proposed poster presentation is: "What combinatoric and probabilistic understanding and thinking do PST have, and how can it be strengthened?" This study, which formally began in the fall of 2015, has had 20 to 30 PST participants per academic year, most of whom were actively involved in the research study for utmost two terms, depending upon their program of study.

The study began with a single task: "How many different two-colour towers that are three cubes high can be built," (modified from Maher and Alhuwali, 2014); and, over time tasks have been added to continue the exploration and strengthening of the PST combinatoric and probabilistic thinking. This evolution of tasks, and the results so far, will be documented through the poster format.

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MATHEMATICAL REASONING IN A MIDDLE-SCHOOL MOBILE SECURITY INTERVENTION

Suzanna Schmeelk[1], Carolyn A. Maher[2], Rasha Abadir[2], Victoria Krupnik[2], and Louise Wilkinson[3]

1 – St. John's University, Queens, New York
2 –Rutgers- The State University of New Jersey, New Brunswick, NJ
3 – Syracuse University, Syracuse, New York

A foundation for understanding cybersecurity risks requires both mathematical and computational reasoning. Little research exists on whether knowledge of basic mathematical concepts, such as counting, combinatorics and probability provide a necessary foundation for building computational and mathematical reasoning. This poster session will describe results of a pilot study with middle-school aged students who were challenged to recognize risks and mitigations.

The poster will present data from the Mobile Security research session in which the students were challenged to recognize mobile scams, high/low risk applications, and build strong passwords. Analyses of the data followed the Powell, et el (2003) analytical model. Findings indicate that key mobile device security concerns require mathematical reasoning skills that can be built by studying mathematical topics in counting, combinatorics, and basic probability (Schmeelk et al, 2024). Earlier studies also suggest that middle-school aged students can successfully learn these concepts in a student-centered, collaborative learning environment (Maher, et al, 2010; Shay, 2008). Addressing mathematical knowledge needed as a foundation for understanding cyber-risks will be reviewed to protect individuals' financial and personal data.

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CULTIVATING STEM AFFINITY THROUGH INFORMAL MATHEMATICS-BASED CODING AND ROBOTICS ACTIVITIES

Mina Sedaghatjou, Uriah Mcclain, and Anton Puvirajah

Rowan University, Rowan University, University Western Ontario

This study delves into the critical role of mathematics as a foundational pillar in fostering STEM (Science, Technology, Engineering, and Mathematics) affinity among underserved adolescents in an informal setting, where the students collaboratively engage in solving problems through coding and robotic activities.

Methods: This mixed-method study involved twelve refugee adolescents aged between 10-13 years who participated in a two-week summer camp focused on coding and robotics. To evaluate the impact of this learning experience, we utilized both qualitative and quantitative methods. Qualitative data were gathered via video recordings, field notes, and focus group interviews.

Results: Using the *Collaborative Engagement (CE)* framework articulated by Sedaghatjou and Rodney (2018), where the intertwining of Affective, Behavioural, and Cognitive dimensions were identified. The *Behavioural Engagement* was found in adolescents who formed affinity groups to actively solve the robotics challenges through hands-on experiences, in which they bridged their theoretical knowledge with practical execution. The *Cognitive Engagement* was identified while learners immersed themselves in abstract conceptualization, thereby enriching group discussions and problem-solving. The *Affective Engagement* was found through adolescents' *enjoyment* emphasizing emotional and attitudinal connections through their reflective observations and joint solution creation to the given tasks. We suggest that these domains play a pivotal role in sustaining interest and affinity in STEM. To quantify this, a paired samples t-test comparing STEM affinity scores before ($M = 51.75$, $SD = 5.50$) and after ($M = 61.00$, $SD = 6.98$) camp participation. We found a statistically significant increase in the STEM affinity scores, $t(11) = 8.60$, $p < .001$.

Conclusion: Using a Freirian lens (1970), we found informal coding, and robotic activities can foster conscientization through critical thinking and personal agency, empowering underserved adolescents to navigate and challenge their past interests, thereby nurturing a genuine affinity for STEM as a pathway to liberation.

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MATHS EDUCATION LABS - ECHOES ON TEACHER TRAINING

Américo Silva[1], Isabel Cabrita[1], Isabel Vale[2,3], and Ana Barbosa[2,3]

[1]CIDTFF – Research Centre on Didactics and Technology in the Education of Trainers, University of Aveiro – Portugal; [2]CIEC, Universidade do Minho– Portugal; [3]Instituto Politécnico de Viana do Castelo – Portugal

Mathematics plays a fundamental role in the sustainability of humanity, but achieving this goal poses serious challenges to the educational process and, consequently, to the training of (future) teachers. Maths Education Labs (MELs) could constitute privileged spaces for structured mathematical experiences, contributing to the development of a solid mathematical competence, essential to address the challenges of the 21st century (Kaushik Das, 2020). In Portugal, there are some of these structures in higher education institutions (HEIs) linked to teacher training, but they are rarely studied.

In this context, a research project was conceived with the main purpose of: i) mapping the existing laboratories; ii) understanding the underlying logic of their creation and how they have evolved; iii) reflecting on representations of (future) teachers about the MELs and iv) evaluating the impact of these structures on the training and teaching practices of (future) teachers. To achieve this, a predominantly qualitative study was chosen, involving MELs administrators, teachers who participated in laboratory-promoted activities and students. Data were collected through inquiry – interviews and questionnaires – and through documents.

Preliminary results allow us to identify four MELs located in four of the 24 mentioned HEIs. These were conceived by teachers/researchers with investigative interests and converging educational perspectives who collaborated on projects of various natures. Conceptually, these units are situated at the confluence of education, research, and community outreach, aiming to foster dialogues among these dimensions to promote a transformative mathematical culture. However, it is observed that the way most students perceive these spaces is overly simplistic and that the impact on their education and teaching practices does not withstand the more directive and 'traditional' experiences they have encountered throughout their academic lives, which, in many instances, still dominate. In this context, it is urgent to continue "*(re)think mathematics education together*", "bearing in mind a transformative mathematics education.

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MANAGING FIGURE AND GROUND IN MATHEMATICS DISCOURSE

Susan Staats, Claire Halpert, and Alyssa Kasahara

University of Minnesota, U.S.A.

"Instead of looking at things, look between things." This comment, widely attributed to artist John Baldessari, calls us to attend to sensemaking experiences that connect figure and ground, positive and negative space, focus and context. Mason describes this in terms of the "structure of attention" (Mason, 2021, p. 11). Roth represents the concepts' dialectical relationship with the vertical bar glyph, as "figure | ground" (2012, p. 19). Students manage figure and ground relationships in many mathematical activities, for example, visualizing relationships within mathematical figures, navigating notation in procedural work, and in explaining conjectures to each other. However, the potential of analysing figure and ground relationships in mathematical sensemaking has not been fully realized.

This poster offers a five-dimensional multimodal discourse framework with examples for analysing figure and ground mathematical sensemaking. Removal refers to physically displacing a mathematical object, so that the previous context or ground becomes the new figure. Mimicking removal refers to a gesture that represents physical removal, e.g., briefly covering a mathematical object with a finger or hand. Information structure is a broad area of linguistics research that tracks changing figure | ground distinctions amongst emerging new topics, relevance, or foci (Hilpert, 2021). Sentence placeholders invoke figure and ground through comments like "two times what plus something," that describe a mathematical genre that students consider relevant to a task. Finally, poetic structures adjust figure and ground through the interaction of repeated and changing phrases in mathematical comments (Staats, 2021). This multimodal discourse framework grows from appreciation | critique of the linguistic channel, which though powerfully expressive, can only specify ideas and cannot make them disappear. The five tools will increase analytical attention to students' dynamic organization of figure and ground in mathematical sensemaking.

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EXPLORING THE EFFECTIVENESS OF TRANSFORMING MATHEMATICS-GROUNDING ACTIVITIES INTO DIGITAL EXPERIMENT ACTIVITIES

Tai-Yih Tso[1], Shu-Hao Hsu[1], and Feng-Lin Lu[2]

[1]National Taiwan Normal University, Taiwan
[2]National Academy for Educational Research, Taiwan

Education researchers developed a series of mathematics-grounding activities to address the significant disparities in mathematical achievement and the phenomenon of high performance paired with low motivation among Taiwanese students (Yang, Lin, & Tso, 2022). These activities aim to cultivate students' mathematical thinking through hands-on engagement and tasks gradually increasing complexity. However, time and space constraints, particularly during the COVID-19 pandemic, limited their suitability for independent study. In response, we transformed these mathematics-grounding activities into digital mathematics experiment activities, incorporating elements of mathematical experiments and microworlds to enable independent student exploration. An example involving the circumcenter concept at the junior high level showed that digital experiment activities significantly enhanced students' learning outcomes, motivation, and confidence. After the experimental intervention, qualitative research with six students of diverse preparedness levels indicated that most could develop a dynamic understanding of the circumcenter concept, focusing on the circumcircle. This included grasping the properties of equidistance to the circumcenter and employing perpendicular bisectors to locate it, demonstrating that digital mathematics experiment activities encourage active thinking and the development of mathematical reasoning. However, only one student independently constructed a complete and correct proof for higher cognitive skills, such as reasoning and proofing. While understanding that the three perpendicular bisectors intersect at a single point, the others made logical errors in their proofs. This suggests a need for integrating additional activities or learning scaffolds into the digital mathematics experiment activities to assist students in mastering higher-level reasoning tasks, underscoring the importance of further research to refine and expand these approaches.

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ENHANCING ALGEBRA LEARNING IN TAIWANESE JUNIOR HIGH STUDENTS THROUGH A DIGITAL MICROWORLD

Tai-Yih Tso[1], Feng-Lin Lu[2], and Shu-Hao Hsu[1]

[1]National Taiwan Normal University, Taiwan
[2]National Academy for Educational Research, Taiwan

This study proposes an innovative solution to address Taiwanese junior high students' challenges with algebra, which is perceived as more abstract and complex than geometry due to its symbolic representation of numbers, unknowns, variables, and mathematical relations in everyday life (Duval, 2006). We developed a digital algebraic microworld learning environment leveraging digital technology to allow students to manipulate mathematical objects directly, enhancing their understanding and construction of algebraic concepts and procedures. This learning environment features interactive tools and activities designed for students to explore algebraic concepts, such as the cross-multiplication method, thereby facilitating the development of algebraic thinking and problem-solving skills across different representations within and beyond the mathematical world. This approach offers a common cognitive starting point for all students, irrespective of their mathematical achievement levels, focusing on enhancing their algebraic reasoning and problem-solving capabilities. Our study randomly assigned 137 eighth-grade students to different learning groups based on their mathematical achievement levels, measured through school grades and standardized test scores. The results showed that the tiered instructional design and interactive operations significantly improved student performance in the algebraic microworld, especially regarding correct response rates. This instructional design allowed students to gradually master complex algebraic concepts while interactive operations deepened their understanding and application of these concepts. Interestingly, even after accounting for students' prior knowledge, we observed minimal differences in the correct response rates to transfer problems across different achievement groups. This indicates that while students may start from the same point, their learning trajectories and outcomes may differ due to variations in prior knowledge. Therefore, we suggest incorporating additional support strategies and learning scaffolds into the algebraic microworld for students with lower achievement levels to more effectively engage in higher-level argumentation and reasoning activities.

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ENGINEERING DESIGN: A CYCLE TO SOLVE AUTHENTIC PROBLEMS AND DEVELOP COLECTIVE CREATIVITY

Isabel Vale[1,2], Ana Barbosa[1,2], and Isabel Cabrita[3]

[1]Instituto Politécnico de Viana do Castelo – Portugal; [2]CIEC, Universidade do Minho– Portugal; [3]CIDTFF – Research Centre on Didactics and Technology in the Education of Trainers, University of Aveiro – Portugal

Tasks play a fundamental role in students mathematical learning, being the starting point for developing different skills, such as creativity and problem-solving. And, if they challenge students to solve them, they lead to an understanding of concepts and stimulate fluency, flexibility and originality as components of creative thinking (Leikin, 2009). Also, the importance of STEAM education in preparing students to deal with the challenges of society is now an international recommendation. One possible way is through the Engineering Design (ED) process (English & King, 2015) which starts from authentic problem solving, enables an articulated mobilization of STEAM areas, in which tasks and collaborative work play a crucial role.

This article reports part of a study with 45 elementary preservice teachers (for children 3-12 years), during a learning experience in the context of a didactical course, which aims to analyze the potentialities, performance and difficulties underlying the use of ED in solving challenging authentic hands-on problems, as well as to understand whether the tasks used promote the students' collective creativity. We adopted an interpretative methodology and data was collected through participant observation, documents, artefacts and photos.

Preliminary results allow us to identify: positive reaction of the participants to this experience solving real-world problems, working collaboratively; that all participants solved the problem, with persistence and motivation in the collective creation of a prototype according to the request; that the ED cycle is similar to Polya´s model but more adequate in this situation; some difficulties to recognize particular concepts underlying the artefacts' construction, with representation from Math and Science; that the ED cycle proved to be useful in solving the used problems, but with some of its stages being merged; that productions allow us to identify some dimensions of creativity, where originality stands out.

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SATISFACTION AND SATISFACTORINESS IN MATHEMATICS FOR SECOND YEAR UNIVERSITY STUDENTS

Jacopo Vicini

Università del Piemonte Orientale, Italy

This study explores the determinants of students' wellbeing at university, focusing on the students' experiences with the university environment and their successful adaptation to the requests of the courses. Two concepts help understanding students' wellbeing: satisfaction and satisfactoriness. Satisfaction is the contentment occasioned by one's activity. It is a subjective personal feeling and it is fundamental to keep the student willing to work for passing the exams (Göksoy, 2017). Satisfactoriness is the achievement of an acceptable level of performance by the student (Lofquist & Dawis, 1969). Monitoring students' satisfaction and satisfactoriness is particularly interesting in the case of STEM topics, like for instance Mathematics. There are several studies (e.g. Andrà, Magnano, Brunetto & Tassone, 2022) showing how the attitude towards mathematics, before and after the start of the University courses, changes in a significant way.

The participants were 3 groups of students (15-20 students per group), randomly selected from 3 different classes of undergraduate courses in Environmental Sciences, Civil Engineering and Psychology. They were all enrolled in the second year of their university studies. Second year university students are interesting because they can be considered "halfway": they have ended the first year, which in terms of motivation and adaptation has requested some effort, but they are not yet in the third and last year, when the (hopefully successful) end of studies seems to be reachable and feasible. Even distribution of gender had been considered in selecting the sample. All the participants were interviewed after one of their mathematics lectures.

It emerges that the students enrolled in Engineering and Environmental Sciences tend to focus on satisfaction, i.e. on personal interest and on marks at the exams, whilst those enrolled in Psychology tend to connect their performances with the social environment. Moreover, female students show higher ex-post payoffs.

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TRENDS AND PREDICTORS OF MATH ANXIETY IN CANADA: THE ROLE OF LEARNING STRATEGIES

Haoyi Wang

University of Pennsylvania

Math anxiety, characterized by feelings of tension and apprehension that interferes with math problem-solving in various contexts (Richardson & Suinn, 1972), can reduce working memory resources and disrupt cognitive processing (e.g., Ashcraft, 2002). The 2012 PISA report revealed an upward trend in math anxiety among Canadian students compared to their counterparts in 2003. However, it remains an open question whether this trend has persisted over the past decade. This study aims to investigate this trend and identify specific learning strategies associated with varying levels of math anxiety. The research questions are: (1) Has the math anxiety level among Canadian 15-year-olds changed over the past decade? (2) What patterns emerge in the relationship between different learning strategies and these students' math anxiety? (3) How does the frequency of each learning strategy predict these students' math anxiety?

To address these questions, this study analysed secondary data from the PISA assessments of Canadian 15-year-olds conducted in 2012 and 2022. The dependent variable was the math anxiety index ($ANXMAT$). The independent variables measured the frequency of different learning strategies in the following areas: *Elaboration*, *Collaborative Learning*, *Self-regulated Learning*, *Time Management*, and *Self-control*.

The findings showed a significant increase in student math anxiety levels, rising from 0.01 standard deviations above the OECD student population mean in 2012 to 0.16 in 2022 ($p < .001$). Additionally, all tested learning strategies exhibited a significant inverse correlation with math anxiety ($p < .001$), suggesting a potential association between these strategies and lower levels of math anxiety. Thirdly, a cross-validated, weighted generalized additive mixed model demonstrated that all tested learning strategies, except for *Time Management*, significantly predict student math anxiety ($p < .001$). Notably, *Collaborative Learning* emerged as the most predictive factor for lower math anxiety levels ($b = -0.118, SE = 0.009, t(15290) = -12.859$).

Accordingly, there is an urgent need for effective practices to address this prevalence of math anxiety. *Collaborative Learning* can be promising for predicting and potentially reducing this anxiety. The next step involves exploring how instructional factors interact with learning strategies to develop comprehensive interventions.

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CREATION AND VALIDATION OF THE ALGEBRA CONCEPT INVENTORY IN THE TERTIARY CONTEXT

Claire Wladis[1], Kathleen Offenholley[1], Benjamin Sencindiver[2], Nils Myszkowski[3], and Geillan D. Aly[1]

[1]City University of New York; [2]University of Texas, San Antonio; [3]Pace University

In college, taking algebra can prevent degree completion. One reason for this is that algebra courses in college tend to focus on procedures disconnected from meaning-making (e.g., Goldrick-Rab, 2007). It is critical to connect procedural fluency with conceptual understanding (Kilpatrick, et al., 2001). Several instruments test algebraic proficiency, however, none were designed to test a large body of algebraic conceptions and concepts. We address this gap by developing the *Algebra Concept Inventory* (*ACI*), to test college students' conceptual understanding in algebra.

A total of 402 items were developed and tested in eight waves from spring 2019 to fall 2022, administered to 18,234 students enrolled in non-arithmetic based mathematics classes at a large urban community college in the US. Data collection followed a common-item random groups equating design. Retrospective think-aloud interviews were conducted with 135 students to assess construct validity of the items.

2PL IRT models were run on all waves; 63.4% of items (253) have at least moderate, and roughly one-third have high or very high discrimination. In all waves, peak instrument values have excellent reliability ($R \geq 0.9$). Convergent validity was explored through the relationship between scores on the ACI and mathematics course level. Students in "mid"-level courses scored on average 0.35 SD higher than those in "low"-level courses; students in "high"-level courses scored on average 0.35 SD higher than those in "mid"-level courses, providing strong evidence of convergent validity. There was no consistent evidence of differential item functioning (DIF) related to examinee characteristics: race/ethnicity, gender, and English-language-learner status.

Results suggest that algebraic conceptual understanding, conceptualized by the ACI, is measurable. The final ACI is likely to differentiate between students of various mathematical levels, without conflating characteristics such as race, gender, etc.

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EXPLORING STUDENTS' GRAPHING MEANINGS USING EYE-TRACKING TECHNOLOGY

Erin Wood, Sohei Yasuda, and Kevin C. Moore

University of Georgia

Students' abilities to reason about graphs as quantities varying in tandem (i.e., covariation) are critical for their success in STEM fields (Karagöz Akar et al., 2022). Yet, predominant instructional approaches do not sufficiently target such reasoning. This leads to persistent student difficulties in STEM courses and leaves much to be understood about how to support student reasoning and success. Further complicating the matter, a majority of the research on students' graphing meanings has relied on traditional methodologies that are time and energy intensive. In order to contribute to not only our knowledge on how to support students' construction of productive graphing meanings, but also field methodologies, we pursue the research questions of:

- In what ways can eye-tracking technology be used to complement current methodologies for investigating and supporting students' graphing meanings?
- In what ways is attentional focus related to the students' graphing meanings?

The advancement of eye-tracking technology opens new possibilities for mathematics education research, with researchers illustrating its promise for investigating student cognition including in the area of graphing (e.g., Thomaneck et al., 2022). Our research responds to the above questions by building on this work along with extant generalizations of students' meanings for graphs (e.g., Karagöz Akar et al., 2022).

We pair eye-tracking technology with traditional methodologies (e.g., semi-structured clinical interviews) to provide insights into undergraduate students' graphing meanings, including the extent their meanings entail productively reasoning about covarying quantities. Our results provide insights into the ways in which students' graphing meanings correspond to their attentional focus. For example, covariational reasoning involves coordinating the foci of a displayed graph and its projection onto axes. Alternative meanings for graphs involve privileging the displayed graph irrespective of axes. Our initial results also suggest that eye-tracking technology can generate artifacts for use in instruction. Such artifacts enable educators a tool to draw students' attention to specific graphing elements for the purpose of engendering covariational reasoning. In our poster, we discuss our methodological design and affordances, our results to date, and how we are pushing this initial work forward.

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THE CONVERGENCE OF FORMAL AND INFORMAL MATHEMATICS LEARNING: USING PLANETARY SCIENCE TO BRIDGE THE GAP AND ENGAGE WITH DIVERSE COMMUNITIES

Woolard, T., Novak, M. A., and Heldmann, J.

Morgan State University, Johns Hopkins Applied Physics Laboratory, NASA Ames

Convergence education describes the use of complex topics where learners apply knowledge and skills using a blended approach across multiple disciplines (Interagency Working Group on Convergence, 2022). This study explores the transition of convergence education from theory to practice using hands-on activities to investigate planetary science mission tasks from NASA's RESOURCE project to engage students of color in informal math learning. Primary and secondary math teachers were selected to pilot and share their perceptions of a protocol and framework created to facilitate the design of exemplar convergent lessons.

Teachers participated in a facilitated professional learning community (PLC) to create a series of activities that can be implemented in either a formal or informal learning environment that emphasized the practice of math skills within real-life applications. Data was collected through interviews and observations of PLC collaboration sessions as teachers utilized the framework and protocol to organize and center their planning and resources to seamlessly integrate several disciplines within an activity. The 4E convergence framework helped teachers brainstorm how to: engage students with a connection, expose them to a complex issue, experience the science, and employ the math skill in a relevant and appropriate way. Once brainstorming was complete, teachers engaged in a think tank protocol, which included: identification of a mandated math skill/standard, selection of a relevant engineering design/activity, connection of the activity to a planetary mission task, identification of a parallel student-centered application, and collaboration for peer feedback.

Teachers shared that the think tank protocol for teacher professional learning and the 4E convergence framework for lesson development significantly assisted them in their work of growth and collaboration. This study suggests the use of both the think tank protocol and 4E convergence framework can be tools to aid math teachers in the development of highly engaging lessons and activities while still adhering to the emphasis of math standards required by curricular guides.

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TAILORING STRATEGIES TO ENHANCE READING COMPREHENSION AND ACHIEVEMENT IN GEOMETRY

Chao-Jung Wu

National Taiwan Normal University

Reading text or worked examples is one of the ways students learn mathematics. Understanding geometry texts must integrate words, symbols, and figures. *Reading out* symbols (e.g., ∠APC) in text maintains them in working memory and allows *text-to-figure correspondence* to identify the geometric object on the figure. Based on embodied cognition, *tracing* a geometric object with a finger could lead attention to perceive the object in a more spatially oriented way. *Ask yourself* what the article's main idea is before finishing reading, as this is more likely to promote a mental model. The four strategies were translated into tips and a 15-minute lesson, accompanied by 11 additional 12-minute exercises of geometric reading and problem-solving.

The experimental program was implemented in five ninth-grade classes (n = 133) from two schools in Taiwan, with six other classes using the original instruction as a control group (n = 164). Geometric reading comprehension and geometric achievement were conducted to evaluate the effectiveness of the strategies. Regarding the PISA's 2018 Framework of Reading Literacy, the *geometric reading comprehension* test measured locating information, understanding, as well as evaluating and reflecting after reading four geometric texts. *The geometry achievement* test was similar to the school exam, including conceptual understanding, calculation, and problem-solving. All experimental and control group students were given a *prior knowledge* (PK) measurement before the program and completed the two tests at the end of the program.

To investigate the strategy effects on students of different abilities, participants were divided into high- and low-ability groups based on the median scores on the PK. Because the control group was marginally better than the experimental group in terms of PK (p = .075), we used the PK score as the covariate in two 2-way ANCOVAs that tested the effect of group, ability, and group x ability on geometric reading comprehension and achievement (all ANCOVA assumptions were met). Results revealed the comprehension of the experiment group was better than the control group, $F(1, 263) = 4.92$, $p = .027$, $\eta^2 = .018$, indicating the strategies promote geometry reading comprehension for both abilities. In addition, the interaction effect on the achievement was significant, $F(1, 281) = 6.92$, $p = .009$, $\eta^2 = .024$. After a simple main effects analysis, the low-ability experimental group learned better than the control group (p = .030), while the high-ability group had no group effect. The results suggested that tailoring strategies focusing on geometric multimodal reading processes not only enhanced the geometric reading comprehension of both high- and low-ability students but also promoted the geometric achievement of low-ability students.

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UNIVERSITY OF NEW ZEALAND

THE UNIVERSITY OF AUCKLAND
Te Whare Wānanga o Tāmaki Makaurau
NEW ZEALAND

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